On the structure of general course of electromagnetism

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Abstract

The structure of general course of electricity and magnetism as the relativistically invariant theory of electromagnetic field grounded on the experimental basis has been discussed. The principal problem is to give proof to the set of Maxwell's equations and show the invariance of these equations under Lorentz transformations. Our long practice of teaching confirms expediency of the course, made according to the represented scheme.

Key words: electricity, magnetism, Maxwell equations, invariance, Lorentz transformation.

1. Introduction

In modern general courses of electricity and magnetism relativistic notions are widely used. The interpretation of magnetic interaction as a relativistic phenomenon takes the central place. In most cases the realization of this idea is limited by applying the relativity principle to Coulomb's law for some examples, natural for general and elementary physics courses. Such approach gives the regularities for uniformly moving charges (for stationary fields) while for arbitrarily moving charges the problem is not actually considered. Therefore, erroneous view arises that in electrodynamics everything is obtained by Coulomb's law and Lorentz transformations, as it is mentioned in Feynman's lectures [1].

On the contemporary level of education and science general course of electricity and magnetism (mainly for future physicists) could be constructed as a relativistically invariant fundamental theory of electromagnetic field grounded on the experimental basis. Of course the corresponding didactic elaboration is necessary to observe the principle of accessibility. The main problem of such treatment is to ground the set of Maxwell's equations bonded with the invariance of these equations under Lorentz transformations.

As a rule, this invariance is not discussed in the courses of general physics since we can not consider the Maxwell's equations in covariant form (this question is a subject of theoretical physics), and to confirm their relativistic invariance in three-dimensional space is quite cumbersome mathematically as compared to the four-dimensional space mathematical formalism. However the argumentation of Maxwell's equations invariance under Lorentz transformations in three-dimensional space has independent physical and didactic importance because this invariance connects Maxwell's equations to each other. The students who had studied theoretical electrodynamics were given exercises and questions to use the invariance of Maxwell's equations under Lorentz transformations in three-dimensional space (e.g., "What can be said about the magnetic field source taking into account the invariance of Gauss's law under Lorentz transformations?"). The considerable number of students could not solve such tasks though they had studied this invariance in four-dimensional space with due mathematical formalism. This fact manifests the "stenographic nature" of mathematical language: studying first the invariance of Maxwell's equations under Lorentz transformations by four-dimensional space formalism, many students can not use and comprehend it in simple cases. So it is more preferable from both physical and didactic point of view to study these fundamental properties first in the course of general physics on the simple mathematical level and then in the course of theoretical physics. Paper [2] and discussions about it [3-7] indicate that this is not trivial.

We tried to work out the methods, which make it easier to prove mathematically the invariance of Maxwell's equations under Lorentz transformations and suit the course of general physics.

2. Structure of physics and basic principles

The main disadvantage of traditional general courses is that they follow the way of historical development of electromagnetism and the set of Maxwell's equations is formed only at the end of the courses. Thus, this set does not serve as the means of investigation. As a result, the future specialist can not comprehend the electromagnetic phenomena on the basis of fundamental equations within general physics frame (we would have the same discrepancy if Newton's laws of motion had bean studied at the end of mechanics course). But putting forward the Maxwell's equations in the modern general courses does not mean that they should be chosen as basic statements. If every part of physics is based on its "own" laws (e.g., mechanics – on Newton's laws, electromagnetism – on Maxwell's equations and so on), it will be impossible to see physics as a whole reflecting the unity of Nature – Nature "does not know" that we have divided it into mechanics, electricity etc. The unity of physical picture of the universe is one of the main methodological principles for us to construct general and elementary physics courses. This unity is manifested in basis of physics therefore the most general principles and laws must be taken as main statements. But how can we choose them?

It is necessary to analyse the structure of physics. We are guided by Wigner's conception [8]. According to it our knowledge of Nature is hierarchical. The first step of this hierarchy is formed by phenomena of Nature that serve as "raw material" for the second step – for the laws of Nature. The laws of Nature establish correlation between phenomena and it becomes possible to predict the unknown phenomena by means of the known ones. The third and the highest step is formed by principles of invariance (symmetry) for which the laws of nature make "raw material". There is a profound analogy between the relation of laws and phenomena on the one side and the principles of invariance and laws relation on the other side. The principles of invariance correlate the laws: we can determine the unknown laws on basis of the known ones. This determines the universal significance of the principles of invariance.

According to this conception we choose the basic statements. From the third step we take the principle of relativity (of course, we need the principle of inertia and properties of time-space symmetry for its formulation – all this is studied in the course of mechanics). From the second step – Lorentz force by which we can define characteristics of electromagnetic interaction; the law of charge conservation; Gauss's law as a generalization of Coulomb's law and superposition principle for the arbitrarily moving charges (for any electric field); nonexistence of magnetic charges. Of course in the general physics course these statements should be studded as the generalization of experimental results.

By mean of basic statements we give proof to the complete set of Maxwell's equations and its invariance under Lorentz transformations which will be compact in that case if we study electromagnetic phenomena separately first in vacuum and then in medium (their parallel studying is one of the main disadvantages of traditional courses, it contradicts the didactic principle step by step transition from simple to difficult: electromagnetic properties of substance are so complex to understand that is preferable to study ectromagnetic phenomena first in vacuum and only afterwards in medium). Of course mathematical level of general physics course must be within the frame of elementary three-dimensional vector analysis. We have developed the appropriate course based on traditions of the didactics of physics Georgian school, founded by M. Mirianashvili [9].

