

Image Compression Using Discrete Cosine Transform

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Abstract

Image compression is the application of [data compression](#) on [digital images](#). Image compression can be [lossy](#) or [lossless](#). In this paper it is being attempted to implement basic JPEG compression using only basic MATLAB functions. In this paper the lossy compression techniques have been used, where data loss cannot affect the image clarity in this area. Image compression addresses the problem of reducing the amount of data required to represent a digital image. It is also used for reducing the redundancy that is nothing but avoiding the duplicate data. It also reduces the storage area to load an image. For this purpose we are using JPEG. JPEG is a still frame compression standard, which is based on, the Discrete Cosine Transform and it is also adequate for most compression applications. The discrete cosine transform (DCT) is a mathematical function that transforms digital image data from the spatial domain to the frequency domain.

Keywords: Image Compression, JPEG, DCT, Fourier transform, Spatial Domain, Frequency Domain, Quantization,

INTRODUCTION

Compressing an image is significantly different than compressing raw binary data. Of course, general purpose compression programs can be used to compress images, but the result is less than optimal. DCT has been widely used in signal processing of image. The one-dimensional DCT is useful in processing one-dimensional signals such as speech waveforms. For analysis of two-dimensional (2D) signals such as images, we need a 2D version of the DCT data, especially in coding for compression, for its near-optimal performance. **JPEG** is a commonly used standard method of compression for photographic images. The name JPEG stands for Joint Photographic Experts Group, the name of the committee who created the standard. JPEG provides for lossy compression of images.

Image compression is the application of [data compression](#) on [digital images](#). In effect, the objective is to reduce redundancy of the image data in order to be able to store or [transmit](#) data in an efficient form. The best image quality at a given bit-rate (or compression rate) is the main goal of image compression. The main objectives of this paper are Reducing the image storage space, Easy maintenance and providing security, Data loss cannot effect the image clarity, Lower bandwidth requirements for transmission, Reducing cost.

PAST WORK DONE:

Fourier Theory: The tool that converts a spatial (real space) description of an image into one in terms of its frequency components is called the Fourier transform. The new version is usually referred to as the Fourier space description of the image. The corresponding inverse transformation, which turns a Fourier space description back into a real space one, is called the inverse Fourier transform. The major disadvantage of Fourier theory is computationally very complex.

Discrete Fourier Transform: In [mathematics](#), the discrete Fourier transform (DFT), occasionally called the [finite Fourier transform](#), is a transform for [Fourier analysis](#) of finite-domain [discrete-time signals](#). It is widely employed in [signal processing](#) and related fields to analyze the frequencies contained in a sampled [signal](#), to solve [partial differential equations](#), and to perform other operations such as [convolutions](#). The DFT can be computed efficiently in practice using a Fast Fourier transform (FFT) algorithm. DCTs are equivalent to DFTs of roughly twice the length, operating on real data with [even](#) symmetry (since the Fourier transform of a real and even function is real and even), where in some variants the input and/or output data are shifted by half a sample. There are eight standard DCT variants, of which four are common.

METHODOLOGY: One of the most popular and comprehensive continuous tone, still frame compression standards is the JPEG standard. In the JPEG base line coding system which is based on the discrete cosine transform and is adequate for most compression applications, the input and output images are limited to eight bits, while the quantized DCT coefficient values are restricted to 11 bits. The discrete cosine transform (DCT) is a mathematical function that transforms digital image data from the spatial to the frequency domain. For an $M \times N$ image, the spatial domain represents the color value of each pixel. The frequency domain considers the image data as a two-dimensional waveform and represents the waveform in terms of its frequency components. A DCT-based method is specified for “lossy” compression.

IMAGE PROCESSING: Image processing is any form of [information processing](#) for which the input is an image, such as photographs or frames of video the output is not necessarily an image, but can be for instance a set of features of the image. Most image-processing techniques involve treating the image as a two-dimensional signal and applying standard [signal-processing](#) techniques to it. The field of digital image processing refers to processing digital images by means of a digital computer. A digital image is composed of a finite number of elements, each of which has a particular location and value. These elements are referred to as picture elements, image elements, pels and pixels. Pixel is the term most widely used to denote the elements of a digital image. The computer representation of an image is often made as an array of pixel values. For a monochrome image, the value is a single intensity of the pixel ranging between black and white. For a color image, there are three numbers stored for each pixel typically this is the red, green, and blue intensities of the pixel, but other color models are possible. All together, a color image of size $n \times m$ consists of $n \times m \times 3$ values. Usually these intensities are stored as integers between 0 and 255 that is 8 bits.

