Statistical Modeling of Mobile Radio Channel in Real-time Multi-path Fading Environments

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Abstract
The goal of this paper is to model and simulate the characteristics behavior of mobile radio channel in real-time multi-path settings. This approach ensures accurate analysis and prediction of important network design features that impact the performance of communication systems.

Keywords*: fading, level crossing rate, average fade duration, co-channel interference*

1. INTRODUCTION

The high demand for wireless applications has promoted significant development of wireless network technologies, especially in the latest generation of cellular voice and data networks and, more recently, ad-hoc data networks. A crucial aspect in radio networks is the computation of the effect of co-channel interferences in radio links. The amount of interference that can be tolerated determines the required separation distance between co-channel cells and therefore also the efficiency of the network.

In a radio communication system, it is known that when the signal is transmitted, it degrades due to several factors; for instance, the level of signal decreases with the distance, the signal is further attenuated by objects placed in the medium, it may also be affected by other signals or by noise. When a transmission is engaged over wireless channels, the radio receiver typically observes multiple attenuated and time delayed copies of the transmitted signal that are responsible for time-varying random fluctuations of the signal to noise ratio (SNR). The signal at the receiver antenna (downlink) fluctuates due to various causes such as an object obstructing the radio path when the mobile moves into a specific area, multi-path propagation produced by reflection, refraction or diffraction of the electromagnetic waves that combine constructively in some places and destructively in others. As a result, the mobile moves through zones with different signal levels, causing a fluctuation of the signal received. This fluctuation phenomenon at the receiver can be so severe as to produce a signal which is below the sensitivity of the receiver, thus rendering the communication impossible. This phenomenon is known as fading. The same effect occurs when the mobile is transmitting and the base station is receiving (uplink) [1].

Fading (here referred to as small-scale fading) is expressed by a mathematical model that describes the distortion during its propagation through an operating environment. Therefore, fading describes the rapid fluctuations of amplitudes, phases, or multi-path delays of a radio signal over a short period of time or distance. Fading occurs when two or more versions of a transmitted signal arrives the receiver with small time difference. These signals, or multi-path waves on arrival at the receiver's antenna combines to produce a widely varying signal in phase as well as in amplitude, which depends on the distribution of the intensity, relative propagation time of the waves, and the transmitted signal bandwidth.

It has been observed by [2] that the establishment of routes with unreliable links, largely attributed to the fine grain variation in signal, is a major factor in diminishing the end-to-end performance of well established protocols. The unreliability often causes connectivity loss during critical data packet transmission. Such a loss in connectivity immediately results in maintenance activities and subsequent discovery of routes, which creates excessive systems overhead and congestion. While there are many reasons that received power may rapidly fluctuate, the most significant fluctuation in wireless networks is due to multi-path fading [3][4]. The consequence of mobility causes this fading, which causes multiple copies of some transmission on two or more paths of different lengths. The copies can either reinforce or partially cancel out each other. A common illustration of multi-path fading is the experience of stopping at a traffic light and hearing an FM broadcast degenerate into static, while the signal is re-acquired after the vehicle moves only a fraction of a meter. The loss of the broadcast is caused by the vehicle stopping at a point where the signal experienced severe destructive interferences. Cellular phones can also exhibit similar momentary fades.

Fading channel models are often used to model the effects of electromagnetic transmission of information over the air in cellular networks and broadcast communication. Fading channel models are also used in underwater acoustic communications to model the distortion caused by the water. Mathematically, fading is usually modeled as a time-varying random change in the amplitude and phase of the transmitted signal [5]. Existing methods that closely model multi-path fading require large amount of detail information about the operating environment and is complex to implement in real life networks [6]. As a result, modeling techniques are limited in general applications.

In this paper, we propose a robust statistical model for characterizing communication link performance in wireless networks under multi-path fading environment. We study through computer simulation, important system design features such as the fade margin to achieve a given availability; level crossing rate and fading duration that impact network protocol algorithms in real networks.

2. STATISTICAL MODELS FOR MULTI-PATH FADING CHANNELS

Several multi-path models have been suggested to explain the observed statistical nature of a mobile channel. The first model presented by Ossana [7] was based on interference of waves, incident and reflected from the flat sides of randomly located buildings. Although Ossana’s model predicts flat fading power spectra that were in agreement with measurements in suburban areas, it assumes the existence of a direct path between the transmitter and receiver, and is only limited to a restricted range of reflection angles. Ossana’s model is therefore rather inflexible and inappropriate for urban areas where the direct path is almost always blocked by buildings or other obstacles.

