A New Method to Attribute Reduction of Decision Systems with Covering Rough Sets

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Abstract

Attribute reduction is an important issue of rough set theory. It has been proven that finding the minimal reduct of an information system is a NP-hard problem, so is finding the minimal reduct of an incomplete information system. Main reason of causing NP-hard is combination problem. In this paper, we theoretically study an attribute reduction algorithm. It based on results of Chen Degang et al in consistent and inconsistent covering decision system. The time complexity of this algorithm is O(|Δ||U|2). An illustrative example is provided that shows the application potential of the algorithm

Keywords: *attribute reduction, family of covering rough sets, consistent covering decision system, inconsistent covering decision system Introduction*

1. Introduction

Rough set theory is a mathematical tool to deal with vagueness and uncertainty of imprecise data. The theory introduced by Pawlak in 1982 has been developed and found applications in the fields of decision analysis, data analysis, pattern recognition, machine learning, and knowledge discovery in databases. While the equivalence relation is too harsh to meet and is extended to tolerance relation and similarity relation. For example, equivalence relation can't be established based on the null value of attribute. In incomplete information systems, which relations are established can be the base of further study for rough computation, knowledge reduction and rule extraction. On this basis, the covering theory of the generalized rough set is study deeply.

Cheng Degang et al. [1] have defined consistency and inconsistency covering decision system and their attribute reduction. They gave an algorithm to compute reducts from decision systems. Their method based on discernibility matrix. But, sometime we only need to find an attribute reduction. So we propose an algorithm which is finding a minimal attribute reduct of incomplete information decision system.

2. Some relevant concepts and results

In this section, we first recall the concept of a cover and present an example suitable for covering rough sets, and then review the existing research on covering rough sets. Finally, propose the definition of induced cover and reveal three basic relations between two objects with respect to the induced cover.

One kind of suitable data set for covering rough sets is the information systems that some objects have multiple attribute values for a given attribute. This kind of data set is available when some objects have multiselections of attribute values for a given attribute. So we have to list all the possible attribute values. One example of this kind of data set is the combination of several information systems. This is illustrated with the following example.

Example 2.1 ([1]) Let us consider the problem of evaluating credit card applicants. Suppose $U = \{x_1,.., x_9\}$ is a set of nine applicants, E={education; salary} is a set of two attributes, the values of ''education'' are {best; better; good}, and the values of ''salary'' are {high; middle; low}. We have three specialists {A, B, C} to evaluate the attribute values for these applicants. It is possible that their evaluation results to the same attribute values may not be the same, listed below:

For attribute ''education''

A: best $=\{x_1 , x_4 , x_5 , x_7\}$, better $= \{x_2 , x_8\}$, good$= \{x_3 ; x_6 ; x_9\}$

B: best $=\{x_1 , x_2 , x_4 , x_7 , x_8\}$, better $= \{x_5\}$, good$= \{x_3 , x_6 , x_9 \}$

C: best $=\{x_1 , x_4 , x_7 \}$, better $= \{x_2 , x_8\}$, good$= \{x_3 , x_5 , x_6 , x_9 \}$

For attribute ''salary''

A: high $=\{x_1 , x_2 , x_3\}$, middle $= \{x_4, x_5, x_6, x_7, x_8\}$, low$= \{x_9\}$
B: high $=\{x_1 , x_2 , x_3\}$, middle $= \{x_4, x_5, x_6, x_7\}$, low$= \{x_8, x_9\}$
C: high $= \{x_1 , x_2 , x_3\}$, middle $= \{x_4, x_5, x_6, x_8\}$, low$= \{x_7, x_9\}$

Table 1 Classification by evaluation of all three specialists

Salary	Education		
	Best	Better	Good
High	$\{x_1, x_2\}$	$\{x_2\}$	$\{x_3\}$
Middle	$\{x_4, x_5, x_7, x_8\}$	$\{x_5, x_8\}$	$\{x_5, x_6\}$
Low	$\{x_7, x_8\}$	$\{x_8\}$	$\{x_9\}$

Suppose the evaluations given by these specialists are of the same importance. If we want to combine these evaluations without losing information, we should union the evaluations given by each specialist for every attribute value as shown in Table 1. This classification is not a partition, but a cover, which reflects a kind of uncertainty caused by the differences in interpretation of data.

