ISSN 1512-1461

VIEW OF SPACE AND TIME FROM STANDPOINT OF DIFFERENTIAL EQUATIONS

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Abstract
Algebraic theory of a differential equation describing some process, generates proper geometry, field geometry. The localization of group parameters is performed. From the scalar curvature, a single Lagrangian is derived for Maxwell, Yang-Mills, Dirac and Einstein equations for strong gravitation. In this case, in the first approximation there arise standard interaction terms and even mass terms. In subsequent approximations the equations of Maxwell and equations of Dirac become non-linear. As to usual gravitation, though it is involved in the field theory developed in the paper, it has an absolutely different nature than all other fields. The alternative properties of the algebraic theory of differential equations allow us to conclude immediately that all fields must be quantized. An exception is a gravitation field whose quantization is meaningless. The developed theory suggests the existence of the double world. There exists only a gravitational interaction between these worlds, all other interactions are absent.

Keywords:
Test field, Relativism, Unified field theory, Processes and anti-processes, Field quantization.

Introduction

In the present paper we continue to study geometry of space based on properties of motion of field that we started in [1]. In Chapter A of present paper, the observer, who still remains in his process, performs the localization of the parameters of the Lorentz and dynamic groups. Note that the localization is carried out with respect to the points of the space-time as well as with respect to the components of the test field. Because of localization the compensating fields are introduced in the theory with the aid of tetrad formalism. There arises a nontrivial algebraic relationship between the curvature tensor components and the connectedness coefficients.

From the curvature components we construct in a nonstandard manner the scalar curvature and then write the action integral. An approximate calculation of this integral with respect to the internal space brings to the well known Lagrangians of fundamental fields.

In Chapter B, the last one, we investigate the questions arising when the observer passes from one process to the other one. In that case we observe the transformation not only of the frames of reference but also of the systems of calculus. Moreover, the algebro-geometric objects of one process transform to the same objects of the other process. These transformations allow the observer to reveal the gravitational interaction of the processes.

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As shown in [2-3], for each calculus generated by a differential equation there always exists its alternative calculus. Then, along with the geometry constructed in a standard calculus, there appears an alternative geometry. The ignoring of this geometry means the ignoring of certain general properties of motion of real processes that were found thanks to the algebraic properties of differential equations. Hence we come face to face with an alternative field theory and therefore with the antiworld.

A. Compensating Fields

After performing the localization of group (5.1)[1], we construct in a standardmanner the geometry of the curved total space.

1. Group localization

Let some process with the observer inside be given. As shown in the preceding paragraph, in his process the observer finds a system of calculus, a frame of reference and a group acting in the complex of spaces. For simplicity, we assume that the differential equation describing this process is written – in the system of calculus of this process – in form (5.9)[1]. The group acting in the complex of spaces is $\overset{\circ}{G}$.

Assume now that the observer has decided to describe some other process in his system of calculus and frame of reference. For this, he must probe by means of his test field (5.9)[1] the process he wants to investigate. But he can do so if the interaction between his own process and the process to be investigated is realizable. Otherwise he will not see that other process.

It is assumed that the processes are not violated while interacting. They undergo smooth deformation. This means that the process, in which the observer is, preserves locally its internal algebro-geometric structure. In other words, the Lorentz group and the dynamic group with their representations must remain unchanged. But for the interacting processes, this condition is realizable only if the parameters of these groups become functions of the points of the total space. Thus we come to the Yang-Mills idea about the localization of the parameters of the group $\overset{\circ}{G}$.

Using Yang-Mills idea, we perform the localization of the parameters of the group $\overset{\circ}{G}$ with respect to the points of the total space. However, while doing so, we must be careful and keep in mind that the internal space is directly connected with the test field. Test fields are in their turn described by differential equations. The localization of the group actually implies that the algebraic operations in the space of solutions must not be violated locally. It is understood that the same is true for a test field as well. But as follows from transformation (8.10) [4], to preserve these operations, the group acting in the external space must not depend on a solution of the equation, i.e. on the coordinates of the internal space. Therefore the parameters of the group $\overset{\circ}{G}$ have to be replaced by arbitrary infinitesimal functions as follows:

$$\xi^{\nu}_{\sigma} \to \Theta^{\nu}_{\sigma}(x),\ \xi^{\nu} \to \Theta^{\nu}(x),$$

$$\begin{aligned} \xi^{a} &\to \Theta^{a}(x, w, w^{+}) = \Theta^{a}(y), \\ \xi &\to \Theta(x, w, w^{+}) = \Theta^{a}(y). \end{aligned} \tag{1.1}$$

Here we have somewhat ignored the derivation from (8.10) [4] and do not provide the functions $\Theta^{a}(y)$ and $\Theta(y)$ with an additional algebraic structure. The obtained localized group is denoted by G.

ISSN 1512-1461

Let us write explicitly the action of the group G in the total space. From (5.1)[1] and (1.1) we have

$$\overline{x}^{\nu} = x^{\nu} + \Theta^{\nu}_{\sigma}(x)x^{\sigma} + \Theta^{\nu}(x),$$

$$\overline{w}^{k} = w^{k} + T^{k}_{n}w^{n}, \quad (1.2)$$
$$\overline{w}^{+}_{k} = w^{+}_{k} - w^{+}_{n}T^{n}_{k}.$$

Nonzero elements of the matrix T^{A}_{B} have the form

$$T^{\nu}_{\tau} = \Theta^{\nu}_{\tau}(x),$$

$$T^{k}_{n} = \frac{1}{8}\Theta^{\nu}_{\sigma}(x)[\gamma_{\nu}, \gamma^{\sigma}]^{k}_{n} + \Theta^{a}(y)\sigma_{a}{}^{k}_{n} + i\Theta(y)\delta^{k}_{n}, \quad (1.3)$$
$$T^{\overline{k}}_{\overline{n}} = -T^{n}_{k}$$

As follows from (5.7)[1], the standard vector field F^{A} transforms by the rule

$$\overline{F}^{A} = F^{A} + T^{A}_{B}F^{B}. \quad (1.4)$$

As seen from (1.1), unlike the parameters of the Lorentz group, the parameters ξ^{a} and ξ of the group $\overset{\circ}{G}$ are localized not only with respect to the points of the external space, but also with respect to the points of the internal space. Under the action of the group G, transformations of geometric objects contain the parameters of this group. Then the dependence of Θ^{a} and Θ on w, w^{+} can be interpreted as an inclusion of the interaction with the test field. Note that such interactions are of arbitrary form. Let us give more attention to (1.4). From (1.1–4) it follows that the field F is a function of the points of the total space. If F^{A} is assumed to be an analytic function of w, w^{+}, then we can write

$$F^{A}(y) = F^{A}(x) + F^{A}_{k}(x)w^{k} + F^{A}_{\overline{k}}(x)w^{+}_{k} + F_{k}{}^{A}_{n}(x)w^{k}w^{n} + \cdots \quad (1.5)$$

It is appropriate to remind that the obtained geometry is constructed on the basis of the field motion logic. This geometry completely reflects all the properties of the test field. Then from equality (1.5) we conclude that the field $F(y)$ is the superposition of the fields $F^{A}(x)$, $F^{A}_{B}(x)$, $F^{A}_{F}(x), \ldots$ interacting with test fields.

If in (1.2) the parameters $\Theta^{a}(y)$ and $\Theta(y)$ are expanded into powers of w and w^{+}, then from (1.2), (1.4–5) we can obtain the transformation rule for fields $F^{A}(x)$, $F^{A}_{k}(x), \ldots$.

From (5.6)[1] we find the transformation rule for the dynamic vector field Φ^{A}:

$$\overline{\Phi}^{A} = \Phi^{A} + T^{A}_{B}\Phi^{B} + \Theta^{a}(y)h_{a}{}^{A}_{B}\Phi^{B}. \quad (1.6)$$

2. A covariant derivative for standard vectors

In this paragraph we will ignore dynamic vectors and consider the action of the group G realized by transformation (1.2), (1.4).

(1) To the vector F^{A} we apply the operator $\partial/\partial y^{A}$. Assuming that under the action of the group G the derivatives of F^{A} obey the tensor rule of transformation, we come by a standard technique to the compensating derivatives

$$\widetilde{\nabla}_{A}F^{B} = \frac{\partial F^{B}}{\partial y^{A}} + \widetilde{\Gamma}_{A}{}^{B}_{C}F^{C}. \quad (2.1)$$

The compensating field $\widetilde{\Gamma}_{A}(= \Gamma_{A}{}^{B}_{C})$ is extended to the Lie algebra of group (1.2). Then nonzero matrix elements $\widetilde{\Gamma}_{A}$ take the form

$$\widetilde{\Gamma}_{A}{}^{\nu}_{\tau} = \widetilde{\Gamma}_{A}{}^{\nu}_{\tau},$$

ISSN 1512-1461

$$\widetilde{\Gamma}_{A}{}_{n}^{k} = \frac{1}{8}\widetilde{\Gamma}_{A}{}_{\tau}^{\nu}[\gamma_{\nu}, \gamma^{\tau}]_{n}^{k} + \widetilde{B}_{A}^{a}\sigma_{a}{}_{n}^{k} + i\widetilde{\varphi}_{A}\delta_{n}^{k}, \tag{2.2}$$

$$\widetilde{\Gamma}_{A}{}_{\overline{n}}^{\overline{k}} = -\widetilde{\Gamma}_{A}{}_{k}^{n}.$$

Under the action of the group G, the compensating derivatives (2.1) undergo transformation according to the rule

$$\overline{\widetilde{\nabla}}_{A}\overline{F}^{B} = \widetilde{\nabla}_{A}F^{B} - \frac{\partial Q^{E}}{\partial y^{A}}\widetilde{\nabla}_{E}F^{B} + T_{E}^{B}\widetilde{\nabla}_{A}F^{E}, \tag{2.3}$$

where T_{B}^{A} is matrix (1.3). Transformation (1.2) can be shortened to

$$\overline{y}^{A} = y^{A} + Q^{A}(y).$$

From (1.4) and (2.1-3) we easily find the transformation rule for compensating fields

$$\overline{\widetilde{\Gamma}}_{A}{}_{\tau}^{\nu} = \widetilde{\Gamma}_{A}{}_{\tau}^{\nu} + \Theta_{\sigma}^{\nu}\widetilde{\Gamma}_{A}{}_{\tau}^{\sigma} - \Theta_{\tau}^{\sigma}\widetilde{\Gamma}_{A}{}_{\sigma}^{\nu} - \frac{\partial Q^{B}}{\partial y^{A}}\widetilde{\Gamma}_{B}{}_{\tau}^{\nu} - \frac{\partial Q_{\tau}^{\nu}}{\partial y^{A}},$$

$$\overline{\widetilde{B}}_{A}^{a} = \widetilde{B}_{A}^{a} + C_{bc}^{a}\Theta^{b}\widetilde{B}_{A}^{c} - \frac{\partial Q^{B}}{\partial y^{A}}\widetilde{B}_{B}^{a} - \frac{\partial \Theta^{a}}{\partial y^{A}}, \tag{2.4}$$

$$\overline{\widetilde{\varphi}}_{A} = \widetilde{\varphi}_{A} - \frac{\partial Q^{B}}{\partial y^{A}}\widetilde{\varphi}_{B} - \frac{\partial \Theta}{\partial y^{A}},$$

where $c_{a}{}_{c}^{b}$ are the structural constants of the dynamic group.

Assume that

$$\widetilde{\Gamma}_{A}{}_{\tau}^{\nu}\overset{\circ}{g}_{\nu\sigma} + \widetilde{\Gamma}_{A}{}_{\sigma}^{\nu}\overset{\circ}{g}_{\tau\nu} = 0. \tag{2.5}$$

Since $\Theta_{\tau}^{\nu}\overset{\circ}{g}_{\tau\nu} + \Theta_{\sigma}^{\nu}\overset{\circ}{g}_{\tau\nu} = 0,$ condition (2.5) does not contradict transformation (2.4). Then it is not difficult to verify that there holds

$$\widetilde{\Gamma}_{AC}^{E}\overset{\circ}{g}_{EB} + \widetilde{\Gamma}_{AB}^{E}\overset{\circ}{g}_{CE} = 0, \tag{2.6}$$

where $\overset{\circ}{g}_{AB}$ is the metric tensor (1.21)[1].

From (2.3) it immediately follows that the following equality is true:

$$\widetilde{\nabla}_{A}T_{C}^{B} = 0 \tag{2.7}$$

(2) Let us form the operator $dy^{A}\widetilde{\nabla}_{A}$ which we can be called the operator of an absolute differential. Indeed, applying this operator to standard objects (a scalar, a vector, a tensor), we obtain the corresponding absolute differential of this object, since it obeys the transformation rule when (1.2–4) and (2.3) are fulfilled.