A model where the statistical characteristics of the electromagnetic fields of the received signal at the mobile are deduced from scattering was developed by [8]. The model is based on scattering and is widely used. It assumes a fixed transmitter with a vertically polarized antenna. The field incident on the mobile antenna is assumed to be comprised of N azimuthal plane waves with arbitrary carrier phases, arbitrary azimuthal angles of arrival, and each wave having equal average amplitude. It should be noted that the equal average amplitude assumption is based on the fact that in the absence of a direct line of sight path, the scattered components arriving at the receiver will experience similar attenuation over small-scale distances.

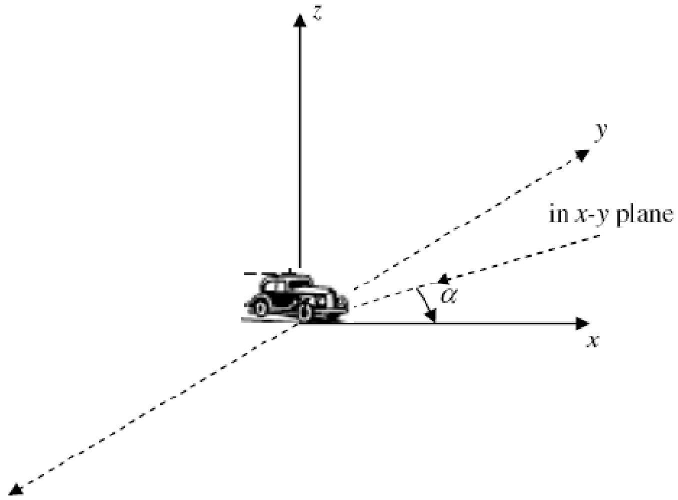

Figure 1. Illustrating plane waves arriving at random angles

Figure 1 shows a diagram of plane wave's incident on a mobile traveling at a velocity in the x-direction. The angle of arrival is measured in the x-y plane with respect to the direction of motion. Every wave that is incident on the mobile undergoes a Doppler shift due to the motion of the receiver and arrives simultaneously at the receiver. That is, no excess delay due to multi-path is assumed for any of the waves (flat fading assumption). For the *nth* wave arriving at an α_n angle to the x-axis, the Doppler shift in Hertz is given by

$$f_n = \frac{v}{\lambda}\cos\alpha_n \quad (1)$$

where λ is the wavelength of the incident wave.

The vertically polarized plane waves arriving at the mobile have *E* and *H* field components given by

$$E_z = E_0 \sum_{n=1}^{N} C_n \cos(2\pi f_c t + \theta_n) \quad (2)$$

$$H_x = -\frac{E_0}{\eta} \sum_{n}^{N} C_n \sin\alpha_n \cos(2\pi f_c t + \theta_n) \quad (3)$$

$$H_y = -\frac{E_0}{\eta} \sum_{n}^{N} C_n \cos\alpha_n \cos(2\pi f_c t + \theta_n) \quad (4)$$

where E_0 is the real amplitude of local average E-field (assumed constant), C_n, a real random variable representing the amplitude of individual waves, η is the intrinsic impedance of the space (377Ω), and f_c is the carrier frequency. The random phase of the nth arriving component, θ_n is given by

$$\theta_n = 2\pi f_n t + \phi_n \quad (5)$$

The amplitudes of the E-and H-field are normalized such that the ensemble average of C_n becomes

$$\sum_{n=1}^{N} \overline{C_n^2} = 1 \quad (6)$$

Since the Doppler shift is negligible compared to the carrier frequency, the three field components may be modeled as a narrow band random processes. The three components E_z, H_x and H_y can be approximated as Gaussian random variables [9] if N is sufficiently large. The phase angles are assumed to have a uniform probability density function *pdf* on the interval ($0, 2\pi$]. Based on the analysis by [10], the E-field can be expressed in an in-plane quadrature form

$$E_z(t) = T_c(t)\cos(2\pi f_c t) - T_s(t)\sin(2\pi f_c t) \quad (7)$$

where

$$T_c(t) = E_0 \sum_{n=1}^{N} C_n \cos(2\pi f_n t + \phi_n) \quad (8)$$

and

$$T_s(t) = E_0 \sum_{n=1}^{N} C_n \sin(2\pi f_n t + \phi_n) \quad (9)$$

both $T_c(t)$ and $T_s(t)$ are Gaussian random processes denoted as T_c and T_s respectively, at any time *t*, T_c and T_s are uncorrelated zero-mean Gaussian random variables with an equal variance given by

$$\overline{T_c^2} = \overline{T_s^2} = \overline{|E_z|^2} = \frac{E_0^2}{2} \quad (10)$$

