All Comparison Analysis in Internet Traffic Sharing Using Markov Chain Model in Computer Networks

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Abstract

Naldi (2002) presented a Markov chain model based analysis for the user's behaviour in a simple scenario of two competitors. The model is applied to predict influence of both parameters (blocking probability and initial preference) on the traffic distribution between the operators. It is also shown that smaller blocking competitors can be benefited from call-by-call basis assumption. In this paper this criteria of Call-by-call attempt is converted into two call attempts and new mathematical results are derived. A comparative study between call-attempts is made with Naldi (2002) expressions. It is found that, by two-call attempt model, the operator gains more traffic than one-call attempt.

Keywords: *Markov chain model, Transition probability, Initial preference, Blocking probability, Call-by-call basis, Two call basis, Internet Service Provider [operators], Quality of service (QoS).*

1. Introduction

A user of internet services has a big proposition among all throw out the world. These services are provided by operators (Internet Service Providers) by the help of wide area network in a regain. A broad band connectivity is easier and few attempts one can achieve the call connection but dialup based connectivity often takes a large number of call attempts to be connected.

Naldi (2002) has opened up the problem of internet traffic sharing evaluation Shukla and Gadewal (2007) have shown the application of Markov Chain model to the modelling of space division switches. In similar type of contribution Shukla et. al. (2007) have the modelling approach for know-out switches. Shukla and Thakur (2008 a,b,c) have useful contribution for modelling of internet traffic sharing phenomena between two operators in competitive markets.

Shukla and Tiwari (2009) have a modelling approach for Internet Traffic in presence of rest state. Shukla & Thakur (2007, 2009) have studded the Cyber Crime behaviour of internet user under Markov chain modelling approach.

The model of Naldi (2002) is based on dial-up setup in which the user behaviours is assumed as following systems:

1.1. System-I

(a) Suppose two operators O_1 and O_2 are in competition in the market.
(b) The user initially chooses one of the two operators (indicated as O_1 and O_2) with probability *p* and *1-p* (initial shares) respectively.
(c) The probability *p* can take into account all the factors that may lead the user to choose one of the two operators as his first choice, including the range of services it offers and past experience.

(d) After each failed attempt the user has two choices: he can either abandon (with probability p_A) or switch to the other operator for a new attempt.
(e) Switching between the two operators is performed on a call-by-call basis and depends just on the latest attempts.
(f) During the repeated call attempt process the blocking probability L_1 and L_2 (i.e. the probability that the call attempt through the operator O_1 and O_2 fails) and the probability of abandonment p_A stay constant.

The transition diagram of a behaviour system-I is in Fig. 1.1 is listed here

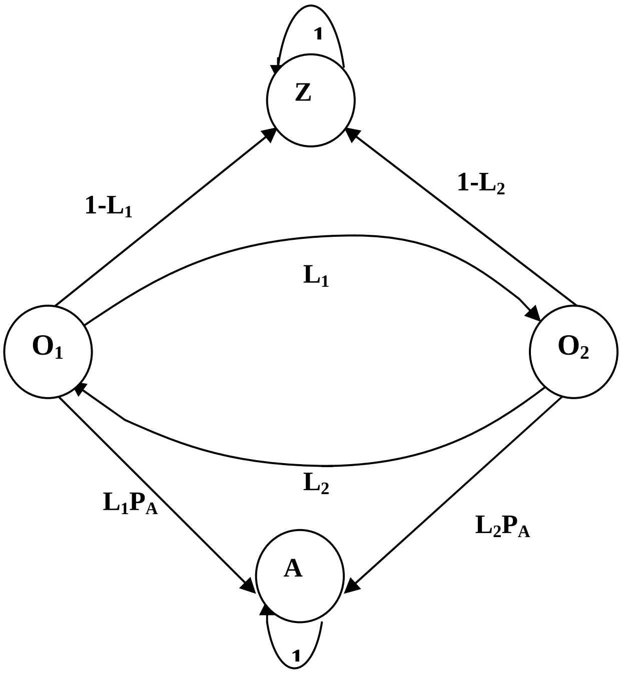

Fig. 1.1

The limitation of system-I by Naldi (2002) is the assumption of connecting attempts on call-by-call basis. If this assumption released a bit then we have another system definition for user's behaviour as described below.