Applications of Image processing are [Photography](#) and printing, Satellite image processing, [Machine Vision](#), [Medical image processing](#), [Face detection](#), feature detection, face identification, [Microscope image processing](#), [Car](#) barrier detection

COMPRESSION: Compression is nothing but reducing the size of the data to reduce the amount of space required to store the data. Compression itself may refer to [Data compression](#), encoding information using fewer bits such as [Image compression](#), [Audio data compression](#) & [Video compression](#) and [Bandwidth compression](#), in telecommunications. Compression can be categorised in two broad ways:

Lossless Compression: Data is compressed and can be reconstituted (uncompressed) without loss of detail or information. These are referred to as bit-preserving or reversible compression systems also Lossless compression frequently involves some form of *entropy encoding* and are based in information theoretic techniques.

Lossy Compression: The aim is to obtain the best possible fidelity for a given bit-rate or minimizing the bit-rate to achieve a given fidelity measure. Video and audio compression techniques are most suited to this form of compression. If an image is compressed it clearly needs to

uncompressed (decoded) before it can viewed/listened to. Some processing of data may be possible in encoded form however. Lossy compression use source encoding techniques that may involve transform encoding, differential encoding or vector quantisation.

The advantage of lossy methods over [lossless](#) methods is that in some cases a lossy method can produce a much smaller compressed file than any known lossless method, while still meeting the requirements of the application. Lossy methods are most often used for compressing sound, images or videos. The compression ratio (that is, the size of the compressed file compared to that of the uncompressed file) of lossy video codecs are nearly always far superior to those of the audio and still-image equivalents. Audio can often be compressed at 10:1 with imperceptible loss of quality, video can be compressed immensely (e.g. 300:1) with little visible quality loss. Lossy compressed still images are often compressed to 1/10th their original size, as with audio, but the quality loss is more noticeable, especially on closer inspection.

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Original Cropped Lenna Image (File size: 12KB)	Cropped Lenna Image, Compressed (File size: 85% smaller, 1.8KB)	Cropped Lenna Image, Highly Compressed (File size: 96% smaller, 0.56KB)

Figure 1: Example of Lossy Compression

The above images show the use of lossy compression to reduce the [file size](#) of the image. The image is an excerpt of the image of [Lenna](#), a [defacto](#) industry-standard [test image](#). The first picture is 12,249 bytes. The second picture has been compressed ([JPEG](#) quality 30) and is 85% smaller, at 1,869 bytes. Notice the loss of detail. The third picture has been highly compressed ([JPEG](#) quality 5) and is 96% smaller, at 559 bytes. The [compression artifacts](#) are much more noticeable and the loss of detail is great.

We can classify compression by the why it employs redundancy or by the method it compresses the data. Most methods for irreversible, or "lossy" digital image compression, consist of three main steps: Transform, quantizing and coding, as illustrated in figure .

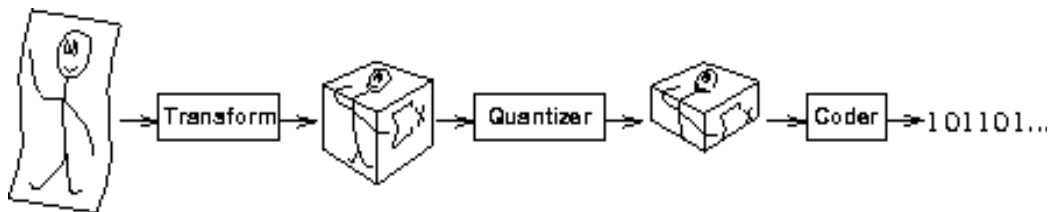


Figure 2: The three steps of digital image compression.