From (1.2) it follows that the coordinates w and w^{+} of the internal space transform as components of a standard vector, i.e. the tensor property of the test field is not violated. Hence we can apply to them the operator of an absolute differential. As a result we obtain

$$D^{k} = dy^{A}\widetilde{\nabla}_{A}w^{k} = dw^{k} + dy^{A}\widetilde{\Gamma}_{A}{}_{n}^{k}w^{n}, \tag{2.8}$$

$$D_{k}^{+} = dy^{A}\widetilde{\nabla}_{A}w_{k}^{+} = dw_{k}^{+} - dy^{A}\widetilde{\Gamma}_{A}{}_{n}^{k}w_{n}^{+},$$

which in the sequel will be called canonical differentials. As will be seen below, this property of the internal space strongly affects the structure of the arisen curved geometry.

(3) Along with (2.8), we introduce the canonical differential

$$D^{\nu} = dx^{\nu} + dy^{A}\omega_{A}^{\nu} \tag{2.9}$$

and assume that under the action of the group G it transforms by the rule

$$\overline{D}^{\nu} = D^{\nu} + \Theta_{\sigma}^{\nu}D^{\sigma}. \tag{2.10}$$

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From (2.9–10) it is not difficult to find the transformation rule for ω_A^ν :

$$\overline{\omega}_A^\nu = \omega_A^\nu + \Theta_\sigma^\nu \omega_A^\sigma - \frac{\partial Q^B}{\partial y^A}\omega_B^\nu - \frac{\partial \Theta_\tau^\nu}{\partial y^A}x^\tau - \frac{\partial \Theta^\nu}{\partial y^A}. \tag{2.11}$$

Let us consider the transformation of ω_k^ν and $\omega_{\overline{k}}^\nu$. With (1.1) taken into account, from (2.11) we obtain

$$\overline{\omega}_\xi^\nu = \omega_\xi^\nu + \Theta_\sigma^\nu \omega_\xi^\sigma - \frac{\partial Q^\eta}{\partial \omega \xi}\omega_\eta^\nu, \tag{2.12}$$

where ξ, η runs through the indexes and overlined indexes of the internal space. This transformation implies that ω_ξ^ν is not a compensating field and if in some system of coordinates $\omega_\xi^\nu = 0$, then under the action of group G this object remains zero. Therefore in the sequel it will be assumed that

$$\omega_k^\nu = 0, \quad \omega_{\overline{k}}^\nu = 0. \tag{2.13}$$

In that case, the transformation rule for ω_τ^ν is

$$\overline{\omega}_\tau^\nu = \omega_\tau^\nu + \Theta_\sigma^\nu \omega_\tau^\sigma - \frac{\partial Q^\sigma}{\partial x^\tau}\omega_\sigma^\nu - \frac{\partial \Theta_\sigma^\nu}{\partial x^\tau}x^\sigma - \frac{\partial \Theta^\nu}{\partial x^\tau}, \tag{2.14}$$

where $Q^\sigma = \Theta_\nu^\sigma(x)x^\nu + \Theta^\sigma(x)$. By virtue of this reasoning we assume that

$$\omega_\tau^\nu = \omega_\tau^\nu(x). \tag{2.15}$$

As seen from (1.1) and (2.14), under the action of the group G the dependence of ω_τ^ν only on x remains. Then the canonical differential (2.9) stops to be influenced by the algebro-geometrical structure of the internal space, which is in complete agreement with the ideology of transformation (8.10) from [4] and (1.1).

(4) Let us introduce tetrads

$$e_A^\nu = \delta_A^\nu + \omega_A^\nu,$$

$$\begin{aligned} e_A^k &= \delta_A^k + \widetilde{\Gamma}_{A}{}_n^k w^n, \\ e_A^{\overline{k}} &= \delta_A^{\overline{k}} - \widetilde{\Gamma}_{A}{}_k^n w_n^+, \end{aligned} \tag{2.16}$$

where δ_A^B is the Kronecker symbol and ω_A^ν has form (2.13) and (2.15). Using (2.16), the canonical differential (2.8-9) can be written in the form

$$D^A = e_B^A dy^B. \tag{2.17}$$

Now we introduce the operators

$$\nabla_A = \widetilde{e}_A^B \widetilde{\nabla}_B, \tag{2.18}$$

where

$$\widetilde{e}_B^A e_C^B = \delta_C^A, \tag{2.19}$$

which in the sequel will be called canonical compensating derivatives. From (2.16) and (2.19) we obtain

$$\begin{aligned} \widetilde{e}_A^k &= \delta_A^k - \Gamma_{A}{}_n^k w^n, \\ \widetilde{e}_A^{\overline{k}} &= \delta_A^{\overline{k}} + \Gamma_{A}{}_k^n w_n^+, \end{aligned} \tag{2.20}$$

where we have introduced the notation

$$\Gamma_{A}{}_C^B = \widetilde{e}_A^E \widetilde{\Gamma}_{E}{}_C^B. \tag{2.21}$$

At that,

$$\Gamma_{A}{}_n^k = \frac{1}{8}\Gamma_{A}{}_\tau^\nu [\gamma_\nu, \gamma^\tau]_n^k + B_A^a \sigma_{a}{}_n^k + i\varphi_A \delta_n^k,$$

$$\Gamma_{A}{}_\tau^\nu = \widetilde{e}_A^B \widetilde{\Gamma}_{B}{}_\tau^\nu, \tag{2.22}$$

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$$B_A^a = \widetilde{e}_A^B \widetilde{B}_B^a,$$
$$\varphi_A = \widetilde{e}_A^B \widetilde{\varphi}_B.$$

The object $\Gamma_{A\tau}^{\ \nu}$ will be defined later. As to B_A^a and φ_A, these fields are irreducible and therefore their definition does not need further improvement.(2.18) contains the operators

$$\widehat{D}_A = \widetilde{e}_A^B \frac{\partial}{\partial y^B}, \tag{2.23}$$

which we call canonical operators.

Taking (2.4) and (2.11) into account, under the action of the group G it is not difficult to derive the transformation rules for D^A and $\widehat{D}_A$:

$$\overline{D}^A = D^A + T_B^A D^B, \tag{2.24}$$
$$\overline{\widehat{D}}_A = \widehat{D}_A + T_A^B \widehat{D}_B,$$

where T_B^A is (1.3).

If in the dynamic space we introduce the bases D^A and $\widehat{D}_A$, which we call canonical, then we obtain transformation (1.4) for standard vectors. On the other hand, in the same space we can introduce the standard bases dy^A and $\partial/\partial y^A$. Then the tensor values written in these bases transform by means of the matrix

$$\delta_B^A + \frac{\partial Q^A}{\partial y^B},$$

where $\overline{y}^A = y^A + Q^A(y)$ is (1.2) written in the shortened form. Thus the tetrads e_B^A realize the transition from a standard basis to a canonical one, while $\widetilde{e}_B^A$ is an inverse transformation. Note that e_B^A transform with respect to the upper index, while $\widetilde{e}_B^A$ with respect to the lower index by rule (2.24), and with respect to the remaining indexes by the rule of a standard basis:

$$\overline{e}_B^A = e_B^A + T_C^A e_B^C - \frac{\partial Q^C}{\partial y^B} e_C^A, \tag{2.25}$$
$$\overline{\widetilde{e}}_{B^A} = \widetilde{e}_{B^A} - T_B^C \widetilde{e}_C^A + \frac{\partial Q^A}{\partial y^C} e_B^C.$$

In view of (2.20) and (2.25), we can rewrite rule (2.4), by which compensating fields transform in the canonical basis, as follows:

$$\overline{\Gamma}_{A\sigma}^{\ \tau} = \Gamma_{A\sigma}^{\ \tau} - T_A^B \Gamma_{B\sigma}^{\ \tau} + \Theta_\nu^\tau \Gamma_{A\sigma}^{\ \nu} - \Theta_\sigma^\nu \Gamma_{A\nu}^{\ \tau} - \widehat{D}_A \Theta_\sigma^\tau,$$

$$\overline{B}_A^a = B_A^a - T_A^B B_B^a + C_{bc}^{\ a} \Theta^b B_A^c - \widehat{D}_A \Theta^a, \tag{2.26}$$
$$\overline{\varphi}_A = \varphi_A - T_A^B \varphi_B - \widehat{D}_A \Theta,$$

where $C_{bc}^{\ a}$ are structural constants from (4.13)[1].

(5) It is not difficult to verify that the following equalities are valid:

$$\nabla_A w^k = \delta_A^k,$$

$$\nabla_A w_k^+ = \delta_A^{\overline{k}}, \tag{2.27}$$
$$\nabla_A \gamma^\nu = 0,$$

where δ_B^A is the Kronecker symbol and γ^ν is the Dirac matrix. Note that in the case of differentiation of γ^ν, it is represented as a tensor of third rank, i.e. $\gamma^{AB}_{\ \ C}$.

Since the matrices $h_a(w, w^+)$ of the dynamic group are constructed by means of w, w^+ and γ^ν, it is

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not difficult to verify by (2.27) that

$$\nabla_A h_a = \frac{\partial h_{a c}^{\ B}}{\partial y^A}. \tag{2.28}$$

Taking into account equalities (2.6) and (2.27) we easily find

$$\nabla_A \overset{\circ}{g}_{BC} = \frac{\partial \overset{\circ}{g}_{BC}}{\partial y^A}, \tag{2.29}$$

where $\overset{\circ}{g}_{AB}$ is the metric tensor (1.21)[1].

3. Consistency with a metric

We continue to investigate the properties of a covariant derivative acting on standard vectors.

(1) For the consistency with (4.5)[1] we introduce the metric in the curved space in the canonical basis

$$Ds^2 = \overset{\circ}{g}_{\nu\tau} D^\nu D^\tau + 2\frac{H}{\rho} D_k^+ D^k, \tag{3.1}$$

where D^A is (2.17).

We introduce the covariant derivative

$$\overset{\circ}{\nabla}_A F^B = \nabla_A F^B + \overset{\circ}{m}{}_{AC}^{B} F^C, \tag{3.2}$$

where F^A is a standard vector field and $\overset{\circ}{m}{}_{AC}^{B}$ are Christoffel coefficients (6.6)[1].

With (2.29) taken into account it is easy to verify that the following equality is valid:

$$\overset{\circ}{\nabla}_A \overset{\circ}{g}_{BC} = 0. \tag{3.3}$$

This means that the covariant derivative (3.2) is consistent with metric (3.1).

(2) It is obvious that in the standard basis a metric tensor has the form

$$\widetilde{g}_{AB} = \overset{\circ}{g}_{EL} e_A^E e_B^L, \tag{3.4}$$

where $\overset{\circ}{g}_{AB}$ is a metric tensor in the canonical basis and e_B^A is tetrad (2.16).

Let us introduce the covariant derivative in the standard basis

$$\overset{\circ}{\widetilde{\nabla}}_A \widetilde{F}_B = \frac{\partial \widetilde{F}^B}{\partial y^A} + T_{AC}^B \widetilde{F}^C. \tag{3.5}$$

We require the fulfillment of the equality

$$\overset{\circ}{\widetilde{\nabla}}_A \overset{\circ}{g}_{BC} = 0. \tag{3.6}$$

A solution of equation (3.6) with respect to T_{AC}^B has the form

$$T_{AC}^B = \overset{\circ}{T}{}_{AC}^{B} + \widetilde{M}_{AC}^B, \tag{3.7}$$

where $\overset{\circ}{T}{}_{AC}^{B}$ are Christoffel coefficients in the standard basis

$$\overset{\circ}{T}{}_{AC}^{B} = \frac{1}{2}\widetilde{g}^{BE}\left(\frac{\partial \widetilde{g}_{AE}}{\partial y^C} + \frac{\partial \widetilde{g}_{EC}}{\partial y^A} - \frac{\partial \widetilde{g}_{AC}}{\partial y^E}\right), \tag{3.8}$$

and $\widetilde{M}_{AC}^B$ satisfies the condition

$$\widetilde{M}_{AB}^E \widetilde{g}_{BC} + \widetilde{M}_{AC}^E \widetilde{g}_{BE} = 0. \tag{3.9}$$

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(3) As has already been said in (4) of Section *2*, the transition from a standard basis to a canonical one and vice versa is realized by the tetrads e^A_B and $\widetilde{e}^A_B$. If $\widetilde{F}^A$ is the vector represented in the standard basis, in the canonical basis it takes the form

$$F^A = e^A_B \widetilde{F}^B. \tag{3.10}$$

Hence, the covariant derivatives (3.2) and (3.5) are related by

$$\widetilde{\overset{\circ}{\nabla}}_A \widetilde{F}^B = e^E_A \widetilde{e}^B_L \overset{\circ}{\nabla}_E F^L. \tag{3.11}$$