2.1. Covering rough sets and induced covers Headings

Definition 2.1 Let U be a universe of discourse, C a family of subsets of U. C is called a cover of U if no subset in C is empty and $\cup C = U$.

Definition 2.2 Let C = {C1, C2, ..,Cn} be a cover of U. For every $x\in U$, let $Cx = \cap\{Cj: Cj \in C, x\in Cj\}$. $Cov(C) = \{Cx: x\in U\}$ is then also a cover of U. We call it induced over of C.

Definition 2.3 Let $\Delta= \{Ci: i=1,m\}$ be a family of covers of U. For every $x\in U$, let $\Delta x= \cap\{Cix: Cix\in Cov (Ci), x\in Cix\}$ then $Cov (\Delta) = \{\Delta x: x\in U\}$ is also a cover of U. We call it the induced cover of Δ.

Clearly Δ_x is the intersection of all the elements in every C_i including x, so for every $x\in U$, Δ_x is the minimal set in Cov(Δ) including x. If every cover in Δ is an attribute, then $\Delta_x= \cap\{C_{ix}: C_{ix}\in Cov(C_i), x\in C_{ix}\}$ means the relation among C_{ix} is a conjunction. Cov(Δ) can be viewed as the intersection of covers in Δ. If every cover in Δ is a partition, then Cov(Δ) is also a partition and Δ_x is the equivalence class including x. For every $x, y \in U$, if $y \in \Delta_x$, then $\Delta_x \supseteq \Delta_y$, so if $y \in\Delta_x$ and $x \in\Delta_y$, then $\Delta_x=\Delta_y$. Every element in Cov(Δ) can not be written as the union of other elements in Cov(Δ). We employ an example to illustrate the practical meaning of C_x and Δ_x.

Example 2.2 ([1]) In Example 2.1 if let Δ= {C1, C2}, where C1 denotes the attribute ''education'' and C2 denotes the attribute ''salary'', then

C_1={C_{11}={x_1, x_2, x_4, x_5, x_7, x_8} (best), C_{12}={x_2, x_5, x_8} (better), C_{13} = {x3, x5, x6, x9} (good)}

C_2={C_{21}={x_1, x_2, x_3} (high), C_{22}={x_4, x_3, x_6, x_7, x_8} (middle), C_{23}= {x_7, x_8, x_9} (lower)}

We have C_{1x5} = {x_5} = $C_{11} \cap C_{12} \cap C_{13}$, which implies the possible description of x_5 is {(best $\vee$ better $\vee$ good} according to attribute ''education''. Δ_{x8} = ($C_{11} \cap C_{12}$) $\cap$ ($C_{22} \cap C_{23}$) which implies the possible description of x_8 is {(best $\vee$ better) $\wedge$ (middle $\vee$ lower)}.

For every $X \subseteq U$, the lower and upper approximation of X with respect to Cov(Δ) are defined as follows:

$$\underline{\Delta}(X) = \cup\{\Delta_x : \Delta_x \subseteq X\}, \quad (1)$$

$$\overline{\Delta}(X) = \cup\{\Delta_x : \Delta_x \cap X \neq \varnothing\} \quad (2)$$

The positive, negative and boundary regions of X relative to Δ are computed using the following formulas respectively:

$$POS_\Delta(X) = \underline{\Delta}(X), NEG_\Delta(U - \overline{\Delta}(X),$$
$$BN_\Delta(X) = \overline{\Delta}(X) - \underline{\Delta}(X) \quad (3)$$

Clearly in Cov(Δ), Δ_x is the minimal description of object x.

Theorem 2.1 ([1]) Supposing U is a finite universe and Δ = {C_i: i=1,..m} be a family of covers of U, the following statements hold:

(1) $\Delta_x = \Delta_y$ if and only if for every $C_i \in \Delta$ we have $C_{ix} = C_{iy}$.