Compression basically employs redundancy in the data. It is achieved by the removal of one or more of three basic data redundancies

- 1) Coding Redundancy: which is present when less than optimum (i.e. the smallest length) code words are used.
- 2) Interpixel Redundancy: which results from correlations between the pixels of an image.
- 3) Psychovisual Redundancy: which is due to data that is ignored by the human visual system (i.e. visually nonessential information).

JPEG provides for lossy compression of images. JPEG's proposed standard aims to be generic and to support a wide variety of applications for continuous-tone images. To meet the differing needs of many applications, the JPEG standard includes two basic compression methods, each with

various modes of operation. A DCT-based method is specified for “lossy” compression, and a predictive method for “lossless” compression.

JPEG, unlike other formats like PPM, PGM, and GIF, is a lossy compression technique; this means visual information is lost permanently. The key to making JPEG work is choosing what data to throw away. It works best on natural images (scenes). This tutorial describes general JPEG compression for grayscale images however, JPEG compresses color images just as easily.

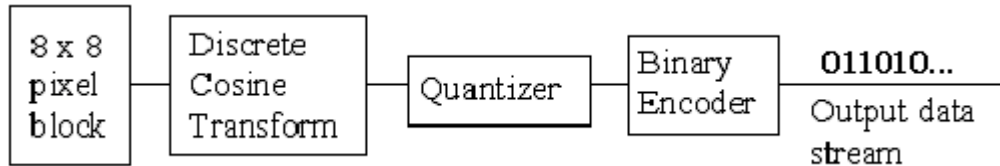


Figure 3: Block Diagram of JPEG Compression

As can be seen in the simplified block diagram of figure 3 the compression itself is performed in four sequential steps 8x8 sub image extractor, DCT computation, quantization, and variable length code assignment. Figure 3 describes the JPEG process. JPEG divides up the image into 8 by 8 pixel blocks, and then calculates the [discrete cosine transform \(DCT\)](#) of each block. A [quantizer](#) rounds off the DCT coefficients according to the [quantization matrix](#). This step produces the "lossy" nature of JPEG, but allows for large compression ratios. [JPEG's compression technique](#) uses a variable length code on these coefficients, and then writes the compressed data stream to an output file (*.jpg). For decompression, JPEG recovers the quantized DCT coefficients from the compressed data stream, takes the inverse transforms and displays the image. Figure 1 shows this process. In [computing](#), JPEG (pronounced JAY-peg) is a commonly used standard method of [compression](#) for photographic images.

The discrete cosine transform (DCT) helps separate the image into parts (or spectral sub-bands) of differing importance (with respect to the image's visual quality). The DCT is similar to the discrete Fourier transform: it transforms a signal or image from the spatial domain to the frequency domain. The input image is N by M. $f(i, j)$ is the intensity of the pixel in row i and column j . $F(u, v)$ is the DCT coefficient in row k_1 and column k_2 of the DCT matrix.

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Figure 4: Discrete Cosine Transform (DCT)

DESIGN : JPEG compression involves the following:

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Figure 5 : JPEG Compression

JPEG Algorithm:A basic jpeg compression (gray level image) algorithm:

- 1) Take an image (2D matrix) and divide it to 8x8 matrices
 - 2) For each matrix (8x8) use the DCT conversion (from the signal processing toolbox). We will get an (8x8) matrix as an answer
 - 3) Build an 8x8 matrix, which is the sum off all the matrices, such that
- $$\text{sum_matrix} = A + B + C + \dots$$
- 4) Sort elements of the 8x8 matrix from the highest to the smallest and get the indices list.
 - 5) Sum the last matrix with part of the elements which have the higher coefficients, until you have a sufficient ratio (lets say 80%).

For example: `idx = sort (sum_matrix (:));`

`part_of_energy = sum_matrix (idx (1:20));`

`all_energy = sum_matrix (:);`

ratio = part_of_energy/all_energy;

6) Save the partial list of indices, number of matrices (rows, lines) and from each matrix from step (2) save only these coefficients (Remember the order you save them) and now this is the compressed data.

To Reconstruct

1) Build matrices of step (2) by the zero command: A = zero (8,8); B = zero (8,8); ... (better to use a for loop...)

2) In each matrix, store the coefficients in the right places, such that the new A matrix is equal to the A matrix from step (2) of the encoding, except for the zeros where you don't know the coefficients.