The envelop of the received E-field, $E_z(t)$, is given by

$$|E_z(t)| = \sqrt{T_c^2(t) + T_s^2(t)} = r(t) \quad (11)$$

Since T_c and T_s are Gaussian random variables, it can be shown through a Jacobean transformation that the random received signal envelop *r* has a Rayleigh distribution given by

$$p(r)=\begin{cases}\dfrac{r}{\sigma^2}\exp(-\dfrac{r^2}{2\sigma^2}) & 0\le r\le\infty \\ 0 & r<0\end{cases} \tag{12}$$

where

$$\sigma^2 = E_0^2 / 2$$

3. EVALUATION OF THE MODEL PARAMETERS

Fade Margin

As earlier explained in section 1, for any radio link, the communication channel has a multi-path structure. Signals come from different secondary sources surrounding the receiver. Basically, because either the radio port and subscriber unit antennas are located far above ground (roof top), the mobility of vehicles or pedestrians have a negligible contribution on the received signal. Movement in the subscriber antenna surrounding clutter will be the main contributor to the received signal strength fluctuation. Thus, the only explanation to the time fluctuation of the signal comes from foliage and atmospheric layers movement. The latter may be neglected for short distance links (< 20 km). In the absence of the wind, the received power will be constant over long periods of time. During windy weather, one expects a change of the channel attenuation with time, the speed of change being related with the wind speed. The maximum Doppler shift f_c of the received signal relates the signal fluctuation with the speed *v* of moving objects in the surrounding clutter that modify the propagation channel structure (see equation. 1).

Availability

Availability is an important parameter when designing wireless links. In a broad sense, availability is defined as the probability of the event ε, whose received power *r* is greater than threshold (*R*), where the threshold *R* represents the signal power below which the communication quality (for instance the bit error rate (BER)) is unacceptably low. Instead of using the absolute threshold *R* which depends on the received power, the fade margin (FM) is considered. The fade margin (in dB) indicates how much below the average power the threshold *R* [11] and is given by:

$$\text{FM (dB)} = -10.\log_{10}\frac{threshold}{average\ received\ \ power} \tag{13}$$

The fade margin is related to the required link availability. For Rayleigh channels, the received signal power is uniformly spread between paths. For such a channel, the availability is given by

$$\text{Availability} = P(\rho_{dB} > FM_{dB}) = \exp(-10\frac{-FM_{dB}}{10}) \tag{14}$$

where ρ_{dB} is the specified signal level.

The Level Crossing Rate

The level crossing rate (LCR) is defined as the expected rate at which the Rayleigh fading envelop, normalized to the local *rms* signal level, crosses a specified level in a positive going direction. The number of level crossings per second using equation (12) is given by

$$N_R = \int_0^\infty \dot{r}p(R,\dot{r})d\dot{r} = \sqrt{2\pi}\, f_m \rho e^{-\rho^2} \tag{15}$$

where $\dot{r}$ is the time derivative of *r(t)*-the slope, $p(R,\dot{r})$ is the joint density function of *r* and $\dot{r}$ at $r=R, f_m$, is the maximum Doppler frequency and $\rho = \frac{R}{R_{rms}}$ is the specified *R* level value, normalized to the local *rms* amplitude of the fading envelop [12]. Equation (15) gives the value of N_R, the average number of level crossings per second at specified levels. The LCR is a function of the mobile speed as is apparent from the presence of f_m in equation (15). There are few crossings at both high and low levels with a maximum occurring rate of $\rho = 1/\sqrt{2}$ (i.e. at a level 3dB below the *rms* level), the signal envelop experiences very deep fades only occasionally, but shallow fades are frequent.