(2) $\Delta_x \supset \Delta_y$ if and only if for every $C_i \in \Delta$ we have $C_{ix} \supseteq C_{iy}$ and there is a $C_i \in \Delta$ such that $C_{i0\,x} \supset C_{i0\,y}$.

(3) $\Delta_x \not\subset \Delta_y$ and $\Delta_y \not\subset \Delta_x$ hold if and only if there are **C_i, C_j** $\in$**Δ** such that $C_{ix} \subset C_{iy}$ and $C_{jx} \supset C_{jy}$ or there is a **C_{i0}** $\in$**Δ** such that $C_{i0\,x} \not\subset C_{i0\,y}$ and $C_{i0\,y} \not\subset C_{i0\,x}$.

2.2. Attribute reduction of consistent and inconsistent decision systems

Definition 2.4 ([1]) Let Δ = {Ci: i=1,..m} be a family of covers of U, D is a decision attribute, U/D is a decision partition on U. If for $\forall x \in U$, $\exists Dj \in U/D$ such that $\Delta x \subseteq Dj$, then decision system (U,Δ,D) is called a consistent covering decision system, and denoted as Cov(Δ)≤ U/D. Otherwise, (U,Δ,D) is called an inconsistent covering decision system. The positive region of D relative to Δ is defined as

$$POS_\Delta(D) = \bigcup_{X \in U/D} \underline{\Delta}(X) \quad (4)$$

Remark 2.1: Let D={d}, then d(x) is a decision function d: U $\rightarrow$ V_d of the universe U into value set V_d. For every x_i, $x_j \in U$, if $\Delta x_i \subseteq \Delta x_j$, then d($x_i$) = d($[x_i]_D$) = d($\Delta_{xi}$) = d($\Delta_{xj}$) = d($x_j$) = d($[x_j]_D$). If d($\Delta_{xi}$) $\neq$ d(Δ_{xj}), then $\Delta_{xi} \cap \Delta_{xj} = \varnothing$, i.e $\Delta_{xi} \not\subset \Delta_{xj}$ and $\Delta_{xj} \not\subset \Delta_{xi}$.

Definition 2.5 ([1]) Let (U,Δ, D= {d}) be a consistent covering decision system. For Ci $\in$Δ , if Cov(Δ-{Ci}) ≤ U/D, then Ci is called superfluous relative to D in Δ , otherwise Ci is called indispensable relative to D in Δ. For every P $\subseteq$ Δ satisfying Cov(P) ≤U/D , if every element in P is indispensable, i.e., for every Ci $\in$P, Cov(Δ-{Ci}) ≤ U/D is not true, then P is called a reduct of D relative to D, relative reduct in short. The collection of all the indispensable elements in D is called the core of Δ relative to D, denoted as CoreD(Δ). The relative reduct of a consistent covering decision system is the minimal set of conditional covers (attributes) to ensure every decision rule still consistent. For a single cover Ci, we present some equivalence conditions to judge whether it is indispensable.

Definition 2.6 ([1]) Suppose U is a finite universe and Δ = {Ci: i=1,..m} be a family of covers of U, Ci $\in\Delta$, D is a decision attribute relative Δ on U and d: U $\rightarrow$ Vd is the decision function Vd defined as d(x) = [x]D. (U,Δ,D) is an inconsistent covering decision system, i.e., POSΔ(D)$\neq$U. If POSΔ(D)=POSΔ-{Ci}(D), then Ci is superfluous relative to D in Δ. Otherwise Ci is indispensable relative to D in Δ. For every P$\subseteq\Delta$, if every element in P is indispensable relative to D, and POSΔ(D)=POSP(D), then P is a reduct of POSΔ(D)=POSΔ-{Ci}(D) relative to D, called relative reduct in short. The collection of all the indispensable elements relative to D in Δ is the core of Δ relative to D, denoted by CoreD(Δ).