3) For each matrix do the inverse transform (IDCT)

4) Compose these inverse-transformed matrices back into a big matrix. This is the reconstructed image.

The major Steps in JPEG coding involve: DCT, Quantization, Zigzag Scan, DPCM on DC component, RLE on AC Components, Entropy Coding.

The Discrete Cosine Transform (DCT): The Discrete Cosine Transform was first proposed by Ahmed et al. (1974), and it has been more and more important in recent years. DCT has been widely used in signal processing of image data, especially in coding for compression, for its near-optimal performance. The discrete cosine transform helps separate the image into parts (or spectral sub-bands) of differing importance (with respect to the image's visual quality). The DCT is similar to the discrete Fourier transform: it transforms a signal or image from the spatial domain to the frequency domain.

The general equation for a 2D (N by M image) DCT is defined by the following equation:

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and the corresponding *inverse* 2D DCT transform is simple $F^{-1}(u,v)$, i.e.: Where

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The basic operation of the DCT is as follows:

The input image is N by M. $f(i,j)$ is the intensity of the pixel in row i and column j. $F(u, v)$ is the DCT coefficient in row k1 and column k2 of the DCT matrix.

For most images, much of the signal energy lies at low frequencies; these appear in the upper left corner of the DCT. Compression is achieved since the lower right values represent higher frequencies, and are often small - small enough to be neglected with little visible distortion. The DCT input is an 8 by 8 array of integers. This array contains each pixel's gray scale level. 8 bit pixels have levels from 0 to 255. Therefore an 8 point DCT would be

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QUANTIZATION

DCT-based image compression relies on two techniques to reduce the data required to represent the image. The first is quantization of the image's DCT coefficients; the second is entropy coding of the quantized coefficients. Quantization is the process of reducing the number of possible values of a quantity, thereby reducing the number of bits needed to represent it. Entropy coding is a technique for representing the quantized data as compactly as possible. We will develop functions to quantize images and to calculate the level of compression provided by different degrees of quantization. We will not implement the entropy coding required to create a compressed image file. We need to quantization to throw out bits

Example: 101101 = 45 (6 bits).

Truncate to 4 bits: 1011 = 11.

Truncate to 3 bits: 101 = 5.

Quantization error is the main source of the Lossy Compression.

Uniform quantization: Divide by constant N and round result ($N = 4$ or 8 in examples above). Non powers-of-two gives fine control (e.g., $N = 6$ loses 2.5 bits)

Quantization Tables: In JPEG, each $F[u, v]$ is divided by a constant $q(u,v)$. Table of $q(u,v)$ is called quantization table.

16	11	10	16	24	40	51	61
12	12	14	19	26	58	60	55
14	13	16	24	40	57	69	56
14	17	22	29	51	87	80	62
18	22	37	56	68	109	103	77
24	35	55	64	81	104	113	92
49	64	78	87	103	121	120	101
72	92	95	98	112	100	103	99

Eye is most sensitive to low frequencies (upper left corner), less sensitive to high frequencies (lower right corner) Standard defines 2 default quantization tables, one for luminance (above), and one for chrominance. Quality factor in most implementations is the scaling factor for default quantization tables. Custom quantization tables can be put in image/scan header.

Zigzag Scan:The purpose of the Zig-zag Scan is To group low frequency coefficients in top of vector. Maps 8×8 to a 1×64 vector

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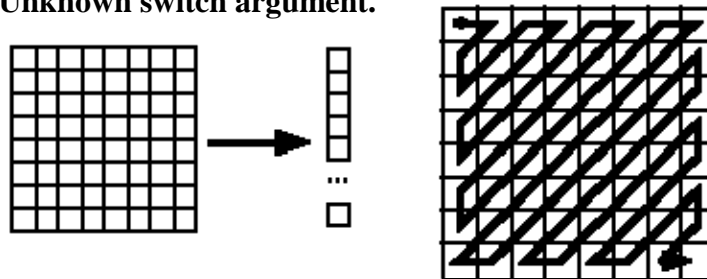


Figure 6:

ZigZag Scan

Differential Pulse Code Modulation (DPCM) on DC component:

Here we see that besides DCT another encoding method is employed: DPCM on the DC component at least. DC component is large and varied, but often close to previous value (like lossless JPEG). Encode the difference from previous 8×8 blocks – DPCM.