Using (2.1), (2.18), (2.21), from equality (3.11) with (3.10) taken into account we obtain

$$\widetilde{e}^B_E \frac{\partial e^E_C}{\partial y^A} + e^L_A \widetilde{e}^B_E e^B_C (\Gamma_L{}^E_F + \overset{\circ}{m}_L{}^E_F) = \overset{\circ}{T}_A{}^B_C + \widetilde{M}^B_{AC}.$$

After lengthy but simple transformations we obtain

$$\Gamma_A{}^B_C - \overset{\circ}{\Gamma}_A{}^B_C = M_A{}^B_C, \tag{3.12}$$

where

$$\overset{\circ}{\Gamma}_A{}^B_C = \frac{1}{2} \overset{\circ}{g}{}^{BE} [\overset{\circ}{g}_{LE} \Gamma_A{}^{-L}_C + \overset{\circ}{g}_{AL} \Gamma_E{}^{-L}_C + \overset{\circ}{g}_{CL} \Gamma_E{}^{-L}_A],$$

$$\Gamma_A{}^{-B}_C = e^B_E (\widehat{D}_A \widetilde{e}^E_C - \widehat{D}_C \widetilde{e}^E_A), \tag{3.13}$$

$$M_A{}^B_C = \widetilde{e}^L_A \widetilde{e}^F_C e^B_E \widetilde{M}_L{}^E_F.$$

By (3.13) it is easy to prove

$$\overset{\circ}{\Gamma}_A{}^E_B \overset{\circ}{g}_{EC} + \overset{\circ}{\Gamma}_A{}^E_C \overset{\circ}{g}_{BE} = 0. \tag{3.14}$$

Using (2.6), (2.21) and (3.14), from (3.12) we obtain

$$M_A{}^E_B \overset{\circ}{g}_{EC} + M_A{}^E_B \overset{\circ}{g}_{BE} = 0.$$

Thus we have shown that (3.12) is consistent with equality (3.9).

(4) From (2.22) it follows that $\Gamma_A{}^k_n$ is expressed in terms of $\Gamma_A{}^\nu_\tau$, where B^a_A and φ_A are irreducible objects. $\overset{\circ}{\Gamma}_A{}^B_C$ is uniquely defined from (3.13) by means of the tetrads e^A_B and $\widetilde{e}^A_B$. But since $M_A{}^B_C$ is a free tensor field, we choose it so that equality (3.12) is fulfilled. An exception from this rule is $M_A{}^\nu_\tau$. As has been said in (4) of Section *2*, $\Gamma_A{}^\nu_\tau$ is the unknown object which needs to be defined. Therefore by (3.12) we can write

$$\Gamma_A{}^\nu_\tau = \overset{\circ}{\Gamma}_A{}^\nu_\tau + M_A{}^\nu_\tau.$$

It is not necessary that $M_A{}^\nu_\tau$ be present as a free tensor field in the discussed theory and that is why it is discarded.

Summarizing our discussion and taking into account (2.13), (2.15–16), from (3.13) we finally obtain

$$\Gamma_\sigma{}^\nu_\tau = \frac{1}{2} \overset{\circ}{g}{}^{\nu\mu} [\overset{\circ}{g}_{\lambda\mu} \Gamma_\sigma{}^{-\lambda}_\tau + \overset{\circ}{g}_{\sigma\lambda} \Gamma_\mu{}^{-\lambda}_\tau + \overset{\circ}{g}_{\tau\lambda} \Gamma_\mu{}^{-\lambda}_\sigma],$$

$$\Gamma_k{}^\nu_\tau = \frac{1}{2} \frac{H}{\rho} \overset{\circ}{g}{}^{\nu\mu} \Gamma_\mu{}^{-\bar{k}}_\tau, \tag{3.15}$$

$$\Gamma_{\bar{k}}{}^\nu_\tau = \frac{1}{2} \frac{H}{\rho} \overset{\circ}{g}{}^{\nu\mu} \Gamma_\mu{}^{-k}_\tau,$$

where h/ρ is the cofactor from metric (3.1). As can be easily noted, $\Gamma_\sigma{}^\nu_\tau$ is expressed only through the tetrads $e^\nu_\tau(x)$ and $\widetilde{e}^\nu_\tau(x)$. For $\Gamma_\sigma{}^\nu_\tau(x)$ expression (3.15) is the well known result for the curved space-time represented in terms of tetrad formalism.

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4. Covariant derivatives for dynamic vectors

In the preceding paragraphs we have constructed a covariant derivative when in the set of spaces the group G acts according to rules (1.2) and (1.4). We have excluded from the consideration dynamic vectors which transform by rule (1.6).

Let us introduce a covariant derivative in the canonical basis for the dynamic vector Φ^A in the form

$$\Delta_A\Phi^B = \widehat{D}_A\Phi^B + K_{A}{}^{B}_{C}\Phi^C, \tag{4.1}$$

where the operator $\widehat{D}_A$ is (2.23).

Under the action of the localized group G the covariant derivative (4.1) represented in the canonical basis transforms by rule (6.2)[1] as follows:

$$\overline{\Delta}_A\overline{\Phi}^B = \Delta_A\Phi^B - T^C_A\Delta_C\Phi^B + (T+h)^B_C\Delta_A\Phi^C, \tag{4.2}$$

where we have introduced the notation

$$h = \Theta^A(y)h_a.$$

From (4.1–2) we find in a standard manner the transformation rule for connectedness coefficients $K_{A}{}^{B}_{C}$:

$$\overline{K}_{A}{}^{B}_{C} = K_{A}{}^{B}_{C} - T^E_A K_{E}{}^{B}_{C} + [T+h, K_A]^B_C - \widehat{D}_A(T+h)^B_C. \tag{4.3}$$

Assume that $K_{A}{}^{B}_{C}$ has the form

$$K_{A}{}^{B}_{C} = \Gamma_{A}{}^{B}_{C} + N_{A}{}^{B}_{C} + m_{A}{}^{B}_{C}, \tag{4.4}$$

where $\Gamma_{A}{}^{B}_{C}$ is (2.22) and (3.15), while $m_{A}{}^{B}_{C}$ is (6.5)[1].

Let us substitute (4.4) into (4.3). Using (2.26) and (6.14)[1] and keeping in mind that transformations are infinitesimal, we obtain

$$\overline{N}_A = N_A - T^C_A N_C + [T+h, N_A] - \widehat{D}_A h + \Theta^a\frac{\partial h_a}{\partial y^A} + [h, \Gamma_A],$$

where $N_A(= N_{A}{}^{B}_{C})$ and $\Gamma_A(= \Gamma_{A}{}^{B}_{C})$. Since

$$\nabla_A h = \widehat{D}_A h + [\Gamma_A, h],$$

where $h = \Theta^a h_a$, we can write

$$\nabla_A(\Theta^a h_a) = \nabla_A\Theta^a \cdot h_a + \Theta^a\nabla_A h_a.$$

On the other hand, $\nabla_A\Theta^a = \widehat{D}_A\,\Theta^a$. By (2.28) we obtain

$$\nabla_A\, h = \widehat{D}_A\Theta^a \cdot h_a + \Theta^a\frac{\partial h_a}{\partial y^a}.$$

Then for N_a we have

$$\overline{N}_a = N_A - T^C_A N_C + [T+h, N_A] - \widehat{D}_A\Theta^a \cdot h_a. \tag{4.5}$$

Assume now that

$$N_A = B^a_A \cdot h_a. \tag{4.6}$$

The substitution of expression (2.26) into $\overline{N}_A = \overline{B}^a_A \cdot \overline{h}a$ gives

$$\overline{N}_A = \overline{B}^a_B\overline{h}_a = B^a_B\overline{h}_a - T^B_A B^a_B h_a + C^a_{bc}\Theta^b B^c_A h_a - \widehat{D}_A\Theta^a \cdot h_a. \tag{4.7}$$

Equating (4.5) and (4.7), we obtain linear homogeneous functions with respect to B^a_A in the right- and left-hand parts. Obtained equality must be identically fulfilled with respect to B^a_A. In that case we obtain

$$\overline{h}_a - ha + C^c_{ba}\Theta^b h_c = [T+h, h_a]. \tag{4.8}$$

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Since

$$[h_a, h_b] = C^c_{ab}\, h_c, \tag{4.9}$$

from (4.8) we find

$$\overline{h}^a = h_a + [T, h_a].$$

This transformation coincides with (6.16)[1]. We have thus come to the conclusion that (4.8) is identically fulfilled. This means that assumption (4.6) is not contradictory.

If we substitute (4.6) and (4.4) into (4.1), then the covariant derivative extended to dynamic vectors takes the final form

$$\Delta_A \Phi^B = \overline{D}_A \Phi^B + (\Gamma_{A\,C}^{\,B} + B^a_A h_{a\,C}^{\,B} + m_{A\,C}^{\,B})\Phi^C. \tag{4.10}$$

It is obvious that (4.10) can be written as follows:

$$\Delta_A \Phi^B = \nabla_A \Phi^B + (B^a_A h_a + m_A)^B_C \Phi^C. \tag{4.11}$$

From (4.10) it immediately follows that the introduction of dynamic vectors does not bring about the formation of new compensating fields.

Thus, as follows from (2.20), (2.22) and (3.15), $\omega^\nu_\tau(x)$, $B^a_A(x, w, w^+)$, $\varphi_A(x, w, w^+)$ are independent fields in the obtained geometry. All other geometric objects are expressed through them.

5. Curvature tensor

Let us rewrite the covariant derivative (4.10) in the invariant form [7]:

$$\nabla_X Y = X^A \Delta_A Y^B \cdot \widehat{Y}_B, \tag{5.1}$$

where $Y^A, \widehat{Y}_A$ are self-conjugate dynamic objects, i.e. if Y^A transforms by rule (1.6), then for $\widehat{Y}_A$ we have

$$\overline{\widehat{Y}}_A = \widehat{Y}_A - (T + \Theta^a h_a)^B_A \widehat{Y}_B. \tag{5.2}$$

As to X^A in (5.1), according to (3) of Section *5* from [1], the vector X^A is only a standard one.

(1) The invariant form of curvature is defined in the form [7]

$$R(X, Y)Z = [\nabla_X, \nabla_Y]Z - \nabla_{[X,Y]} Z. \tag{5.3}$$

From (5.1) it immediately follows that in (5.3) X, Y are standard vectors, while Z is a dynamic vector. Using (5.1), equality (5.3) allows us to write the components of the curvature tensor

$$R_{AB} = \widehat{D}_A E_B - \widehat{D}_B E_A + [E_A, E_B] - \Gamma_{A\,B}^{-E} E_E, \tag{5.4}$$

where E_A are matrices with the following elements:

$$E_{A\,C}^{\,L} = \Gamma_{A\,C}^{\,L} + B^a_A h_{a\,C}^{\,L} + m_{A\,C}^{\,L}. \tag{5.5}$$

At that, in (5.4) the object $\Gamma_{A\,B}^{-E}$ is defined from (3.13), while the operator $\widehat{D}_A$ — from (2.23).

Using (1.4), (1.6) and (5.1-2), from (5.3) we easily find the transformation rule for the curvature tensor under the action of the group G :

$$\overline{R}_{AB} = R_{AB} - T^C_A R_{CB} - T^C_B R_{AC} + [T + \Theta^a h_a, R_{AB}], \tag{5.6}$$

where $R_{AB}\ (= R_{AB\,E}^{\ \ C})$.