2.3. Some results of Chang et al

Theorem 2.2 ([1]) Suppose Cov(Δ)$\leq$ U/D, Ci $\in\Delta$, Ci is then indispensable, i.e., Cov(Δ-{Ci}) $\leq$ U/D is not true if and only if there is at least a pair of xi, xj $\in$U satisfying d(Dxi)$\neq$ d(Dxj), of which the original relation with respect to Δ changes after Ci is deleted from Δ.

Theorem 2.3 ([1]) Suppose Cov(Δ)$\leq$ U/D,P $\subseteq\Delta$, then Cov(P)$\leq$ U/D if and only if for xi, xj $\in$U satisfying d(Δxi) $\neq$ d(Δxj), the relation between xi and xj with respect to Δ is equivalent to their relation with respect to P, i.e., Δxi $\not\subset$ Δxj and Δxj $\not\subset$ Δxi $\Leftrightarrow$ Pxi$\not\subset$ Pxj, Pxj $\not\subset$ Pxi.

Theorem 2.4 ([1]) Inconsistent covering decision system (U,Δ, D = {d}) have the following properties:

(1) For $\forall$xi$\in$U, if $\Delta_{xi} \subset POS_{\Delta}(D)$, then $\Delta_{xi} \subseteq [D]_{xi}$; if $\Delta_{xi} \not\subset POS_{\Delta}(D)$, then for $\forall x_k \in U$, $\Delta_{xi} \subseteq [x_k]_D$ is not true.

(2) For any P$\subseteq\Delta$, $POS_P(D)= POS_{\Delta}(D)$ if and only if

$$\underline{P}(X) = \underline{\Delta}(X) \qquad (5)$$

for $\forall$X$\in$U/D.

(3) For any P$\subseteq\Delta$, $POS_P(D)= POS_{\Delta}(D)$ if and only if

$$\forall x_i \in U,\ \Delta_{xi} \subseteq [x_i]_D \Leftrightarrow P_{xi} \subseteq [x_i]_D \qquad (6)$$

2.4. Two theorems as a base for new algorithm

Theorem 2.5 Let (U,Δ,D={d}) be a covering decision system. P $\subseteq\Delta$, then we have:

a. (U,Δ,D={d}) is a consistent covering decision system when it holds:

$$\sum_{x\in U} \frac{|\Delta_x \cap [x]_D|}{|\Delta_x|} = |U| \qquad (7)$$

b. Suppose Cov(Δ)$\leq$ U/D, Ci $\in\Delta$, Ci is then indispensable, i.e., Cov(Δ-{Ci}) $\leq$ U/D is true if and only if

$$\sum_{xi\in U}\sum_{xj\in U} \left|(\Delta_{xi} \cap \Delta_{xj} \cup (P_{xi} \cap P_{xj})\right| \left|d(\Delta_{xi}) - d(\Delta_{xj})\right| = 0 \qquad (8)$$

Where $Cov(\Delta\text{-}\{C_i\})=\{P_x : x\in U\}$, $Cov(\Delta)= \{\Delta_x : x \in U\}$

Proof:

a. By define of a consistent covering decision system, clearly for every x$\in$U, $\Delta_x \subseteq [x]_D$ is always true, thus we have

$$\left|\Delta_x \cap [x]_D\right| = \left|\Delta_x\right| \quad (9)$$

i.e

$$\sum_{x \in U} \frac{\left|\Delta_x \cap [x]_D\right|}{\left|\Delta_x\right|} = \left|U\right| \quad (10)$$

b. Let Cov(Δ-{C_i})={P_x : x∈U} = Cov(P), Cov(Δ)= {Δ_x : x ∈U, by theorem 2.3, P is a reduct or C_i is indispensable, for xi, xj ∈U satisfying d(Δxi) ≠ d(Δxj), the relation between xi and xj with respect to Δ is equivalent to their relation with respect to P, i.e., Δxi ⊄ Δxj and Δxj ⊄ Δxi ⇔ Pxi⊄ Pxj, Pxj ⊄ Pxi. Follow remark 2.1, If $d(\Delta_{xi}) \neq d(\Delta_{xj})$, then $\Delta_{xi} \cap \Delta_{xj} = \varnothing$, i.e

$$\left|(\Delta_{xi} \cap \Delta_{xj}) \cup (P_{xi} \cap P_{xj})\right| = 0 \quad (11)$$