Run Length Encode (RLE) on AC components: Yet another simple compression technique is applied to the AC component: 1×64 vector has lots of zeros in it. Encode as (skip, value) pairs, where skip is the number of zeros and value is the next non-zero component. Send (0,0) as end-of-block sentinel value.

Entropy Coding: DC and AC components finally need to be represented by a smaller number of bits. Categorize DC values into SSS (number of bits needed to represent) and actual bits.

Value	SSS
0	0
-1,1	1
-3,-2,2,3	2
-7..-4,4..7	3

For example if DC value is 4, 3 bits are needed. Send off SSS as Huffman symbol, followed by actual 3 bits. For AC components (skip, value), encode the composite symbol (skip, SSS) using the Huffman coding.

RESULTS & TESTING: The following table.1 shows the test reports in the image compression using DCT . Figure 7 shows the original image and the restored images with different coefficients. Figure 8 & Figure 9 shows the two dimensional DCT for the given image. SNR of a compressed image against the level of compression is being calculated and shown as a graph in figure 10. This project has been tested for all possible situations on MATLAB environment on Windows XP and finally produced an 8×8 Compressed DCT image. One of the main problems and

the criticism of the DCT is the blocking effect. In DCT, images are broken into blocks of 8x8 or 16x16 or bigger. The problem with these blocks is that when the image is reduced to higher compression ratios, these blocks become visible. This has been termed as the blocking effect. This image is compressed using 8x8 blocks and only 4 coefficients are retained. The blocking effect is very prominent in this image shown in the figure11.

Table-1 : Test Reports in the Image compression using DCT

Test case no	Input	Expected Behavior	Observed Behavior	Status P = Passed F = Failed
1	Input image of size 128x128	Input image is converted into class double	-Do-	P
2	Input image with double class	Converting the pixel values to zeroes	-Do-	P
3	128x128 image with zero pixel values	128x128 image converted into 8x8 Matrix image	-Do-	P
4	8x8 Block image	8x8 DCT image	-Do-	P
5	Applying quantization on 8x8 DCT image	We obtain frequency coefficients	-DO-	P
6	Applying zig zag scan to the frequency coefficients	Sorting the frequency coefficients from lower to higher	-DO-	P
7	Applying DPCM on DC components	Encoding the difference from previous 8x8 blocks DPCM	-Do-	P
8	Applying RLE on AC components	Skipping the zero value components	-DO-	P