1.2. System-II

(a) The user initially chooses one of the two operators (indicated as O_1 and O_2) with probability *p* and *1-p* (Initial shares) respectively.
(b) The probability *p* can take into account all the factors that may lead the user to choose one of the two operators as his first choice, including the range of services it offers and past experience.
(c) After each failed attempt the user has two choices: he can either abandon (with probability p_A) or switch to the other operator for a new attempt.
(d) The switching between two operators is on two call basis, which means if call attempt on O_1 is failed then user is allowed to make one more call attempts with O_1, if this also fails them user is to move to O_2 for next attempts. Similar happens for operators O_2.
(e) During the repeated call attempt process the blocking probability L_1 and L_2 (i.e. the probability that the call attempt through the operator O_1 and O_2 fails) and the probability of abandonment p_A stay constant.

The transition diagram of a behaviour system-II is in Fig. 1.2 are

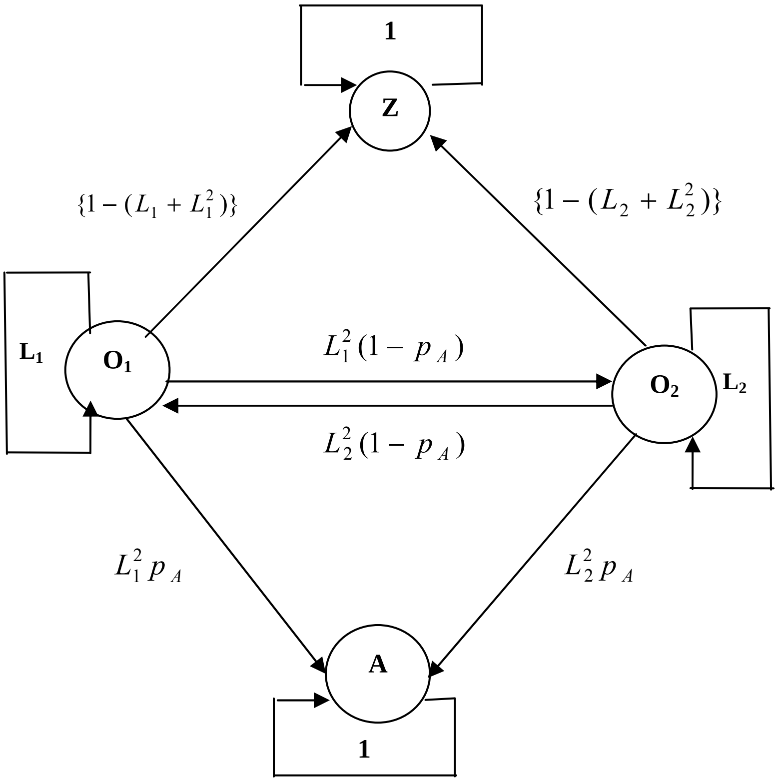

Fig. 1.2

The transition probability matrices are i.e. Fig. 1.3 & Fig. 1.4

System-I

$$
\begin{array}{cc}
 & \text{States} \\
 & X^{(n)} \\
\begin{array}{c}\text{States}\\ X^{(n-1)}\end{array} &
\left[\begin{array}{c|cccc}
 & O_1 & O_2 & Z & A \\
\hline
O_1 & 0 & L_1(1-P_A) & 1-L_1 & L_1P_A \\
O_2 & L_2(1-P_A) & 0 & 1-L_2 & L_2P_A \\
Z & 0 & 0 & 1 & 0 \\
A & 0 & 0 & 0 & 1
\end{array}\right]
\end{array}
$$

Fig. 1.3 (Transition probability matrix for system-I)

System-II

$$\xleftarrow{\qquad\qquad} \text{States} \xrightarrow{\qquad\qquad}$$

$$
\begin{array}{c}
\\
\\
\end{array}
X^{(n)}
$$

$$
\begin{array}{c}
\Big\uparrow \\
\text{States} \\
X^{(n-1)} \\
\Big\downarrow
\end{array}
\left[
\begin{array}{c|cccc}
 & O_1 & O_2 & Z & A \\ \hline
O_1 & L_1 & L_1^2(1-P_A) & \{1-(L_1+L_1^2)\} & L_1^2P_A \\
O_2 & L_2^2(1-P_A) & L_2 & \{1-(L_2+L_2^2)\} & L_2^2P_A \\
Z & 0 & 0 & 1 & 0 \\
A & 0 & 0 & 0 & 1
\end{array}
\right]
$$