If xi,xj ∈U satisfying d(Δxi) = d(Δxj) then

$$\left|d(\Delta_{xi}) - d(\Delta_{xj})\right| = 0 \quad (12)$$

In other words, it holds:

$$\sum_{xi \in U} \sum_{xj \in U} \left|(\Delta_{xi} \cap \Delta_{xj} \cup (P_{xi} \cap P_{xj})\right| \left|\Delta_{xi} - \Delta_{xj}\right| = 0 \quad (13)$$

This completes the proof.

Theorem 2.6 Let (U,Δ,D={d}) be a inconsistent covering decision system. P ⊆ Δ, $POS_P(D)$ = $POS_{\Delta}(D)$ if and only if ∀xi∈U,

$$\sum_{xi \in U} \left[\left| \frac{\Delta_{xi} \cap [x_i]_D}{\Delta_{xi}} \right| - \left| \frac{P_{xi} \cap [x_i]_D}{P_{xi}} \right| \right] = 0 \quad (14)$$

Proof:

By theorem 2.4, from third condition ∀xi∈U, Δxi ⊆[xi]D ⇔ Pxi ⊆ [xi]D i.e ∀xi∈U,

$$\left|\Delta_{xi} \cap [x]_D\right| = \left|\Delta_{xi}\right| \Leftrightarrow \left|P_{xi} \cap [x]_D\right| = \left|P_{xi}\right| \quad (15)$$

In other words, we have theorem above.

3. Algorithm of attribute reduction

In this section, an algorithm of attribute reduction is presented. Theorem 2.5 and 2.6 are theorical foundation for our proposing. This algorithm finds an approximately minimal reduct.

3.1. Algorithm of attribute reduction in covering decision system:

Input: A covering decision system

S= (U,Δ,D={d}) (16)

Output: One product RD of Δ.

Method

Step 1: Compute

$$CI = \sum_{x \in U} \frac{\left|\Delta_x \cap [x]_D\right|}{\left|\Delta_x\right|} \quad (17)$$

Step 2: If CI = |U| {S is a consistent covering decision system} then goto Step 3 else goto Step 5.

Step 3: Compute

$$\Delta_x, d(\Delta_x), \forall x \in U \tag{18}$$

Step 4: Begin

for each $C_i \in \Delta$ do

if

$$\sum_{xi \in U} \sum_{xj \in U} \left|(\Delta_{xi} \cap \Delta_{xj} \cup (P_{xi} \cap P_{xj})\right| \left|d(\Delta_{xi}) - d(\Delta_{xj})\right| = 0 \tag{19}$$

*{Where Δ - {**C_i** }= {Px : x∈U}}then Δ:= Δ - {**C_i** };*

Endfor;

goto Step 6.

End;

Step 5: Begin

for each $C_i \in \Delta$ do

if

$$\sum_{xi \in U} \left[\left| \frac{\Delta_{xi} \cap [x_i]_D}{\Delta_{xi}} \right| - \left| \frac{P_{xi} \cap [x_i]_D}{P_{xi}} \right| \right] = 0 \tag{20}$$

then Δ:= Δ - {C_i };

{Where Δ - {C_i }= {Px : x∈U}}

Endfor;

End;

Step 6: RD= Δ; the algorithm terminates.

By using this algorithm, the time complexity to find one reduct is polynomial.

At the first step, the time complexity to compute CI is O(|U|).

At the step 2, the time complexity is O(1).

At the step 3, the time complexity is O(|U|).

At the step 4, the time complexity to compute ΣΣ() is O($|U|^2$), from i=1..|Δ|, thus the time complexity of this step is O($|\Delta||U|^2$),

At the step 5, the time complexity is the same as step 4. It is O($|\Delta||U|^2$).