The average Fade Duration

The average fade duration is defined as the average period of time for which the received signal falls below a specified R level. For a Rayleigh fading signal, this is given by

$$\bar{\tau} = \frac{1}{N_R} \Pr[r \le R] \tag{16}$$

where $\Pr[r \le R]$ is the probability that the received signal r is less than R and is given by

$$\Pr[r \le R] = \frac{1}{T} \sum_i \tau_i \tag{17}$$

where τ_i is the duration of the fade and T is the observation interval of the fading signal. The probability that the received signal r is below the threshold R is found from the Rayleigh distribution as

$$\Pr[r \le R] = \int_0^R p(r)dr = 1 - \exp(-\rho^2) \tag{18}$$

where *p(r)* is the *pdf* of the Rayleigh distribution. Therefore, using equations (15) (16) and (17), the average fade duration as a function of ρ and f_m can be expressed as

$$\tau = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}} \tag{19}$$

The average duration of a signal fade helps determine the most likely number of signaling bits that may be lost during a fade. Average fade duration primarily depends upon the mobile speed and decreases as the maximum Doppler frequency f_m becomes large. If a particular fade margin is built into a mobile communication system, it is appropriate to evaluate the receiver performance by determining the rate at which the input signal falls below a given level R, and how long it remains below this level on the average. This is useful for relating SNR during a fade to the instantaneous BER which results.

4. SIMULATION RESULTS

Figure 2 shows a plot of availability versus fade margin. We notice from this figure that for a fade margin of 1dB, the availability is 90.5%. That is, when the subscriber’s antenna has a 0.905 gain, a fade margin of 10dB will guaranty such (90.5%) availability. A logarithmic trend equation is fitted to enable the reader predict new empirical results.

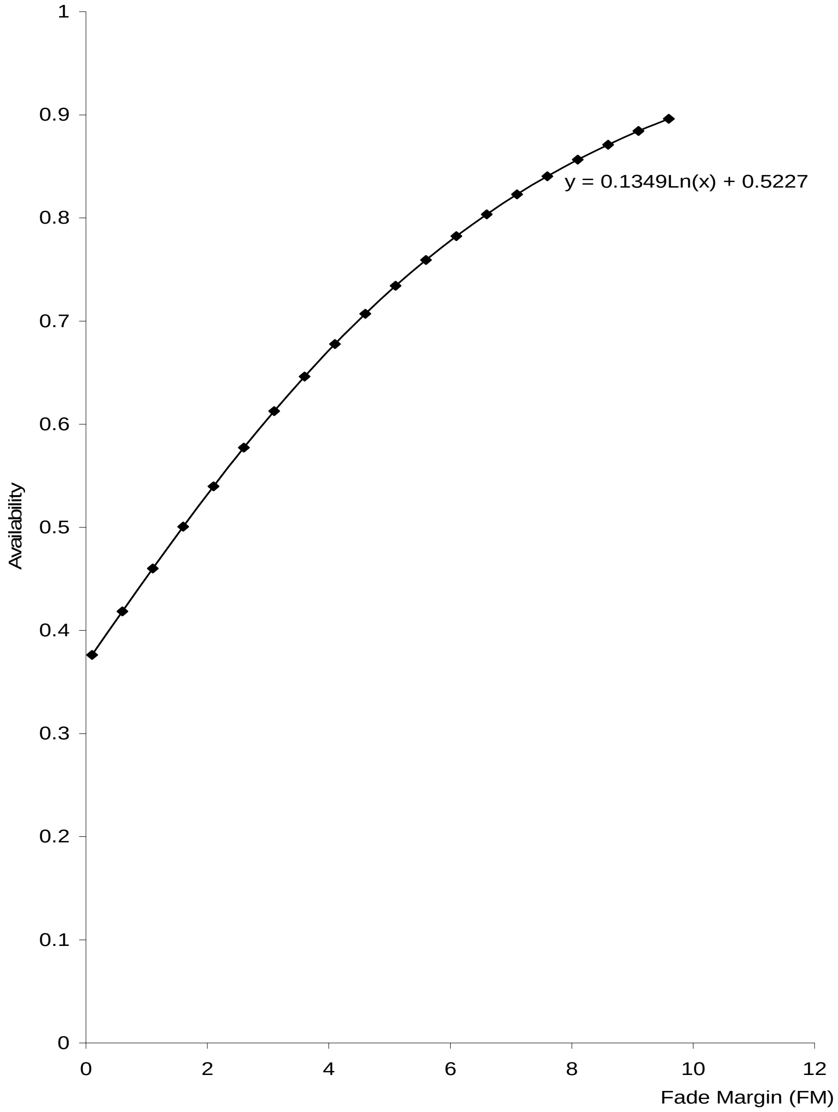

Figure 2. A Plot of Availability vs Fade Margin

Figure 3 is a plot of the normalized level crossing rate (LCR) versus fade margin. This figure shows that fading increases rapidly until it reaches a maximum level at 0.7dB, below the root mean square level. It quantifies how often the fading crosses some threshold $\rho = R / R_{rms}$, where R is the value of the specific level, normalized to the local root mean square *rms* amplitude of the fading rate. A logarithmic trend equation is also fitted to enable the prediction of new empirical values.