(2) As has been noted, in (5.1) X is a standard vector and Y is a dynamic vector. In that case, $\nabla_Y X$ is meaningless. Hence we conclude that in this geometry there exists no torsion of the space which, as is known, is defined in the invariant form as follows:

$$\nabla_X Y - \nabla_Y X - [X, Y].$$

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(3) If in (5.5) we discard the second and the third term, then from (5.4) we obtain

$$\overset{\circ}{R}_{AB}{}^{L}_{C} = \widehat{D}_A \Gamma_B{}^{L}_{C} - \widehat{D}_B \Gamma_A{}^{L}_{C} + [\Gamma_A, \Gamma_B]^{L}_{C} - \Gamma_A{}^{-E}_{B} \Gamma_E{}^{L}_{C}, \tag{5.7}$$

which we call the truncated curvature tensor. Since the matrix Γ_A is extended to the algebra of matrix (1.3), it is easy to find nonzero elements of the matrix $\overset{\circ}{R}_{AB}$:

$$\overset{\circ}{R}_{AB}{}^{\nu}_{\tau} = \widehat{D}_A \Gamma_B{}^{\nu}_{\tau} - \widehat{D}_B \Gamma_A{}^{\nu}_{\tau} + \Gamma_A{}^{\nu}_{\sigma} \Gamma_B{}^{\nu}_{\tau} - \Gamma_B{}^{\nu}_{\sigma} \Gamma_A{}^{\nu}_{\tau} - \Gamma_A{}^{-E}_{B} \Gamma_E{}^{\nu}_{\tau},$$

$$\overset{\circ}{R}_{AB}{}^{k}_{n} = \frac{1}{8} \overset{\circ}{R}_{AB}{}^{\nu}_{\tau} [\gamma_\nu, \gamma^\tau]^{k}_{n} + G_{AB}{}^{k}_{n} + iF_{AB} \delta^{k}_{n}, \tag{5.8}$$

$$\overset{\circ}{R}_{AB}{}^{\overline{k}}_{\overline{n}} = -\overset{\circ}{R}_{AB}{}^{n}_{k},$$

where

$$G_{AB} = \widehat{D}_A B_B - \widehat{D}_B B_A + [B_A, B_B] - \Gamma_A{}^{-L}_{B} B_L, \tag{5.9}$$

$$F_{AB} = \widehat{D}_A \varphi_B - \widehat{D}_B \varphi_A - \Gamma_A{}^{-L}_{B} \varphi_L.$$

Matrices B_A have the form

$$B_A = B^a_A \sigma_a.$$

Using the commutation relations for the representation of a dynamic group

$$[\sigma_a, \sigma_b] = C_{ab}^{\;c} \sigma_c, \tag{5.10}$$

where σ_a are representation generators, the matrix G_{AB} can be rewritten as

$$G_{AB} = G_A{}^{a}_{B} \sigma_a, \tag{5.11}$$

$$G_A{}^{a}_{B} = \widehat{D}_A B^a_B - \widehat{D}_B B^a_A + C_{bc}^{\;a} B^b_A B^c_B - \Gamma_A{}^{-E}_{B} B^a_E.$$

(4) Let us investigate the object Γ^{-C}_{AB} which is defined from (3.13) by means of tetrads. With (2.13–16) taken into account it is not difficult to show that

$$\Gamma_\sigma{}^{-\nu}_{\mu} = e^\nu_\tau (\widehat{D}_\sigma \widetilde{e}^\tau_\mu - \widehat{D}_\mu \widetilde{e}^\tau_\sigma) \Gamma_k{}^{-\nu}_{A} = 0, \tag{5.12}$$

$$\Gamma_{\overline{k}}{}^{-\nu}_{A} = 0.$$

These equalities immediately imply

$$\Gamma_\sigma{}^{-\nu}_{\mu} = \Gamma_\sigma{}^{-\nu}_{\mu}(x). \tag{5.13}$$

Now let us consider $\Gamma_A{}^{-k}_{B}$. After simple calculations, from (2.16) and (2.20) we find

$$\Gamma^{-\overline{k}}_{AB} = -\overset{\circ}{R}{}^{k}_{ABn} w^n + \Gamma^k_{An} \delta^n_B - \Gamma^k_{Bn} \delta^n_A, \tag{5.14}$$

$$\Gamma_A{}^{-\overline{k}}_{B} = \overset{\circ}{R}_{AB}{}^{n}_{k} w^+_n - \Gamma_A{}^{n}_{k} \delta^{\overline{n}}_B + \Gamma_B{}^{n}_{k} \delta^{\overline{n}}_A,$$

where δ is the Kronecker symbol.

Thus equality (5.8) with (5.12–14) taken into account is the defining equation for components of the truncated curvature tensor $\overset{\circ}{R}_{AB}$.

(5) Let us consider the total curvature (5.4). Substituting (5.5) into (5.4) and using (4.9), expression (5.4–5) can be rewritten as

$$\begin{aligned} R_{AB} = \overset{\circ}{R}_{AB} &+ G^a_{AB} h_a + B^a_B \left(\widehat{D}_A h_a + [\Gamma_A, h_a] + [m_a, h_a] \right) \\ &- B^a_A \left(\widehat{D}_B h_a + [\Gamma_B, h_a] + [m_B, h_a] \right) \\ &+ \left[\left(\widehat{D}_A m_B + [\Gamma_A, m_B] \right) - \left(\widehat{D}_B m_A + [\Gamma_B, m_A] \right) + [m_A, m_B] - \Gamma_A{}^{-E}_{B} m_E \right]. \end{aligned} \tag{5.15}$$

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Since $\widehat{D}_A h_a + [\Gamma_A, h_a] = \nabla_A h_a$, we obtain with (2.28) and (6.3)[1] taken into account

$$\nabla_A h_a + [m_a, h_a] = 0.$$

It should be noted that from (6.6)[1] and (6.13)[1] we immediately have

$$m_{\nu}{}_B^A = 0.$$

Then, using (5.14) and (2.22), we obtain

$$\Gamma_{A}{}_B^{-E} m_E = -\overset{\circ}{R}_{AB}{}_n^k w^n m_k + \overset{\circ}{R}_{AB}{}_k^n w_n^+ m_{\overline{k}} + \Gamma_{A}{}_B^C m_C - \Gamma_{A}{}_B^C m_C. \tag{5.16}$$

From (6.15)[1], (1.2) and (1.5) it follows that $m_{A}{}_C^B$ is a tensor value. Since

$$\nabla_A m_B = \frac{\partial m_B}{\partial y^A},$$

from (5.15) with (5.16) taken into account, for components of the complete curvature tensor we finally obtain

$$R_{AB}{}_E^C = \overset{\circ}{R}_{AB}{}_E^C + G^a_{AB} h_a{}_E^C + \overset{\circ}{R}_{AB}{}_n^k w^n m_k{}_E^C - \overset{\circ}{R}_{AB}{}_k^n w_n^+ m_{\overline{k}}{}_E^C + r_{AB}{}_E^C, \tag{5.17}$$

where

$$r_{AB}{}_E^C = \frac{\partial m_B{}_E^C}{\partial y^A} - \frac{\partial m_A{}_E^C}{\partial y^B} + [m_A, m_B]_E^C. \tag{5.18}$$

It is obvious that (5.18) is a space curvature tensor provided that the parameters of acting continuous groups are not localized.

6. An action integral

The aim of our further investigation is to find out what aspects of the physical theory of a field can be explained by the geometric theory expounded here. We will follow the scheme proposed by D. Hilbert. Namely, first we compose the scalar curvature by means of the curvature tensor and then construct the action integral.

(1) From (5.6) it immediately follows that the curvature tensor $R_{AB}{}_E^C$ transforms – with respect to the indexes A and B – by the rules of a standard vector (see Section 5 from [1]) and – with respect to the indexes C and E – by the rules of a dynamic vector. This means that the scalar curvature cannot be constructed by a standard technique because $R_{AB}{}_C^A \overset{\circ}{g}{}^{BC}$ is noninvariant with respect to the dynamic group.

Let us compose a scalar curvature in the form

$$R = R_{AB}{}_E^C a_C^B a_L^E \overset{\circ}{g}{}^{AL}. \tag{6.1}$$

In (5) of Section 5 from [1], we have found that the matrix a_B^A changes standard vectors to dynamic ones and, vice versa, dynamic vectors to standard ones at the cost of violation of the dynamic group. We easily find that (6.1) is invariant with respect to the one-parameter groups (3.10)[1] and (3.15)[1], but is noninvariant with respect to the commutator $g_1 g_2 g_1^{-1} g_2^{-1}$, where g_1 and g_2 are the elements of groups (3.10)[1] and (3.15)[1], respectively. Along with this, (5.17)[1] implies the noninvariance of (6.1) with respect to groups (3.11)[1] and (3.14)[1].

Thus the requirement that (6.1) be invariant narrows the group G to the one-parameter but localized groups (3.10)[1] and (3.15)[1]. Moreover, scalar (6.1) does not admit a simultaneous action of these groups. We have the right to put into action only one of them.

These questions certainly require a more comprehensive study. However, at this stage of the investigation our interest lies in mathematical constructions based on the logic of test field motion and the

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algebraic properties of differential equations.

(2) It is not difficult to verify that the sphere $\rho = w^+ w = const$ is an invariant manifold with respect to the group G. In the theory expounded here the internal space is treated as a ball with fixed radius ℓ_0 :

$$\rho = w^+ w \le \ell_0^2. \tag{6.2}$$

(3) Let us consider $\det e_B^A$. Condition (2.13) immediately implies

$$\det e_B^A = \det e_\tau^\nu \cdot \det e_\eta^\xi, \tag{6.3}$$

where ξ, r runs through the indexes and the overlined indexes of the internal space.

(4) Let us compose the integral

$$L(x) = \frac{1}{V_0} \int_{\rho \le \ell_0^2} R(x, w, w^+) \det e_\eta^\xi dV_w, \tag{6.4}$$

where dV_w is an element of the volume of the internal and the conjugate internal space, V_0 is the volume of ball (6.2). By virtue of the reasoning of (3) of Section *5* from [1], we conclude that dV_w is invariant with respect to the dynamic group acting in the dynamic space. From (2.25) we immediately obtain the invariance of $\det e_\eta^\xi$ with respect to the dynamic group. In the sequel, $L(x)$ will be called the Lagrangian.

Integrating (6.4) with respect to some volume V_4 of the external space, we finally obtain the action integral with respect to this volume

$$S = \int_{V_4} L(x) \det e_\sigma^\nu d^4 x. \tag{6.5}$$

(5) As has been shown in Section *4*, the compensating fields $\omega_\tau^\nu(x)$, $B_A^a(x, w, w^+)$ and $\varphi_A(x, w, w^+)$ are independent fields. To simplify our further calculations, we discard B_A^a and the dynamic group. Thus the summand $\widetilde{m}_{A\,C}^{\;B}$ is excluded from (6.5)[1]. Simultaneously, we require that φ_A be dependent only on x :

$$\varphi_A = \varphi_A(x). \tag{6.6}$$

We impose on the group G the condition under which its action on φ_A preserves its independence of w and w^+. This requirement uniquely brings to the condition

$$\Theta = \Theta(x). \tag{6.7}$$

Then (1.3) and (2.26) immediately imply

$$\overline{\varphi}_\nu = \varphi_\nu - \Theta_\nu^\sigma(x)\varphi_\sigma - \widehat{D}_\nu \Theta(x),$$

$$\begin{aligned} \overline{\varphi}_k &= \varphi_k - T_k^n \varphi_n, \\ \overline{\varphi}_{\overline{k}} &= \varphi_{\overline{k}} + T_n^k \varphi_{\overline{n}}, \end{aligned} \tag{6.8}$$

where

$$T_n^k = \frac{1}{8}\Theta_\sigma^\nu(x)[\gamma_\nu, \gamma^\sigma]_n^k + i\Theta(x)\delta_n^k. \tag{6.9}$$

Under requirement (6.6) the group G narrows and in the wake of this narrowing the compensating field $\varphi_k(x)$, $\varphi_{\overline{k}}(x)$ becomes a tensor value. From (6.8) we conclude that $\varphi_\nu(x)$ can be interpreted as potentials of the Maxwell field, while $\varphi_k(x)$, $\varphi_{\overline{k}}(x)$ as a standard spinor field. Hence for φ_k and $\varphi_{\overline{k}}$ we can introduce the standard notation [5]:

$$\begin{aligned} \varphi_k &= \overline{\Psi}_k(x), \\ \varphi_{\overline{k}} &= \Psi^k(x). \end{aligned} \tag{6.10}$$

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7. Calculation of curvature components

We will calculate the curvature components under the assumptions given in (5) of Section *6*.

(1) Let us consider the components $\overset{\circ}{R}_{\sigma\mu}{}^{\nu}_{\tau}$ of the truncated curvature tensor (5.8) which are represented in the form

$$\overset{\circ}{R}_{\sigma\mu}{}^{\nu}_{\tau} = \overset{\circ\circ}{R}_{\sigma\mu}{}^{\nu}_{\tau} - \Gamma_{\sigma}{}^{-k}_{\mu}\Gamma_{k}{}^{\nu}_{\tau} - \Gamma_{\sigma}{}^{-\bar{k}}_{\mu}\Gamma_{\bar{k}}{}^{\nu}_{\tau}, \tag{7.1}$$

where we have introduced the notation

$$\overset{\circ\circ}{R}_{\sigma\mu}{}^{\nu}_{\tau} = \widehat{D}_{\sigma}\Gamma_{\mu}{}^{\nu}_{\tau} - \widehat{D}_{\mu}\Gamma_{\sigma}{}^{\nu}_{\tau} + \Gamma_{\sigma}{}^{\nu}_{\lambda}\Gamma_{\mu}{}^{\lambda}_{\tau} - \Gamma_{\mu}{}^{\nu}_{\lambda}\Gamma_{\sigma}{}^{\lambda}_{\tau} - \Gamma_{\sigma}{}^{-\lambda}_{\mu}\Gamma_{\lambda}{}^{\nu}_{\tau}. \tag{7.2}$$

Object (7.2) depends only on x (because of (3.15)) and is a standard curvature tensor of the external space (space-time) in tetrad formalism.