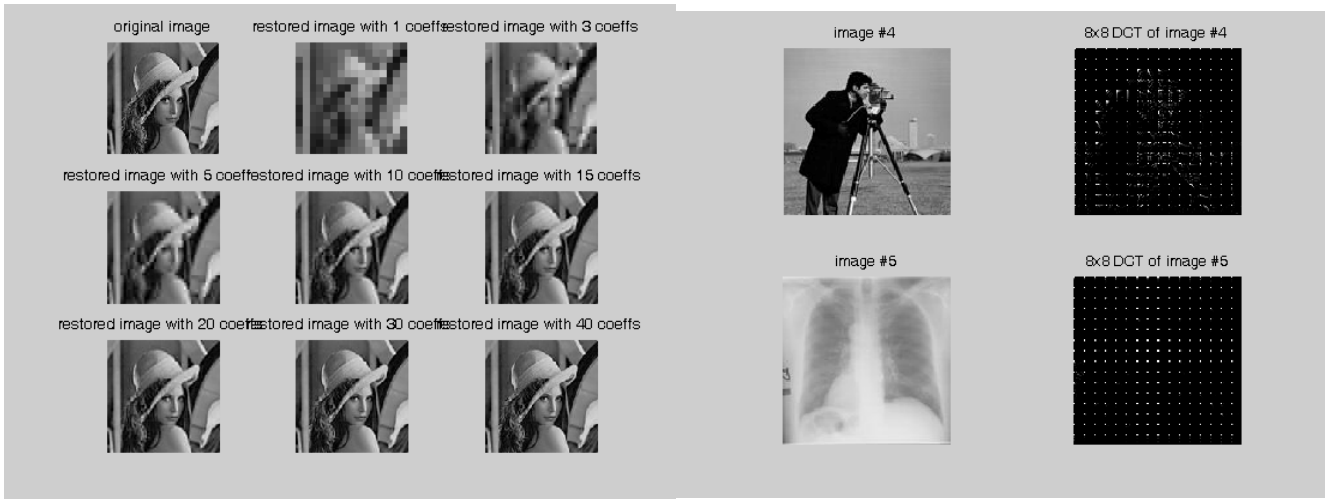


Figure.7)Original Image and restored images with different coefficients

Figure 8) 8x8 DCT of image

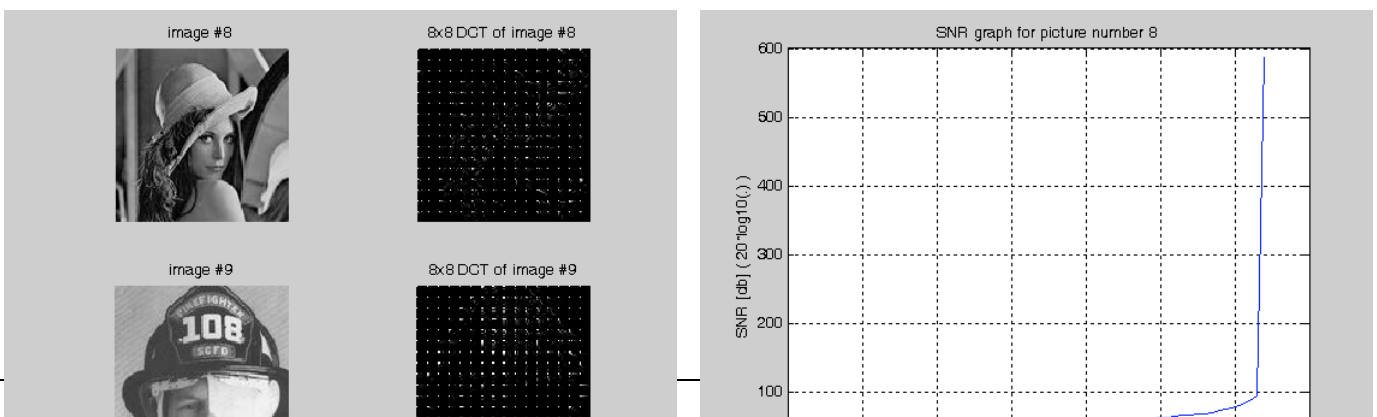


Figure 9) An 8x8 DCT of image

Figure 10) SNR Graph for picture

#8

CONCLUSION

This paper is a project which successfully implemented the JPEG image compression. The system is designed by using MATLAB software. This project has been tested for all possible situations on MATLAB environment on Windows XP and finally produced an 8x8 Compressed DCT image. One of the main problems and the criticism of the DCT is the blocking effect. In DCT, images are broken into blocks of 8x8 or 16x16 or bigger. The problem with these blocks is that when the image is reduced to higher compression ratios, these blocks become visible. This has been termed as the blocking effect. This image is compressed using 8x8 blocks and only 4 coefficients are retained. The blocking effect is very prominent in this image. Let us see with an example

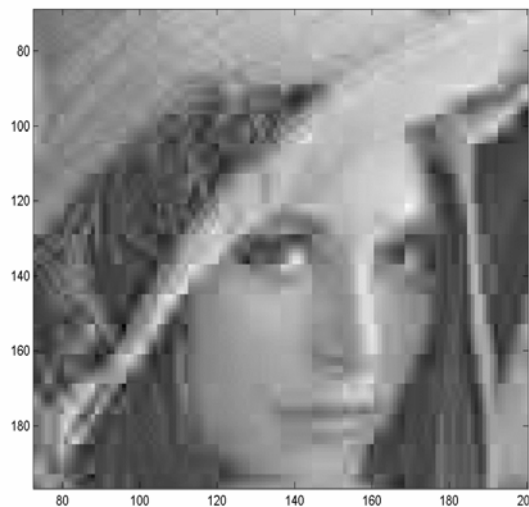


Figure 11) Blocking effect of DCT using 8x8 blocks.
4 coefficients retained.

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