Fig. 1.4 (Transition probability matrix for system-II)

Computation of probabilities under Markov chain model in system-I are the starting conditions (state distribution before the first call attempt) are

$$
\begin{aligned}
&P[X^{(0)}=O_1]=P,\\
&P[X^{(0)}=O_2]=1-P,\\
&P[X^{(0)}=Z]=0,\\
&P[X^{(0)}=A]=0,
\end{aligned}
$$

The state probabilities after the first attempt can be obtained by simple relationships:

$$
\left.
\begin{aligned}
&P[X^{(1)}=O_1]_{System-I}=P[X^{(0)}=O_2]P[X^{(1)}=O_1\mid X^{(0)}=O_2]=(1-p)L_2(1-p_A),\\
&P[X^{(1)}=O_2]_{System-I}=P[X^{(0)}=O_1]P[X^{(1)}=O_2\mid X^{(0)}=O_1]=pL_1(1-p_A),
\end{aligned}
\right\}
$$

after unwrapping the recursions obtain the general relationships
for O$_1$

$$
\begin{cases}
P[X^{(n)}=O_1]_{System-I}=p\sqrt{(L_1L_2)^n}.(1-p_A)^n, & n\ even\\
P[X^{(n)}=O_1]_{System-I}=(1-p)L_2\sqrt{(L_1L_2)^{n-1}}.(1-p_A),^n & n\ odd
\end{cases}
$$

for O$_2$

$$
\begin{cases}
P[X^{(n)}=O_2]_{System-I}=(1-p)\sqrt{(L_1L_2)^n}.(1-p_A)^n, & n\ even\\
P[X^{(n)}=O_2]_{System-I}=pL_1\sqrt{(L_1L_2)^{n-1}}.(1-p_A)^n, & n\ odd
\end{cases}
$$

The details of transition probabilities, for n>0, is the system-II are attempts n=0,1,2,3,4,5,... are classified into four different categories A, B, B, C and D like :
The general expressions of probability of n^{th} attempts for O_1 are:

Type A : when t=4n+1, (e.g. t= 1,5,9,13,17,21,....); (n≥0)

$$P\left[X^{(4n+1)} = O_1\right]_{A\ \ for\ system\ -II} = L_1\left[pL_1^{(3n)}L_2^{(3n)}(1-p_A)^{(2n)}\right]$$

Type B : when t=4n+3, (e.g. t= 3,7,11,15,19,23....); (n≥0)

$$P\left[X^{(4n+3)} = O_1\right]_{B\ for\ system\ -II} = \left[(1-p)L_1^{(3n+1)}L_2^{(3n+3)}(1-p_A)^{(2n+1)}\right]$$

Type C : when t=4n, (e.g. t= 0,4,8,12,16,20,.....); (n>0)

$$P\left[X^{(4n)} = O_1\right]_{C\ for\ system\ -II} = \left[pL_1^{(3n)}L_2^{(3n)}(1-p_A)^{(2n)}\right] When \qquad n > 0$$

Type D : when t=4n+2, (e.g. t= 2,6,10,14,18,22.....); (n>0)

$$P\left[X^{(4n+2)} = O_1\right]_{D\ for\ system\ -II} = \left[(1-p)L_1^{(3n)}L_2^{3(n+1)}(1-p_A)^{(2n+1)}\right] When \qquad n > 0$$

Same as for O_2.

2. Traffic Sharing

Traffic Sharing Difference between “Call-by-Call” and “Two-Call” basis contains following notations.

D_C = Difference due to Call-by-call basis Naldi (2002).

D_T = Difference due to Two-call basis.