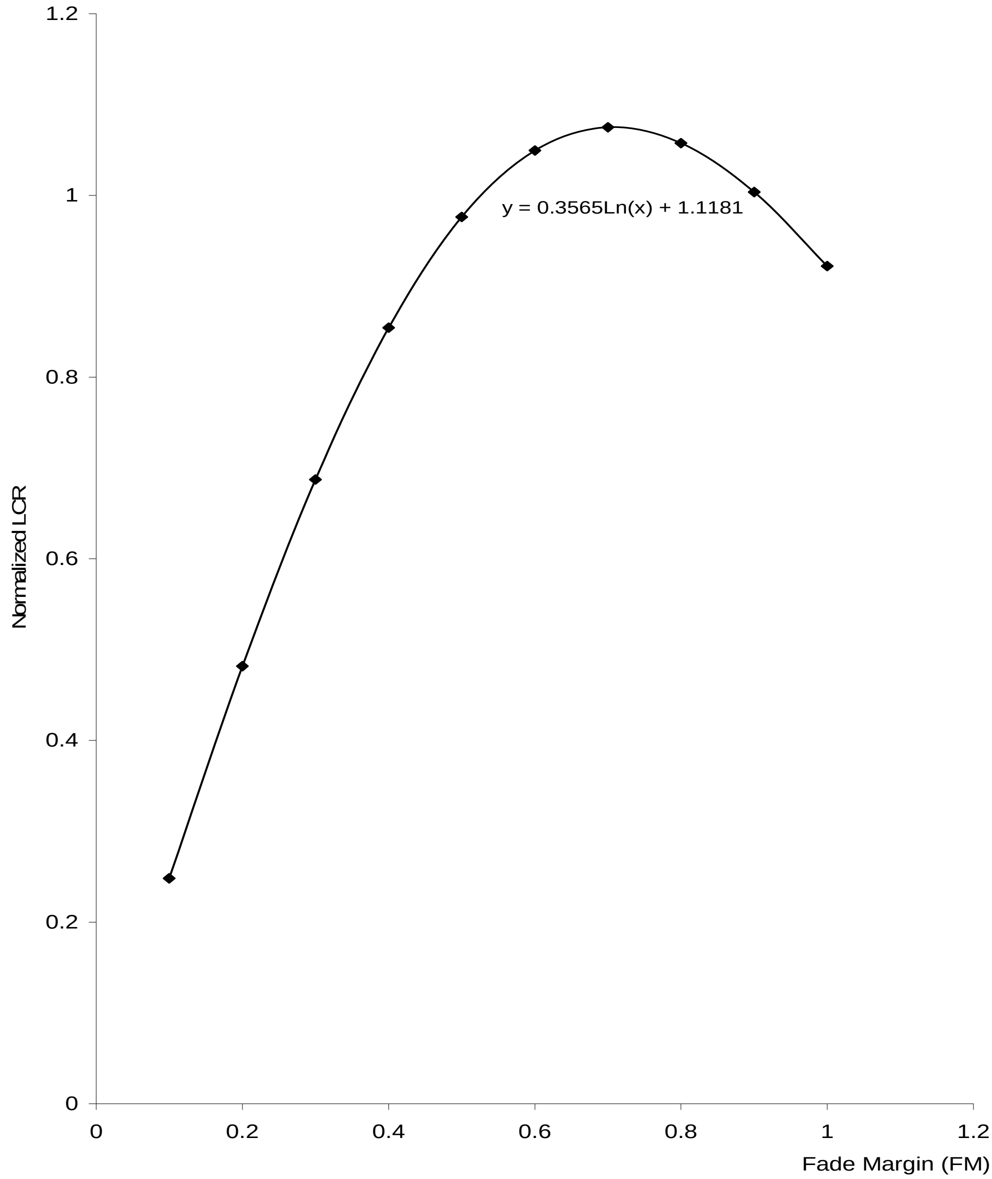

Figure 3. A Plot of LCR versus FM

Plotted in figure 4, is the normalized average fading duration (AFD) versus the fade margin. As seen from this plot, the average fade duration increases steadily with the fade margin. This quantifies how long the signal spends below the threshold ρ . So, at 0.05, the input signal is 0.1dB. We observe that this graph is a power curve and new empirical results could be predicted by using the trend equation fitted in the graph.