At the step 6, the time complexity is O(1).

Thus the time complexity of this algorithm is O($|\Delta||U|^2$) (Where we ignore the time complexity for computing Δ_{xi}, P_{xi}, i= 1..|Δ|).

4. Illustrative Example

Suppose U = {x_1, x_2, .., x_9}, Δ = {C_i, i=1..4}, and

$C_1=\{\{x_1,x_2,x_4,x_5,x_7,x_8\},\{x_2,x_3,x_5,x_6,x_8,x_9\}\}$,

$C_2=\{\{x_1,x_2,x_3,x_4,x_5,x_6\},\{x_4,x_5,x_6,x_7,x_8,x_9\}\}$,

$C_3=\{\{x_1,x_2,x_3\},\{x_4,x_5,x_6,x_7,x_8,x_9\},\{x_5,x_6,x_8,x_9\}\}$,

$C_4=\{\{x_1,x_2,x_4,x_5\},\{x_2,x_3,x_5,x_6\},\{x_4,x_5,x_7,x_8\},\{x_5,x_6,x_8,x_9\}\}$

$U/D=\{\{x_1,x_2,x_3\}, \{x_4,x_5,x_6\}, \{x_7,x_8,x_9\}\}$

where, $\Delta_i=\Delta_{xi}$, P_i là P_{xi} (for short)

Step 1:

$\Delta_1=\{x_1,x_2\}$, $\Delta_2=\{x_2\}$, $\Delta_3=\{x_2,x_3\}$,

we have $d(\Delta_1) = d(\Delta_2) = d(\Delta_3) = 1$, because Δ_1, Δ_1, $\Delta_1 \subseteq \{x_1,x_2,x_3\}$, $\Delta_4=\{x_4,x_5\}$, $\Delta_5=\{x_5\}$, $\Delta_6=\{x_5,x_6\}$,

we have $d(\Delta_4) = d(\Delta_5) = d(\Delta_6) = 2$, because Δ_4, Δ_5, $\Delta_6 \subseteq \{x_4,x_5,x_6\}$, $\Delta_7=\{x_7,x_8\}$, $\Delta_8=\{x_8\}$, $\Delta_9=\{x_8,x_9\}$

we have $d(\Delta_7) = d(\Delta_8) = d(\Delta_9) = 3$, because Δ_7, Δ_8, $\Delta_9 \subseteq \{x_7,x_8,x_9\}$

$CI = 9 \Rightarrow S$ is consistent

Step 2:

$P - \{C_1\}$:

$P_1=\{x_1,x_2\}$, $P_2=\{x_2\}$, $P_3=\{x_2,x_3\}$,

$P_4=\{x_4,x_5\}$, $P_5=\{x_5\}$, $P_6=\{x_5,x_6\}$,

$P_7=\{x_7,x_8\}$, $P_8=\{x_8\}$, $P_9=\{x_8,x_9\}$

$$\sum_{xi\in U}\sum_{xj\in U}\left|(\Delta_{xi}\cap\Delta_{xj}\cup(P_{xi}\cap P_{xj})\right|\left|d(\Delta_{xi})-d(\Delta_{xj})\right|=0 \quad (21)$$

$\Delta=\Delta - \{C_1\}=\{C_2,C_3,C_4\}$.

Step 3:

$P=\Delta - \{C_2\}$

$P_1=\{x_1,x_2\}$, $P_2=\{x_2\}$, $P_3=\{x_2,x_3\}$,

$P_4=\{x_4,x_5\}$, $P_5=\{x_5\}$, $P_6=\{x_5,x_6\}$,

$P_7=\{x_7,x_8\}$, $P_8=\{x_8\}$, $P_9=\{x_8,x_9\}$

$$\sum_{xi\in U}\sum_{xj\in U}\left|(\Delta_{xi}\cap\Delta_{xj}\cup(P_{xi}\cap P_{xj})\right|\left|d(\Delta_{xi})-d(\Delta_{xj})\right|=0 \quad (22)$$