Using (3.15) and (5.14) and performing simple calculations we obtain

$$\overset{\circ}{R}_{\sigma\mu}{}^{\nu}_{\tau} = \overset{\circ\circ}{R}_{\sigma\mu}{}^{\nu}_{\tau} + \frac{H}{2\rho}\overset{\circ}{g}{}^{\nu\lambda}w^{+}_{n}\{\widehat{\overset{\circ}{R}}_{\lambda\tau}, \widehat{\overset{\circ}{R}}_{\sigma\mu}\}^{n}_{k}w^{k}, \tag{7.3}$$

where $\{A, B\} = AB + BA$. From (5.8) we define $\widehat{\overset{\circ}{R}}_{\lambda\tau}{}^{k}_{n}$,:

$$\widehat{\overset{\circ}{R}}_{AB}{}^{k}_{n} = \frac{1}{8}\overset{\circ}{R}_{AB}{}^{\sigma}_{\tau}[\gamma_{\sigma}, \gamma^{\tau}]^{k}_{n} + iF_{AB}\delta^{k}_{n}. \tag{7.4}$$

From (7.4) it immediately follows that (7.3) is a quadratic equation for the components $\overset{\circ}{R}_{\sigma\mu}{}^{\nu}_{\tau}$.

From (1.1) and (2.26) we find that $\Gamma_{k}{}^{\nu}_{\tau}$ and $\Gamma_{\bar{k}}{}^{\nu}_{\tau}$ are tensor objects. Then, performing transformations analogous to the above ones, we obtain

$$\overset{\circ}{R}_{\sigma k}{}^{\nu}_{\tau} = \nabla_{\sigma}\Gamma_{k}{}^{\nu}_{\tau} + \frac{H}{2\rho}\overset{\circ}{g}{}^{\nu\lambda}w^{+}\{\widehat{\overset{\circ}{R}}_{\lambda\tau}, \widehat{\overset{\circ}{R}}_{\sigma k}\}w,$$

$$\overset{\circ}{R}_{mk}{}^{\nu}_{\tau} = \nabla_{m}\Gamma_{k}{}^{\nu}_{\tau} - \nabla_{k}\Gamma_{m}{}^{\nu}_{\tau} - \Gamma_{m}{}^{\nu}_{\mu}\Gamma_{k}{}^{\mu}_{\tau} + \Gamma_{k}{}^{\nu}_{\mu}\Gamma_{m}{}^{\mu}_{\tau} + \frac{H}{2\rho}\overset{\circ}{g}{}^{\nu\lambda}w^{+}\{\widehat{\overset{\circ}{R}}_{mk}, \widehat{\overset{\circ}{R}}_{\lambda\tau}\}w, \tag{7.5}$$

$$\overset{\circ}{R}_{m\bar{k}}{}^{\nu}_{\tau} = \nabla_{m}\Gamma_{\bar{k}}{}^{\nu}_{\tau} - \nabla_{\bar{k}}\Gamma_{m}{}^{\nu}_{\tau} - \Gamma_{m}{}^{\nu}_{\mu}\Gamma_{\bar{k}}{}^{\mu}_{\tau} + \Gamma_{\bar{k}}{}^{\nu}_{\mu}\Gamma_{m}{}^{\mu}_{\tau} + \frac{H}{2\rho}\overset{\circ}{g}{}^{\nu\lambda}w^{+}\{\widehat{\overset{\circ}{R}}_{m\bar{k}}, \widehat{\overset{\circ}{R}}_{\lambda\tau}\}w.$$

We easily obtain $\overset{\circ}{R}_{\bar{m}\bar{k}}{}^{\nu}_{\tau}$ from $\overset{\circ}{R}_{mk}{}^{\nu}_{\tau}$ by putting bars over m and k. $\widehat{\overset{\circ}{R}}_{mk}{}^{n}_{l}$ and $\widehat{\overset{\circ}{R}}_{m\bar{k}}{}^{n}_{l}$ are defined from (7.4). It immediately follows from (3.15) and (5.4):

$$\Gamma_{k}{}^{\nu}_{\tau} = \frac{H}{2\rho}\overset{\circ}{g}{}^{\nu\mu}\widehat{\overset{\circ}{R}}_{\mu\tau}{}^{n}_{k}w^{+}_{n}, \tag{7.6}$$

$$\Gamma_{\bar{k}}{}^{\nu}_{\tau} = -\frac{H}{2\rho}\overset{\circ}{g}{}^{\nu\mu}\widehat{\overset{\circ}{R}}_{\mu\tau}{}^{k}_{n}w^{n}.$$

(2) In (5.9) let us consider $F_{\nu\tau}$ which we write in the form

$$F_{\nu\tau} = \widehat{D}_{\nu}\varphi_{\tau} - \widehat{D}_{\tau}\varphi_{\nu} - \Gamma_{\nu}{}^{-\sigma}_{\tau}\varphi_{\sigma} - \Gamma_{\nu}{}^{-k}_{\tau}\varphi_{k} - \Gamma_{\nu}{}^{-\bar{k}}_{\tau}\varphi_{\bar{k}}. \tag{7.7}$$

Using (5.14) and notation (6.10), we obtain

$$F_{\nu\tau} = \overset{\circ}{F}_{\nu\tau} + \overline{\Psi}\widehat{\overset{\circ}{R}}_{\nu\tau}w - w^{+}\widehat{\overset{\circ}{R}}_{\nu\tau}\Psi, \tag{7.8}$$

ISSN 1512-1461

where

$$\overset{\circ}{F}_{\nu\tau} = \widehat{D}_\nu\varphi_\tau - \widehat{D}_\tau\varphi_\nu - \Gamma_{\nu\tau}^{-\sigma}\varphi_\sigma. \tag{7.9}$$

If it is assumed that φ_ν are potentials of the electromagnetic field, then $\overset{\circ}{F}_{\nu\tau}$ should be interpreted as a strength component in the same representation.

After inserting (7.4) into the right-hand part of (7.8), we obtain the equality by means of which we define $F_{\nu\tau}$:

$$F_{\nu\tau} = \frac{1}{1 - i(\overline{\Psi}w - w^+\Psi)}[\overset{\circ}{F}_{\nu\tau} + \overline{\Psi}\widetilde{\overset{\circ}{R}}_{\nu\tau}w - w^+\widetilde{\overset{\circ}{R}}_{\nu\tau}\Psi], \tag{7.10}$$

where

$$\widetilde{\overset{\circ}{R}}_{AB}{}^k_n = \overset{\circ}{R}_{AB}{}^\sigma_\mu[\gamma_\sigma, \gamma^\mu]^k_n. \tag{7.11}$$

After simple calculations, from (5.9) we find

$$\begin{aligned}
F_{\nu k} &= \frac{1}{1 - i(\overline{\Psi}w - w^+\Psi)}[\nabla_\nu\overline{\Psi}_k + \overline{\Psi}\widetilde{\overset{\circ}{R}}_{\nu k}w - w^+\widetilde{\overset{\circ}{R}}_{\nu k}\Psi],\\
F_{mk} &= \frac{1}{1 - i(\overline{\Psi}w - w^+\Psi)}[\overline{\Psi}\widetilde{\overset{\circ}{R}}_{mk}w - w^+\widetilde{\overset{\circ}{R}}_{mk}\Psi\\
&\quad + \frac{1}{8}(\Gamma_m{}^\nu_\tau\overline{\Psi}_n[\gamma_\nu, \gamma^\tau]^n_k - \Gamma_k{}^\nu_\tau\overline{\Psi}_n[\gamma_\nu, \gamma^\tau]^n_m)],\\
F_{m\bar{k}} &= \frac{1}{1 - i(\overline{\Psi}w - w^+\Psi)}\\
&\quad\times\left[2i\overline{\Psi}_m\Psi^k + \overline{\Psi}\widetilde{\overset{\circ}{R}}_{m\bar{k}}w - w^+\widetilde{\overset{\circ}{R}}_{m\bar{k}}\Psi + \frac{1}{8}(\Gamma_{\bar{k}}{}^\nu_\tau\overline{\Psi}_n[\gamma_\nu, \gamma^\tau]^n_m + \Gamma_m{}^\nu_\tau[\gamma_\nu, \gamma^\tau]^k_n\Psi^n)\right].
\end{aligned} \tag{7.12}$$

It is obvious that (7.3–12) are equations for the curvature components $\overset{\circ}{R}_{AB}{}^\nu_\tau$ and F_{AB}.

(3) We will solve system (7.3–12) approximately, assuming that the constant H is a sufficiently small value. Besides, it is assumed that the spinor fields Ψ and $\overline{\Psi}$ are values of order H^ε, where $\varepsilon > 0$. Applying the method of successive approximations and eliminating summands of higher than first order with respect to H, we obtain

$$\begin{gathered}
\overset{\circ}{R}_{\sigma\mu}{}^\nu_\tau = \overset{\circ\circ}{R}_{\sigma\mu}{}^\nu_\tau + \frac{H}{2\rho}\overset{\circ}{g}{}^{\nu\lambda}w^+\{\widetilde{R}_{\lambda\tau}, \widetilde{R}_{\sigma\mu}\}w,\\
\overset{\circ}{R}_{\sigma k}{}^\nu_\tau = \frac{H}{2\rho}\overset{\circ}{g}{}^{\nu\mu}\nabla_\sigma\widetilde{R}_{\mu\tau}{}^n_k w^+_n,\\
\overset{\circ}{R}_{mk}{}^\nu_\tau = 0,\\
\overset{\circ}{R}_{m\bar{k}}{}^\nu_\tau = -\frac{H}{2\rho}\overset{\circ}{g}{}^{\nu\mu}\widetilde{R}_{\mu\tau}{}^k_m,\\
F_{\nu\tau} = [1 - i(\overline{\Psi}w - w^+\Psi)]^{-1}\\
\times\left[\overset{\circ}{F}_{\nu\tau} + \overset{\circ\circ}{R}_{\nu\tau}{}^\sigma_\mu\left(\overline{\Psi}[\gamma_\sigma, \gamma^\mu]w - w^+[\gamma_\sigma, \gamma^\mu]\Psi\right)\right],\\
F_{\nu k} = [1 - i(\overline{\Psi}w - w^+\Psi)]^{-1}\nabla_\nu\overline{\Psi}_k,\\
F_{mk} = 0,\\
F_{m\bar{k}} = 2i[1 - i(\overline{\Psi}w - w^+\Psi)]^{-1}\overline{\Psi}_m\Psi^k,
\end{gathered} \tag{7.13}$$

ISSN 1512-1461

where

$$\widetilde{R}_{\lambda\tau}{}_k^n = \overset{\circ\circ}{R}_{\lambda\tau}{}_\mu^\sigma[\gamma_\sigma, \gamma^\mu]_n^k + \overset{\circ}{F}_{\lambda\tau}\delta_n^k. \tag{7.14}$$

(4) Let us calculate $\det e_\eta^\xi$ in (6.3-4). Like in (3), the calculation will be performed with accuracy up to first order with respect to H. For this we consider $\det e_\eta^\xi$. By (2.20), (2.22) and (7.6), we can write

$$\det \widetilde{e}_\eta^\xi = \det\left[\delta_\eta^\xi - \delta_n^\xi \Gamma_{\eta k}^{\ n} w^k + \delta_{\overline{n}}^\xi \Gamma_{\eta n}^{\ k} w_k^+\right]$$
$$= \det\left[\delta_\eta^\xi - \frac{1}{8}\Gamma_{\eta\tau}^{\ \nu}\left(\delta_n^\xi[\gamma_\nu, \gamma^\tau]_k^n w^k - \delta_{\overline{n}}^\xi[\gamma_\nu, \gamma^\tau]_n^k w_k^+\right) - i\Psi_\eta\left(\delta_k^\xi w^k - \delta_{\overline{k}}^\xi w_k^+\right)\right].$$

Taking into account that Ψ_η is a value of order H^ε, while $\Gamma_{\eta\tau}^{\ \nu}$ is of order H, we have

$$\det \widetilde{e}_\eta^\xi = \det\left[\delta_\eta^\xi - \frac{1}{8}\Gamma_{\eta\tau}^{\ \nu}\left(\delta_n^\xi[\gamma_\nu, \gamma^\tau]_k^n w^k - \delta_{\overline{n}}^\xi[\gamma_\nu, \gamma^\tau]_n^k w_k^+\right)\right]$$
$$\times \det\left[\delta_\eta^\xi - i\Psi_\eta\left(\delta_k^\xi w^k - \delta_{\overline{k}}^\xi w_k^+\right)\right]$$

The second determinant is calculated exactly and found to be equal to

$$1 - i(\overline{\Psi}w - w^+\Psi).$$

The first determinant is calculated with an accuracy up to first order with respect to H:

$$\det e_\eta^\xi = \left[1 - i\left(\overline{\Psi}w - w^+\Psi\right)\right]^{-1}$$
$$\times\left(1 + \frac{1}{8}\Gamma_{n\tau}^{\ \nu}[\gamma_\nu, \gamma^\tau]_k^n w^k - \frac{1}{8}\Gamma_{\overline{n}\tau}^{\ \nu}[\gamma_\nu, \gamma^\tau]_n^k w_k^+\right).$$

Using (7.6) with an accuracy up to first order with respect to H, we finally obtain

$$\det e_\eta^\xi = \left[1 - i\left(\overline{\Psi}w - w^+\Psi\right)\right]^{-1} + \frac{H}{16\rho}\overset{\circ}{g}^{\nu\mu} w^+\left\{\widetilde{R}_{\mu\tau}, [\gamma_\nu, \gamma^\tau]\right\} w, \tag{7.15}$$

where $\widetilde{R}_{\mu\tau}$ is (7.14), and $\{A, B\} = AB + BA$.