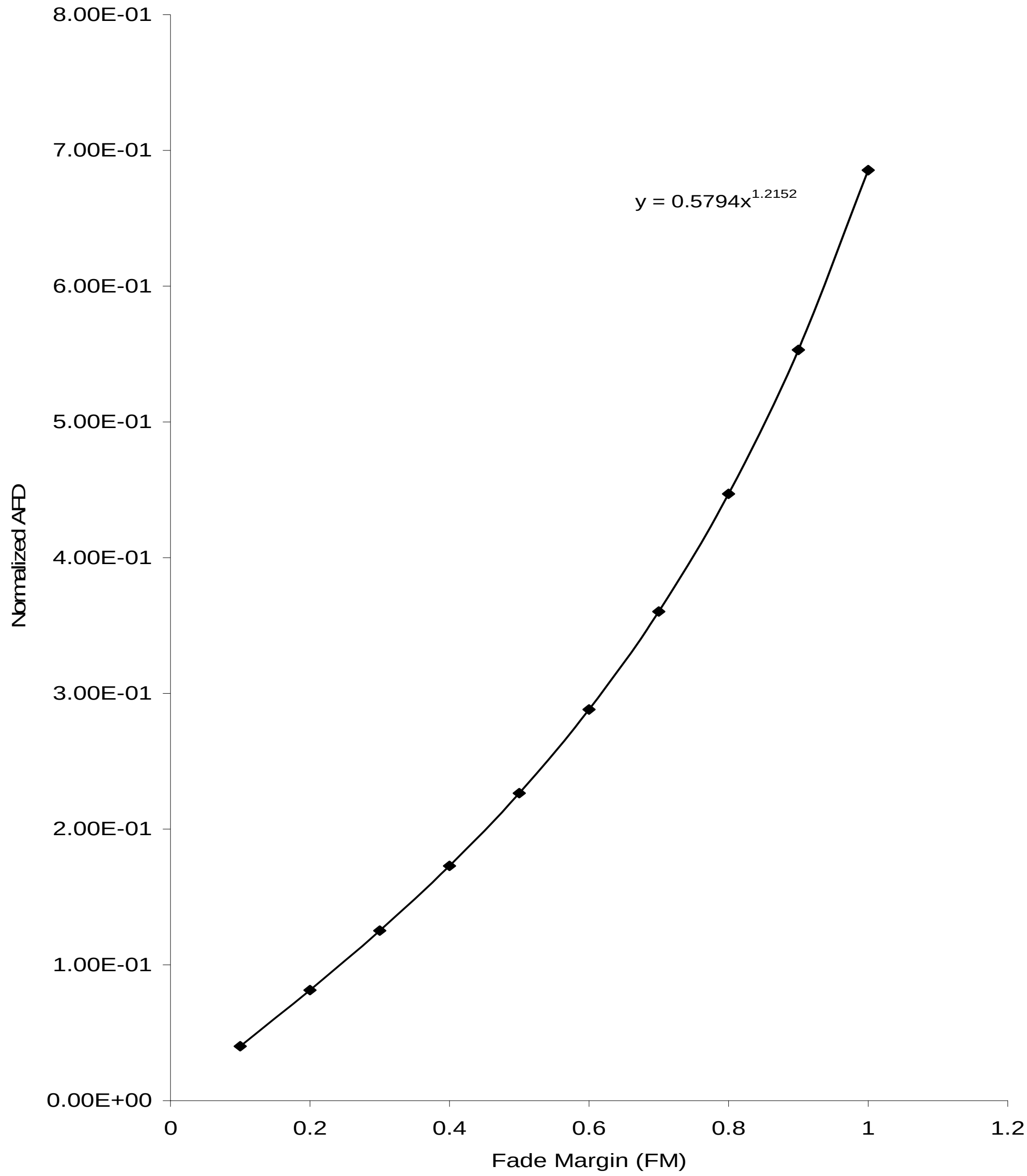

Figure 4. A Plot of AFD vs FM

5. CONCLUSION

A model that predicts the actual behaviour of propagation phenomena is essential for accurate design and simulation of robust radio communication systems. In this paper, we have studied using statistical modeling, the radio link communication characteristics in a multi-path environment. The proposed model was then simulated and results obtained were represented graphically. This representation helped in the analysis of important design aspects that impact the performance of real communication networks.

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