3. Maxwell's equations invariance under Lorentz transformations

It is convenient to write out appropriate Lorentz transformation formulas together. Different variants to deduce them are given in the modern courses of general physics. We are pointing some of them out hoping that the interested reader can easily choose the suitable variant of himself [10-12]. We suppose that axes of moving and rest reference frames are parallel and primed reference frame moves with velocity v along the *x*-axis positive direction.

The transformation formulas for electric $\boldsymbol{E}$ and magnetic $\boldsymbol{B}$ fields are:

$$\begin{aligned} &\boldsymbol{E}_{\parallel} = \boldsymbol{E}'_{\parallel}, \quad \boldsymbol{E}_{\perp} = \gamma\left(\boldsymbol{E}'_{\perp} - \boldsymbol{v}\times\boldsymbol{B}'\right), \\ &\boldsymbol{B}_{\parallel} = \boldsymbol{B}'_{\parallel}, \quad \boldsymbol{B}_{\perp} = \gamma\left(\boldsymbol{B}'_{\perp} + \frac{1}{c^2}\boldsymbol{v}\times\boldsymbol{E}'\right), \end{aligned} \tag{1}$$

where $\parallel$ and $\perp$ indices denote parallel and perpendicular components of fields relative to velocity respectively and $\gamma \equiv \left(1 - v^2/c^2\right)^{-1/2}$.

The transformation formulas for current $\boldsymbol{j}$ and charge ρ densities are:

$$\begin{aligned} &j_x = \gamma\left(j'_x + v\rho'\right), \\ &j_y = j'_y, \ j_z = j'_z, \\ &\rho = \gamma\left(\rho' + \frac{v}{c^2} j'_x\right). \end{aligned} \tag{2}$$

We also need Lorentz transformation formulas for partial derivatives with respect to coordinates and time. As a rule they are not considered in the general physics courses so let us show how to derive them. According to the known Lorentz transformation for coordinates and time we can write $x = x(x', t')$. According to the rules of derivation we have $\partial/\partial x = (\partial/\partial x')(\partial x'/\partial x) + (\partial/\partial t')(\partial t'/\partial x)$. If $(\partial x'/\partial x)_t$ and $(\partial t'/\partial x)_t$ are determined from Lorentz transformations, we will easily find out the transformation rules for partial derivatives with respect to coordinates and time (analogically). The corresponding formulas are:

$$\begin{aligned} &\frac{\partial}{\partial x} = \gamma\left(\frac{\partial}{\partial x'} - \frac{v}{c^2}\frac{\partial}{\partial t'}\right), \\ &\frac{\partial}{\partial y} = \frac{\partial}{\partial y'}, \ \frac{\partial}{\partial z} = \frac{\partial}{\partial z'}, \\ &\frac{\partial}{\partial t} = \gamma\left(\frac{\partial}{\partial t'} - v\frac{\partial}{\partial x'}\right). \end{aligned} \tag{3}$$

Let us write Gauss's law in vacuum

$$\operatorname{div}\boldsymbol{E} = \rho/\varepsilon_0 \tag{4}$$

and the equation expressing nonexistence of magnetic charges

$$\operatorname{div}\boldsymbol{B} = 0. \tag{5}$$

(4) and (5) are Maxwell's scalar equations (basic statements for us). We should emphasize that Gauss's law unlike Coulomb's law is true for arbitrarily moving charges.

According to the principle of relativity equations (4) and (5) are invariant under the Lorentz transformations (l)-(3). If we express divergence in Cartesian coordinates, we shall easily get (as all the inertial reference frames are equivalent we can omit the primes for simplicity):

$$\operatorname{div}\boldsymbol{E} = \frac{\rho}{\varepsilon_0} + v\left[\frac{j_x}{\varepsilon_0 c^2} + \frac{1}{c^2}\frac{\partial E_x}{\partial t} - \left(\operatorname{rot}\boldsymbol{B}\right)_x\right], \tag{6}$$

$$\operatorname{div}\boldsymbol{B} = \frac{v}{c^2}\left[\frac{\partial B_x}{\partial t} + \left(\operatorname{rot}\boldsymbol{E}\right)_x\right]. \tag{7}$$

(4) and (5) laws must have the same form in all inertial frames. Thus, in equations (6) and (7) the expressions in brackets should be zero. It is clear why these expressions contain only *x* projections: we have used transformation formulas for the case when reference frame moves along the *x*-axis. Taking into account isotropy of space we can write

$$\operatorname{rot}\boldsymbol{E} = -\frac{\partial\boldsymbol{B}}{\partial t}, \tag{8}$$

$$\operatorname{rot}\boldsymbol{B} = \frac{1}{\varepsilon_0 c^2}\boldsymbol{j} + \frac{1}{c^2}\frac{\partial\boldsymbol{E}}{\partial t}. \tag{9}$$

We have derived Maxwell's vector equations. Clearly the requirement of their invariance will give us (4) and (5)scalar equations. We have obtained a complete set and proved their invariance under Lorentz transformations.

4. Conclusion

Equation (8) describes phenomenon of Faraday electromagnetic induction and the last term in equation (9) is connected with displacement current. Thus, we have deductively proved relativistic nature of magnetic field, phenomenon of electromagnetic induction and displacement current. It has essential cognitive importance. For example, it becomes clear for students why it was so important for Einstein the phenomenon of electromagnetic induction – his famous article of 1905 about special relativity begins with the discussion of this phenomenon and not Michelson's experiment.

The offered discussion elucidates the essence of principle of relativity: it establishes correlation between laws of Nature that enables us to determine new fundamental laws according to the known ones. To understand physics profoundly it is necessary to comprehend its laws not only towards "natural" fixed laboratory frame but also towards moving inertial frame. This is the essential distinctive aspect of physical mentality.

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