8. Calculation of the Lagrangian

In the preceding paragraph we have found approximate expressions (7.13–15) for curvature component. We want to calculate Lagrangian (6.4) for the fields $\omega_\tau^\nu(x)$, $\varphi_\nu(x)$, $\overline{\Psi}_k(x)$ and $\Psi^k(x)$. In (7.13) we have found curvature components with an accuracy H up to first order. In this paragraph, to avoid cumbersome calculations, it is additionally assumed that $\overset{\circ\circ}{R}_{\nu\sigma}{}_\mu^\tau$ and $\overline{\Psi}$, Ψ are sufficiently small values. This assumption allows us to discard the summands containing $\overset{\circ\circ}{R}_{\nu\sigma}{}_\mu^\tau$ quadratically, and $\overline{\Psi}$, Ψ above second order. But before proceeding to the calculation of Lagrangian (6.4), we should establish some auxiliary relations.

(1) As can be easily verified, the elements of the matrix a_B^A of transformation (3.1)[1] satisfy the equalities

$$a_\xi^\nu a_\tau^\xi = \delta_\tau^\nu,$$
$$a_\eta^\xi = \delta_\eta^\xi - a_\nu^\xi a_\eta^\nu, \tag{8.1}$$
$$a_\eta^\xi = a_\varsigma^\xi a_\eta^\zeta,$$

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where

$$a_k^\nu = i\sqrt{\frac{H}{2}}\frac{1}{\rho}w^+\gamma_k^\nu,$$
$$a_{\overline{k}}^\nu = -i\sqrt{\frac{H}{2}}\frac{1}{\rho}\gamma^{\nu k}w, \tag{8.2}$$
$$a_\nu^k = -i\sqrt{\frac{H}{2}}\gamma_\nu^k w,$$
$$a_\nu^{\overline{k}} = i\sqrt{\frac{H}{2}}w^+\gamma_{\nu k},$$

and ξ, η, ζ run through the indexes and overlined indexes of the internal space.

(2) As is known, the spurs of Dirac matrices γ^ν are equal to zero. The Lie algebra matrices constructed on γ^ν also have spurs equal to zero. Moreover,

$$sp(\gamma^{\nu_1}\dots\gamma^{\nu_m}) = 0 \tag{8.3}$$

when m is an odd number. It is not difficult to verify the validity of the following equalities:

$$sp\,[\gamma^\nu, \gamma^\tau] = 0, \tag{8.4}$$
$$sp([\gamma_\mu, \gamma^\nu][\gamma_\sigma, \gamma^\tau]) = 4(\delta_\mu^\tau\delta_\sigma^\nu - \overset{\circ}{g}_{\mu\sigma}\overset{\circ}{g}{}^{\nu\tau})\delta_k^k.$$

(3) Let us calculate the scalar curvature (6.1) under the restrictions that have been introduced in (4) of Section *6*. Along with this, in (6.1) we discard the summand $r_{AB}{}_E^C$, and denote the remaining part by Λ. By (6.6)[1], (7.13) and (8.1), we can reduce Λ to the form

$$\begin{aligned}\Lambda = &\frac{\rho}{H}\overset{\circ}{R}_{k\overline{n}\sigma}{}^\nu[a_\nu^{\overline{n}}a_{\overline{k}}^\sigma - a_\nu^k a_n^\sigma] + \overset{\circ}{R}_{\nu\mu n}{}^k[a_k^\mu a_\sigma^n - a_{\overline{n}}^\mu a_\sigma^{\overline{k}}]\overset{\circ}{g}{}^{\nu\sigma}\\ &+\frac{2\rho}{H}\overset{\circ}{R}_{m\mu n}{}^k\left[a_k^\mu a_{\overline{m}}^n - a_{\overline{n}}^\mu a_{\overline{m}}^{\overline{k}} - \frac{1}{2\rho}\left(w^n a_k^\mu - w_k^+ a_{\overline{n}}^\mu\right)w^m\right]\\ &+\frac{2\rho}{H}\overset{\circ}{R}_{\overline{m}\mu n}{}^k\left[a_k^\mu a_m^n - a_{\overline{n}}^\mu a_m^{\overline{k}} - \frac{1}{2\rho}\left(w^n a_k^\mu - w_k^+ a_{\overline{n}}^\mu\right)w_m^+\right]\\ &+\frac{2\rho}{H}\overset{\circ}{R}_{m\overline{\ell}n}{}^k\left[a_k^{\overline{\ell}}a_{\overline{m}}^n - a_{\overline{n}}^{\overline{\ell}}a_{\overline{m}}^{\overline{k}} - \frac{1}{2\rho}\left(w^n a_k^{\overline{\ell}} - w_k^+ a_{\overline{n}}^{\overline{\ell}}\right)w^m - \frac{1}{2\rho}(w_k^+ a_{\overline{m}}^n - w^n a_{\overline{m}}^{\overline{k}})w_\ell^+\right].\end{aligned} \tag{8.5}$$

(4) (7.13) and (7.15) contain

$$[1 - i(\overline{\Psi}w - w^+\Psi)]^{-1}.$$

Let us expand this function into powers of $\overline{\Psi}w - w^+\Psi$. Then the product $\Lambda\cdot\det e_r^\xi$ is a polynomial function of w and w^+. Using equalities (8.3–4), it is not difficult to calculate integral (6.4). It is obvious that this procedure is rather cumbersome and therefore we will demonstrate these calculations only for some summands from (8.5).

(5) We take the first summand from (8.5) and multiply it by (7.15). By (7.13–14) and (8.1–2), we obtain

$$\begin{aligned}\Lambda_1 = &\left[\frac{1}{8\rho}\overset{\circ\circ}{R}_{\nu\tau}{}^\sigma{}_\mu w^+\gamma^\tau[\gamma_\sigma, \gamma^\mu]\gamma^\nu w + \frac{i}{2\rho}\overset{\circ}{F}_{\nu\tau}w[\gamma^\tau, \gamma^\nu]w\right]\\ &\times\left[\left(1 - i\left(\overline{\Psi}w - w^+\Psi\right)\right)^{-1} + \frac{H}{16\rho}w^+\left\{\widetilde{R}_{\nu\tau}, [\gamma^\nu, \gamma^\tau]\right\}w\right].\end{aligned} \tag{8.6}$$

Let us substitute (7.14) into (8.6) and remove the brackets. If the obtained expression is substituted into (6.4) and integrated, then the summands, where w and w^+ are not contained simultaneously to an

ISSN 1512-1461

equal degree, vanish. Therefore when unbracketing product (8.6), we discard such summands from the very beginning:

$$\begin{aligned}\Lambda_1 = {} & \frac{1}{8\rho}\overset{\circ\circ}{R}_{\nu\tau}{}^{\sigma}_{\mu} w^+\gamma^\tau[\gamma_\sigma,\gamma^\mu]\gamma^\nu w + \frac{i}{2\rho}\overset{\circ}{F}_{\nu\tau} w^+[\gamma^\tau,\gamma^\nu]w \\ & + \frac{1}{4\rho}\overset{\circ\circ}{R}_{\nu\tau}{}^{\sigma}_{\mu} w^+\gamma^\tau[\gamma_\sigma,\gamma^\mu]\gamma^\nu w\cdot(\overline{\Psi}w)(w^+\Psi) \\ & + \frac{i}{2\rho}\overset{\circ}{F}_{\nu\tau} w^+[\gamma^\tau,\gamma^\nu]w\cdot(\overline{\Psi}w)(w^+\Psi) \\ & + \frac{H}{128\rho^2}\overset{\circ\circ}{R}_{\nu\tau}{}^{\sigma}_{\mu} w^+\gamma^\tau[\gamma_\sigma,\gamma^\mu]\gamma^\nu w\cdot w^+\left\{\widetilde{R}_{\lambda\varepsilon},[\gamma^\lambda,\gamma^\varepsilon]\right\}w \\ & + \frac{H}{32\rho^2}\overset{\circ}{F}_{\nu\tau} w^+[\gamma^\tau,\gamma^\nu]w\cdot w^+\{\widetilde{R}_{\lambda\varepsilon},[\gamma^\lambda,\gamma^\varepsilon]\}w.\end{aligned}$$

The integration of this sum gives

$$\begin{aligned}L_1(x) = {} & \mu_1\left[\overset{\circ\circ}{R}_{\nu\tau}{}^{\sigma}_{\mu} sp(\gamma^\tau[\gamma_\sigma,\gamma^\mu]\gamma^\nu) + \overset{\circ}{F}_{\nu\tau} sp([\gamma^\tau,\gamma^\nu])\right] \\ & + \ell_0^2\mu_2\left[\frac{1}{2}\overset{\circ\circ}{R}_{\nu\tau}{}^{\sigma}_{\mu} sp(\gamma^\tau[\gamma_\sigma,\gamma^\mu]\gamma^\nu) + \overset{\circ}{F}_{\nu\tau} sp([\gamma^\tau,\gamma^\nu])\overline{\Psi}\Psi\right] \\ & + \ell_0^2\mu_3\left[\frac{1}{2}\overset{\circ\circ}{R}_{\nu\tau}{}^{\sigma}_{\mu}\overline{\Psi}\gamma^\tau[\gamma_\sigma,\gamma^\mu]\gamma^\nu\Psi + \overset{\circ}{F}_{\nu\tau}\overline{\Psi}[\gamma^\tau,\gamma^\nu]\Psi\right] \\ & + H\mu_4\overset{\circ\circ}{R}_{\nu\tau}{}^{\sigma}_{\mu} sp(\gamma^\tau[\gamma_\sigma,\gamma^\mu]\gamma^\nu) sp\{\widetilde{R}_{\lambda\varepsilon},[\gamma^\lambda,\gamma^\varepsilon]\} \\ & + H\mu_5\overset{\circ\circ}{R}_{\nu\tau}{}^{\sigma}_{\mu} sp(\gamma^\tau[\gamma_\sigma,\gamma^\mu]\gamma^\nu\{\widetilde{R}_{\lambda\varepsilon},[\gamma^\lambda,\gamma^\varepsilon]\}) \\ & + H\mu_6\overset{\circ}{F}_{\nu\tau} sp([\gamma^\tau,\gamma^\nu]) sp\{\widetilde{R}_{\lambda\varepsilon},[\gamma^\lambda,\gamma^\varepsilon]\} \\ & + H\mu_7\overset{\circ}{F}_{\nu\tau} sp([\gamma^\tau,\gamma^\nu]\{\widetilde{R}_{\lambda\varepsilon},[\gamma^\lambda,\gamma^\varepsilon]\}).\end{aligned}$$

As is known, the matrices $[\gamma^\nu,\gamma^\tau]$ form the closed Lie algebra with traces equal to zero. Then the spur of odd products of such matrices will also be equal to zero. Therefore in $L_1(x)$ the summands containing $[\gamma_\nu,\gamma^\tau]$ and triple products $[\gamma_\nu,\gamma^\tau]$, will vanish.