Using proposed model of both systems, the expressions for traffic sharing (when n→∞) under system-I are:

$$D_{C1} = \overline{P_1} = (1-L_1)\left\{\frac{\{P+(1-P)L_2(1-P_A)}{\left[1-L_1L_2(1-P_A)^2\right]}\right\} \quad \text{for operator } O_1$$

$$D_{C2} = \overline{P_2} = (1-L_2)\left\{\frac{(1-P)+PL_1(1-P_A)}{\left[1-L_1L_2(1-P_A)^2\right]}\right\} \quad \text{for operator } O_2$$

Similar expression of traffic share under system-II are :

$$D_{T1} = \overline{P_1} = \left\{\frac{\{1-(L_1+L_1^2)\}(1+L_1)}{\left[1-L_1^3L_2^3(1-P_A)^2\right]L_1^3(1-P_A)}\right\}\left[PL_1^3(1-P_A)+(1-P)L_1^3L_2^3(1-P_A)^2\right] \quad \text{for operator } O_1$$

$$D_{T2} = \overline{P_2} = \left\{\frac{\{1-(L_2+L_2^2)\}(1+L_2)}{\left[1-L_1^3L_2^3(1-P_A)^2\right]L_2^3(1-P_A)}\right\}\left[(1-P)L_2^3(1-P_A)+PL_1^3L_2^3(1-P_A)^2\right] \quad \text{for operator } O_2$$

While comparing both systems I and II only first operator O1, the numerical difference between traffic sharing is:

$$D_{ifference} = D_{T1} - D_{C1} = p\left[\left\{\frac{\{1-(L_1+L_1^2)\}(1+L_1)}{\left[1-L_1^3L_2^3(1-P_A)^2\right]}\right\} - \left\{\frac{(1-L_1)}{\left[1-L_1L_2(1-P_A)^2\right]}\right\}\right]$$

$$+(1-p)L_2(1-P_A)\left[\left\{\frac{\{1-(L_1+L_1^2)\}(1+L_1)L_2^2}{\left[1-L_1^3L_2^3(1-P_A)^2\right]}\right\} - \left\{\frac{\{(1-L_1)}{\left[1-L_1L_2(1-P_A)^2\right]}\right\}\right]$$

3. Share Loss

As per Naldi (2002) and for system – I the share loss expression ΔP_{C1} , for O_1 is:

$$\Delta p_{C1} = \left(p - \overline{P}_1\right) = \frac{p\left[1-L_1L_2(1-p_A)^2\right] - (1-L_1)\left[p+(1-p)(1-p_A)L_2\right]}{1-L_1L_2(1-p_A)^2}$$

$$= \frac{p\{L_1 + L_2(1-p_A)[1-L_1(2-p_A)]\} - L_2(1-L_1)(1-p_A)}{1-L_1L_2(1-p_A)^2}$$

Under system – II (two – call basis) expression of share loss are:

$$\Delta p_{T1} = \left(p - \overline{P}_1\right) = \frac{pL_1^2\{2 + L_1 - L_1L_2^3(1-p_A)^2\} - (1-p)L_2^3(1-p_A)\{1 + L_1^3 + 2L_1^2\}}{\left[1 - L_1^3L_2^3(1-p_A)^2\right]}$$

4. Simulation Study

Fig. 4.1 to 4.5 are showing the graphical pattern of traffic sharing $\overline{P}_1$ of operator O_1 when blocking probability L_1 of O_1 is very (keeping L_2, p, p_A is fixed) by fig 4.1, one can observe that in a system-II the traffic sharing goes down with a faster rate than system-I. After 50% call blocking the traffic share call blocking reaches to nearly at zero level.

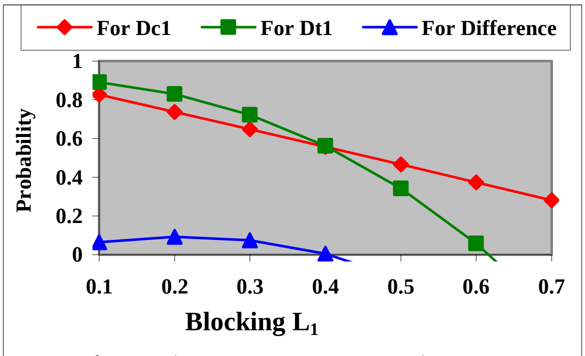

Fig. 4.1 (L_2=0.8, P=0.9, P_A=0.8)