$\Delta=\Delta - \{C_2\}= \{C_3,C_4\}$

Step 4:

$P= \Delta - \{C_3\}$:

$P_1=\{x_1,x_2, x_4,x_5\}$, $P_2=\{x_2\}$, $P_3=\{x_2,x_3,x_5,x_6\}$,

$P_4=\{x_4,x_5\}$, $P_5=\{x_5\}$, $P_6=\{x_5,x_6\}$,

$P_7=\{x_4,x_5,x_7,x_8\}$, $P_8=\{x_5,x_8\}$, $P_9=\{x_5,x_6,x_8,x_9\}$

$$\sum_{xi\in U}\sum_{xj\in U}\left|(\Delta_{xi}\cap\Delta_{xj}\cup(P_{xi}\cap P_{xj})\right|\left|d(\Delta_{xi})-d(\Delta_{xj})\right|\neq 0 \quad (23)$$

(we can see $(\Delta_1\cap\Delta_4)=\emptyset$, but $(P_1\cap P_4)\neq\emptyset$, $|d(\Delta_1)-d(\Delta_4)|\neq 0$)

$\Delta=\{C_3,C_4\}$.

Step 5:

$P=\Delta - \{C_4\}$

$P_1=\{x_1,x_2, x_3\}$, $P_2=\{x_1,x_2,x_3\}$, $P_3=\{x_1,x_2,x_3\}$,

$P_4=\{x_4,x_5,x_6,x_7,x_8,x_9\}$,$P_5=\{x_4,x_5,x_6,x_7,x_8,x_9\}$, $P_6=\{x_4,x_5,x_6,x_7,x_8,x_9\}$

$P_7=\{x_7,x_8,x_9\}$, $P_8=\{x_7,x_8,x_9\}$, $P_9=\{x_7,x_8,x_9\}$

$$\sum_{xi\in U}\sum_{xj\in U}\left|(\Delta_{xi}\cap\Delta_{xj}\cup(P_{xi}\cap P_{xj})\right|\left|d(\Delta_{xi})-d(\Delta_{xj})\right|\neq 0 \qquad (24)$$

(we can see $(\Delta_6\cap\Delta_7)=\varnothing$, but $(P_6\cap P_7)\neq\varnothing$, $|d(\Delta_6)-d(\Delta_7)|\neq 0$)

$\Delta=\{C_3,C_4\}$.

Step 6:

$RD=\{C_3,C_4\}$ is reduct. i.e attributes with respect to C_1, C_2 are deleted.

5. Conclusion

In this paper, we propose an attribute reduction algorithm. It based on results of Chen Degang et al in consistent and inconsistent covering decision system. The time complexity of this algorithm is $O(|\Delta||U|^2)$. Compare with the results of Cheng Degang's the algorithm, our result is compatible. In next time, we are studying algorithms which are developed from the theory of traditional rough sets.

6. References

[1] Chen Degang, Wang Changzhong, Hu Quinghua, A new approach to attribute reduction of consistent and inconsistent covering rough sets, Information Sciences, 177 , 3500-3518, 2007

[2] Guo-Yin Wang, Jun Zhao, Jiu-Jiang An, Yu Wu, Theoretical study on attribute reduction of rough set theory: Comparision of algebra and information views, Proceedings of the Third IEEE International Conference on Cognitive Informatics (ICCI'04), 2004

[3] Ruizhi Wang, Duoqian Miao, Guirong Hu, Descernibility Matrix Based Algorithm for Reduction of Attributes,: 477- 480, IAT Workshops 2006

[4] Yiyu Yao, Yan Zhao, Jue Wang, On Reduct Construction Algorithms, Rough sets and Knowledge Technology, First International Conference, RSKT2006, Proceedings, LNAI 4062, pp 207-304, 2006

[5] Zhi Kong, Liqun Gao, Qingli Wang, Lifu Wang, Yang Li, Uncertainty Measurement Based on General relation, In 2008 American Control Conference, Westin Sealtle Hotel, Seatle, Washington, USA, , FrA102.3:3650-3653, June 11-13,2008

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