Using (8.4) and the equalities $\overset{\circ\circ}{R}_{\nu\tau}{}^{\sigma}_{\mu} = -\overset{\circ\circ}{R}_{\tau\nu}{}^{\sigma}_{\mu}$, $sp(AB) = sp(BA)$, it is not difficult to verify the validity of

$$\overset{\circ\circ}{R}_{\nu\tau}{}^{\sigma}_{\mu} sp(\gamma^\tau[\gamma_\sigma,\gamma^\mu]\gamma^\nu) = 16\overset{\circ\circ}{R},$$

where we have taken into account that $\delta^k_k = 4$, and $\overset{\circ\circ}{R}$ is a classical scalar curvature of the space-time

$$\overset{\circ\circ}{R} = \overset{\circ\circ}{R}_{\nu\tau}{}^{\tau}_{\sigma}\overset{\circ}{g}^{\nu\sigma}. \tag{8.7}$$

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Analogous calculations carried out for other summands in $L_1(x)$ give

$$
\begin{aligned}
L_1(x) = {} & (\widetilde{\mu}_1 + \widetilde{\mu}_2 \ell_0^2 \overline{\Psi}\Psi)\overset{\circ\circ}{R} \\
& + \widetilde{\mu}_3 \ell_0^2 \left(i\overset{\circ}{F}_{\nu\tau}\overline{\Psi}[\gamma^\tau,\gamma^\nu]\Psi + \frac{1}{2}\overset{\circ\circ}{R}_{\nu\tau}{}^{\sigma}{}_{\mu}\overline{\Psi}\gamma^\tau[\gamma_\sigma,\gamma^\mu]\gamma^\nu\Psi \right) \\
& + H\widetilde{\mu}_4 \overset{\circ\circ}{R}_{\nu\sigma}{}^{\sigma}{}_{\mu}\overset{\circ}{F}{}^{\mu}{}_{\tau}\overset{\circ}{g}{}^{\nu\tau} \\
& + H\widetilde{\mu}_5 \overset{\circ}{F}{}^{\nu}{}_{\tau}\overset{\circ}{F}{}^{\tau}{}_{\nu},
\end{aligned}
\tag{8.8}
$$

where

$$
\overset{\circ}{F}{}^{\nu}{}_{\sigma} = \overset{\circ}{g}{}^{\nu\tau}\overset{\circ}{F}_{\sigma\tau}. \tag{8.9}
$$

Note that $\widetilde{\mu}_1,\ldots,\widetilde{\mu}_5$ are numerical coefficients depending only on the dimension of the internal space. By analogous calculations in the same approximation carried out for the second summand in (8.5) we come to the same result as (8.8).

(6) Let us now consider the second and the third summand in (8.5). Multiplying them by (7.15), we obtain

$$
\begin{aligned}
\Lambda_2 = \frac{2\rho}{H} & \left\{ \begin{array}{l} \overset{\circ}{R}_{m\mu n}{}^{k}\left[a^\mu_k a^n_{\overline{m}} - a^\mu_{\overline{n}} a^{\overline{k}}_{\overline{m}} - \frac{1}{2\rho}(w^n a^\mu_k - w^+_k a^\mu_{\overline{n}})w^n\right] \\ +\overset{\circ}{R}_{\overline{m}\mu n}{}^{k}\left[a^\mu_k a^n_m - a^\mu_{\overline{n}} a^{\overline{k}}_m - \frac{1}{2\rho}(w^n a^\mu_k - w^+_k a^\mu_{\overline{n}})w^+_m\right] \end{array} \right\} \\
& \times \left(1 + i(\overline{\Psi}w - w^+\Psi) - (\overline{\Psi}w - w^+\Psi)^2 + \frac{H}{16\rho}w^+\{\widetilde{R}_{\nu\tau},[\gamma^\nu,\gamma^\tau]\}w\right).
\end{aligned}
\tag{8.10}
$$

Let us multiply (8.10) with respect to volume (6.2) of the internal space. It is not difficult to verify that the cofactor of the summands not containing $\overline{\Psi}$ and Ψ is made up of traces of the matrices which are products of an odd number of Dirac matrices γ^ν. In that case, as follows from (8.3), such summands vanish. The summands containing $\overline{\Psi}$ and Ψ linearly have w^+ and w raised to an odd degree as their cofactors. When integrated, such summands also vanish.

Further, from (8.2) it follows that a^ν_k and $a^\nu_{\overline{k}}$ contain the cofactor $\sqrt{H/2}$. This cofactor can be taken out of the brackets of the entire expression. If we leave the summands enclosed in the brackets in the same approximation as in Section 7, then after simple transformations we obtain

$$
\widetilde{\Lambda}_2 = \frac{4\rho}{H}\left[\begin{array}{l} \left(\nabla_\mu\overline{\Psi}_m\right)\left(a^\mu_k a^k_{\overline{m}} - a^\mu_{\overline{k}} a^{\overline{k}}_{\overline{m}} - \frac{1}{2\rho}(w^k a^\mu_k - w^+_k a^\mu_{\overline{k}})w^m\right) \\ +\left(\nabla_\mu\Psi^m\right)\left(a^\mu_k a^k_m - a^\mu_{\overline{k}} a^{\overline{k}}_m - \frac{1}{2\rho}(w^k a^\mu_k - w^+_k a^\mu_{\overline{k}})w^+_m\right) \end{array} \right] \times (w^+\Psi - \overline{\Psi}w). \tag{8.11}
$$

By equalities (8.1-2), we can write

$$
\begin{aligned}
& a^\mu_k a^k_{\overline{m}} - a^\mu_{\overline{k}} a^{\overline{k}}_{\overline{m}} - \frac{1}{2\rho}(w^k a^\mu_k - w^+_k a^\mu_{\overline{k}})w^m \\
& = i\sqrt{\frac{H}{2}}\frac{1}{\rho}\left(\gamma^{\mu m}{}_{n}w^n - \gamma^{\sigma m}{}_{n}w^n\frac{1}{2\rho}w^+[\gamma_\sigma,\gamma^\mu]w - \frac{1}{\rho}w^+\gamma^\mu w w^m\right),
\end{aligned}
\tag{8.12}
$$

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$$a^\mu_k a^k_m - a^\mu_{\overline{k}} a^{\overline{k}}_m - \frac{1}{2\rho}(w^k a^\mu_k - w^+_k a^\mu_{\overline{k}})w^+_m$$
$$= i\sqrt{\frac{H}{2}}\frac{1}{\rho}\left(w^+_n \gamma^{\mu n}{}_m + \frac{1}{2\rho}w^+_n \gamma^{\sigma n}{}_m w^+[\gamma_\sigma, \gamma^\mu]w - \frac{1}{\rho}w^+\gamma^\mu w w^+_m\right).$$

Let us substitute (8.12) into (8.11). The resulting expression is integrated over volume (6.2). Having first symmetrized the coefficients of the polynomial of w^+, w and then using (8.3–4), after simple transformations we finally have

$$L_2(x) = \frac{i\ell_0^2}{\sqrt{H}}\overline{\mu}_6\left[\left(\nabla_\mu \overline{\Psi}\right)\gamma^\mu\Psi - \overline{\Psi}\gamma^\mu\nabla_\mu\Psi\right], \tag{8.13}$$

where $\overline{\mu}_6$ is the numerical coefficient depending on the dimension of the internal space.

(7) Now we will consider the last summand in (8.5). Multiplying it by (7.15), we obtain

$$\Lambda_3 = \frac{2\rho}{H}\overset{\circ}{R}_{m\overline{\ell}n}{}^k\left[a^{\overline{\ell}}_k a^n_{\overline{m}} - a^{\overline{\ell}}_{\overline{n}} a^{\overline{k}}_{\overline{m}} - \frac{1}{2\rho}(w^n a^{\overline{\ell}}_k - w^+_k a^{\overline{\ell}}_{\overline{n}})w^m - \frac{1}{2\rho}(w^+_k a^n_{\overline{m}} - w^n a^{\overline{k}}_{\overline{m}})w^+_\ell\right] \tag{8.14}$$
$$\times\left[1 + i(\overline{\Psi}w - w^+\Psi) - (\Psi w - w^+\Psi)^2 + \frac{H}{16\rho}w^+\{\widetilde{R}_{\nu\tau}, [\gamma^\nu, \gamma^\tau]\}w\right].$$

By the same reasoning and calculation as in (6), we find

$$L_3(x) = \overline{\mu}_7\overset{\circ\circ}{R} + \overline{\mu}_8\frac{\ell_0^2}{H}\overline{\Psi}\Psi, \tag{8.15}$$

where $\overline{\mu}_7, \overline{\mu}_8$ are the numerical coefficients.

(8) Thus the total Lagrangian (6.4) arising on the basis of (8.5) is the sum of (8.8), (8.13) and (8.15). The obtained sum is divided by $\widetilde{\mu}H$, where $\widetilde{\mu}$ is the numerical coefficient of $\overset{\circ\circ}{R}$. ($\widetilde{\mu}H$ could have been introduced in (6.4) from the very beginning as the cofactor of V_0). We retain the previous notation of the obtained Lagrangian.

From the phenomological theories it follows that in the Lagrangian the coefficients of $\overset{\circ}{F}{}^\nu_\tau\overset{\circ}{F}{}^\tau_\nu$ and $i(\nabla_\mu\overline{\Psi}\cdot\gamma^\mu\Psi - \overline{\Psi}\gamma^\mu\nabla_\mu\Psi)$ should be at least of the same order. This leads to the condition

$$\frac{\ell_0^2}{\sqrt{H}} = H. \tag{8.16}$$

Then the Lagrangian takes the final form

$$L(x) = \frac{1}{H}(1 + \mu_1 H\sqrt{H}\overline{\Psi}\Psi)\overset{\circ\circ}{R} + \mu_2\sqrt{H}\left(i\overset{\circ}{F}_{\nu\tau}\overline{\Psi}[\gamma^\nu, \gamma^\tau]\Psi + \frac{1}{2}\overset{\circ\circ}{R}_{\nu\tau}{}^\sigma_\mu\overline{\Psi}\gamma^\tau[\gamma_\sigma, \gamma^\mu]\gamma^\nu\Psi\right) \tag{8.17}$$
$$+ \mu_3\overset{\circ}{g}{}^{\nu\tau}\overset{\circ\circ}{R}_{\nu\sigma}{}^\sigma_\mu\overset{\circ}{F}{}^\mu_\tau + \mu_4\overset{\circ}{F}{}^\tau_\nu\overset{\circ}{F}{}^\nu_\tau + \mu_5 i(\nabla_\mu\overline{\Psi}\gamma^\mu\Psi - \overline{\Psi}\gamma^\mu\nabla_\mu\Psi) + \mu_6\frac{1}{\sqrt{H}}\overline{\Psi}\Psi,$$

where $\mu_1, \ldots, \mu_6$ are numerical coefficients. In (8.17) the objects $\overset{\circ\circ}{R}_{\nu\sigma}{}^\mu_\tau$ and $\overset{\circ\circ}{R}$ are defined from (7.2) and (8.7), $\overset{\circ}{F}{}^\nu_\sigma$ is defined from (7.9) and (8.9), and

$$\nabla_\nu\Psi = \widehat{D}_\nu\Psi + \frac{1}{8}\Gamma_\nu{}^\sigma_\mu[\gamma_\sigma, \gamma^\mu]\Psi + i\varphi_\nu\Psi, \nabla_\nu\overline{\Psi} \tag{8.18}$$
$$= \widehat{D}_\nu\overline{\Psi} - \frac{1}{8}\Gamma_\nu{}^\sigma_\mu\overline{\Psi}[\gamma_\sigma, \gamma^\mu] - i\varphi_\nu\overline{\Psi},$$

where $\Gamma_\sigma{}^\nu_\mu$ has form (3.15).

As has been mentioned, we have performed the same calculations as in Section *7*. In the case of suc-

ISSN 1512-1461

cessive approximations, in the Lagrangian there arise summands of the form $H\overset{\circ\circ}{R}{}^2$, $H\overset{\circ\circ}{R}_{\nu\tau\sigma\mu}\overset{\circ\circ}{R}{}^{\nu\tau\sigma\mu}$, $\ell_0 H^{-1}(\overline{\Psi}\Psi)^2$, $\ell_0 H^{-1}(\overline{\Psi}\gamma_\nu\Psi)(\overline{\Psi}\gamma^\nu\Psi)$, $(\overset{\circ}{F}\overset{\circ}{F})^2$ and so on. This implies that the equations of Maxwell and equations of Dirac appear to be non-linear objects.

(9) For simplicity, in (8.17) we discard the summand containing μ_1, μ_3 and the second summand containing μ_2. Then we have

$$L(x) = \frac{1}{H}\overset{\circ\circ}{R} + \mu_2 i\sqrt{H}\overset{\circ}{F}_{\nu\tau}\overline{\Psi}[\gamma^\tau,\gamma^\nu]\Psi + \mu_4\overset{\circ}{F}{}^\nu_\tau\overset{\circ}{F}{}^\tau_\nu + \mu_5 i\left[\left(\nabla_\mu\overline{\Psi}\right)\gamma^\mu\Psi - \overline{\Psi}\gamma^\mu\nabla_\mu\Psi\right] + \mu_6\frac{1}{\sqrt{H}}\overline{\Psi}\Psi. \tag{8.19}$$

It is obvious that (8.19) is the classical Lagrangian of the interacting strong gravitation, Maxwell and Dirac fields. The summand with μ_2 is the well known anomalous magnetic moment of the Dirac field.