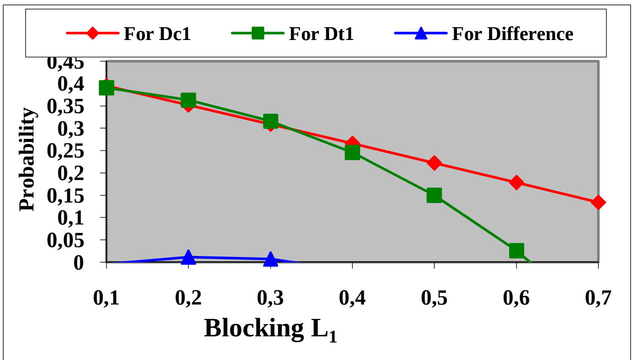

Fig. 4.2. (L_2=0.8, P=0.33, P_A=0.8)

Looking over Fig. 4.2 when p is low (0.33), the similar pattern is found.

While comparing the blocking of opponent, with the increase of L_2, the operator O_1 gains the traffic with relatively slower rate. With reference Fig. 4.4 if the blocking of opponent is high over then the traffic

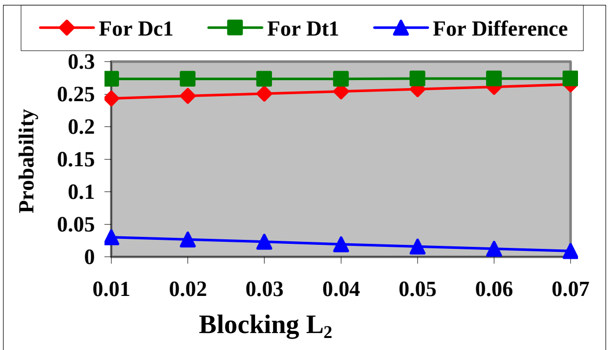

Fig. 4.3. (L_1=0.2, P=0.3, P_A=0.4)

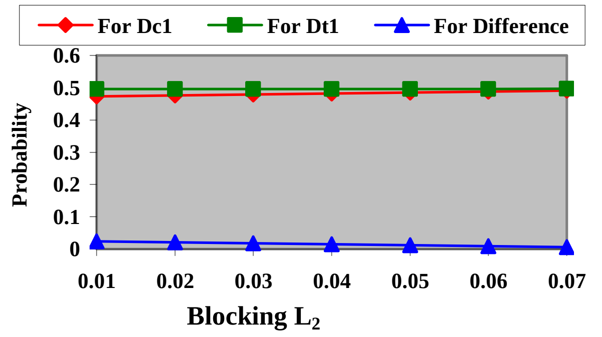

Fig. 4.4. (L_1=0.06, P=0.5, P_A=0.4)

share doesn't change. In other way it is observe that the traffic share is independent of call variance with increasing value of L_2.

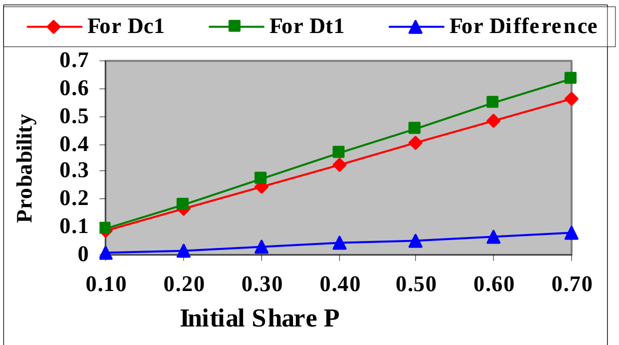

Fig. 4.5. (L_1=0.6, L_2=0.1, P_A=0.9)

The Fig. 4.5 shows the effect of initial market share *p* over both systems. If seems that system-II has little advantages over system-I when *p* is high.

5. Concluding Remarks

Both the systems of user behaviour have shown the little different in traffic sharing because of call difference. The two call based system is not able to bear blocking more than 60 % for operator O_1. The operant blocking, is high provides better traffic share in system-II than system-I for operator O_1. Moreover if initial traffic share is high the system-II reveals more gain in internet traffic than system-I.

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