(10) In Section *6* we have excluded from the consideration the geometric object $B^a_A(x,w,w^+)$. Let us expand B^a_A into powers of w and w^+. As a simple illustration we can investigate the case with $B^a_\nu(x)$, $B^a_k(x)$ and $B^a_{\overline{k}}(x)$. These fields form constructions analogous to $\varphi_\nu(x)$, $\Psi(x)$ and $\overline{\Psi}(x)$, but where the algebra to which they are extended is noncommutative and the dynamic group is specific. $B_\nu(x)$ will bring us to Yang-Mills fields, and $B_k(x)$, $B_{\overline{k}}(x)$ to noncommutative Dirac fields.

(11) From (8.19) it immediately follows that the mass of the Dirac field Ψ is equal to $\mu_6/\left(\mu_5\sqrt{H}\right)$.

As has been said in (3) of this section, from (6.1) we have discarded the summand with $r_{AB}{}^C_E$. Calculations show that when this summand is present in (6.1), in Lagrangians (8.17) and (8.19) there arises the summand $\mu_7 H^{-2}$. Obviously, $\mu_7 H^{-1}$ should be interpreted as the squared mass for strong gravitation, i.e. for the field $e^\nu_\tau(x)$.

Preliminary investigations show that Yang-Mills field $B^a_\nu(x)$ and the noncommutative Dirac field $B^a_k(x)$ also contain mass terms.

The only field having no mass is the Maxwell field $\varphi_\nu(x)$.

(12) Thus, in constructing the theory of compensating fields, the observer is within process (5.9)[1]. Since all the time he uses the calculus of process (5.9)[1], he will not be able to leave this process. He can manipulate only with various states of his process – the consequence of this is the appearance of the Lorentz group and the dynamic group.

(13) Let us return to Lagrangian (8.17) and discard all the summands responsible for the interaction between the fields $e^\nu_\tau(x)$, $\varphi_\nu(x)$ and $\Psi(x)$, $\overline{\Psi}(x)$. As a result we obtain

$$L(x) = \frac{1}{H}\overset{\circ\circ}{R} + \mu_4\overset{\circ}{F}{}^\nu_\tau\overset{\circ}{F}{}^\tau_\nu + \mu_5 i\left(\frac{\partial\overline{\Psi}}{\partial x^\nu}\gamma^\nu\Psi - \overline{\Psi}\gamma^\nu\frac{\partial\Psi}{\partial x^\nu}\right) + \frac{\mu_6}{\sqrt{H}}\overline{\Psi}\Psi. \tag{8.20}$$

In that case, we have the delocalization of the group G, i.e. the restoration of the group $\overset{\circ}{G}$. It is obvious that (8.20) is invariant with respect to the group $\overset{\circ}{G}$. The equations obtained from (8.20) correspond to the fields described in the frames of reference and systems of calculus of the observer from process (5.9)[1] in the absence of interaction (both with the observer's process and with each other).

B. Basic Principles of Process Motion

In this chapter we consider the set of all possible kinds of noninteracting processes. We intend to investigate the properties of this set and establish algebraic relations arising between processes.

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9. Questions of relativity

(1) It has been more than once mentioned that each differential equation, describing free motion of a process, contains a proper double numerical field, where each field acts alternatively. These fields generate in turn alternative calculi and alternative frames of reference [2, 3]. It should be emphasized that, due to the alternativeness of a double field, while being realized the process chooses only one of the fields, thereby uniquely choosing the corresponding calculus and frame of reference. This means that the process evolution takes place according to the algebraic rules dictated by the chosen numerical field.

(2) As has been established in [2, 3], the differential equation of a given process takes a linear form in its calculus and frame of reference. We want to specially remark that we have not studied in detail what concrete linear form this equation has and on what its construction depends. These issues need further careful investigation and therefore we confine our discussion to the above-mentioned remark. Hence, like in the preceding chapters, it is assumed that the equations of the considered processes are reducible to form (1.1)[1].

(3) Being in a given process, the observer begins to investigate the algebraic properties of his process. As was shown in Sections *8, 9* of [2], each process has its own inertial frames of reference. By virtue of the reasoning of Section *1* from [1], we require that the group acting over the inertial frames of reference be the Lorentz group. In this case, the equation of the considered process must be invariant with respect to the action of this group. Besides, we assume that equalities like (1.2)[1] are fulfilled. In other words, the equation of the considered process is an admissible one.

By virtue of the reasoning of Sections *1–6* from [1], the group acting in the set of spaces of the considered process is assumed to be analogous to the group $\overset{\circ}{G}$ (5.1-2)[1]. To simplify our further discussion, we assume that in all considered processes such groups act in the proper calculi of these processes.

(4) Suppose we have two processes π_1 and π_2. Inside the process π_1 is the observer. As said in (13) of Section *8*, the observer can describe the process π_2 in his system of calculus provided that there exists an interaction between both processes. In a certain sense this interaction must be complete; otherwise the observer from π_1 will not see the process π_2 or will see it partly. He writes the differential equation of the process π_2 in the language of the calculus system and reference frame of the process π_1. In the obtained equation the unknown functions are the values by means of which various states of the process π_2 can be described. After this, the observer discards the interaction terms from the arisen equation. The arisen equation of the process π_2 is autonomous, since π_1 and π_2 already interact and there are no other processes.

Let group (5.1–2)[1] (or its analogue) act in the process π_1. Recall that the equation of the process π_1 has form (1.1)[1] and is the invariant of group (5.1–2)[1]. Therefore the calculus system of the process π_1 remains invariant with respect to group (5.1–2)[1]. Then in the equation of π_2, at least the derivatives of the unknown functions get transformed, since the Lorentz group acts on the coordinates of the external space of the process π_1. Naturally, the equation of the process π_2 must be invariant with respect to group (5.1-2)[1].

We call π_2 the process observed from the process π_1 if the equation of π_2 can be described in the system of π_1. If, simultaneously, the equation of the process π_1 can be described in the system of π_2, then π_1 and π_2 mutually observable processes. As shown in [2], mutually observable processes belong to processes of the same class.

(5) Let us consider the set π consisting of mutually observable processes. Let $\pi_1, \pi_2 \in \Pi$. Then,

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as was shown in Sections *8, 9* from [2], there exists a transformation of the process π_1 to π_2 and vice versa. Along with this symmetry property, it is proved that the arisen transformations possess the following transitive property: if the observer passes from π_1 to π_2 and then to π_3, the intermediate transformation falls out and, as a result, bypassing π_2 he passes from π_1 directly to π_3. In addition to symmetry and transitivity, the set Π possesses the reflexivity property: each process from Π is observable within itself.

(6) A process is called elementary if it is one-dimensional and, accordingly, is described in some system by one quasilinear differential equation with partial derivatives of first order.

Let us return to the set Π. Since all processes from Π are mutually observable, they are processes of one and the same class. Let N be the dimension of the internal spaces of processes from Π.

Assume that the observer is in some process $\pi \in \Pi$. Since the set Π consists of mutually observable processes, the observer can write, in his system, the differential equations of all processes.

Now assume that $\pi^* \in \Pi$ is the process whose equation in the calculus system of the process π has the form

$$A_k^\nu(u^k)\frac{\partial u^k}{\partial x^\nu} = F^k(u^k), \quad (k = 1, \dots, N), \tag{9.1}$$

where summation is performed only over ν from 1 to 4. It is obvious that system (9.1) is split into N independent equations. Thus the observer from π can say that the process π^* consists of N elementary processes which do not interact with each other. However, if the observer passes to another process $\widetilde{\pi}$, then in the new system of calculus the equations of elementary processes from π^* can be nonlinearly interrelated. During this transition, the sought functions $u^1, \dots, u^N$ of equation (9.1) do not transform [2]. In other words, in the system of $\widetilde{\pi}$, the equation of π^* has a quasilinear form, while, unlike (9.1), elementary processes do not interact with each other.

Such behavior of the process π^* should not be regarded as exceptional. Generally speaking, any process from Π must possess analogous properties. This however depends on the completeness of the process Π. If Π is complete, we call it the space of mutually observable processes of order N and denote by Π^N.

(7) Let us consider processes from the space Π^N. Transformations arising when one process changes to another process are realized by means of the characteristic functions of differential equations describing this process and have the properties discussed in (5). Under these transformations there arise algebraic objects. In particular, as has been shown in [2], the inertial frames of reference of processes are algebraic objects.

Let us discuss frames of reference in more detail. Let the observer be in a process $\pi \in \Pi^N$. According to Sections *8, 9* from [2], he finds his own N frames of reference, among which only one arbitrarily chosen copy is independent, while the remaining $N-1$ copies are expressed through it. On the other hand, as shown in (6), any process from Π^N consists of N elementary processes. Therefore each elementary process generates its own frame of reference. This is clearly exemplified by (9.1).

According to [2], when passing from π_1 to π_2, the inertial frames of the process π_1 transform to the inertial frames of the process π_2, where $\pi_1, \pi_2 \in \Pi^N$. If, for instance, x_1^ν and x_2^ν are the coordinates of the external spaces of the processes of π_1 and π_2, respectively, then we have

$$x_2^\nu = f_{21}(x_1^\nu). \tag{9.2}$$

Note that if the equation of the process π_2 is nonlinear from the standpoint of the process π_1, then transformation (9.2) is also nonlinear [2]. Hence we conclude that the inertial frames of the process π_1

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are not the inertial frames of the process π_2.

(8) Thus the algebraic theory of differential equations leads to two types of transformations:

a) The observer is in an arbitrarily chosen but fixed process of the space Π^N. Within his process he finds the frame of reference and the system of calculus. Applying the reasoning given in Chapter A, the observer discovers a group of transformations of form (5.1–2) [1] that preserves the invariance of the equation of his process. After that he writes the equations of all processes from Π^N in terms of his frame of reference. Since the processes of the space Π^N do not interact with each other, the observer requires that all processes be invariant with respect to the action of group (5.1–2)[1].

b) Let the processes $\pi_1, \pi_2 \in \Pi^N$. As shown in [2], when passing from π_1 to π_2, a transformation is formed that changes π_1 to π_2. Such transformations acting in the space Π^N satisfy the properties from (5). Concurrently, there arise algebraic objects such as a numerical field, a system of calculus, inertial frames of reference and so on. A concrete algebraic object of one process transforms to the same kind of object of the other process.

10. Existence of the Double World

We continue the investigation of the properties of a double algebraic field and its influence on the hierarchy of physical processes. As has been stated in (1) of Section *9*, each of these algebraic fields acts alternatively and generates its own frame of reference and system of calculus. In [2–3] we have established the existence of an operation transforming the system to an alternative system and called this operation algebraic conjugation (more exactly, φ-conjugation).

For algebraic conjugation, the differential equation of the process transform to the conjugate equation [2]. The obtained equation describes the process which we have called the antiprocess with respect to the considered process.

(1) Let us consider the process described by the standard Dirac equation

$$\gamma^{\nu k}_{\ \ n}\frac{\partial u^n}{\partial x^\nu} = -imu^k. \tag{10.1}$$

As shown in Section 7 from [2], an equation of the antiprocess is written in the a-conjugate form as

$$\widehat{\gamma}^{\widehat{\nu} k}_{\ \ \widehat{n}}\frac{\widehat{\partial}\widehat{u}^{\widehat{n}}}{\widehat{\partial}\widehat{x}^{\widehat{\nu}}} = -i\widehat{m}\widehat{u}^k. \tag{10.2}$$

In (10.1) the summation is performed over the same indexes $(a \dot{+} b = a + b)$, while in equation (10.2) the symbol $\widehat{}$ over the same indexes denotes alternative summation $b = (a^{-1} + b^{-1})^{-1}$.

For the a-conjugation, the values contained in (10.1) and (10.2) are interrelated by the equalities [2]

$$\begin{gathered}
\widehat{x}^\nu = H_0/x^\nu,\\
m\,\widehat{m} = H_0^{-1},\\
\widehat{u}^k = \widehat{\gamma}^{2k}_{\ \ \widehat{n}}\frac{1}{u^{\widehat{n}}},\\
\\
\widehat{\gamma}^{1k}_{\ \ n} = 1/\gamma^{1k}_{\ \ n},\\
\widehat{\gamma}^{2k}_{\ \ n} = -1/\gamma^{2k}_{\ \ n},\\
\widehat{\gamma}^{3k}_{\ \ n} = 1/\gamma^{3k}_{\ \ n},\\
\widehat{\gamma}^{4k}_{\ \ n} = 1/\gamma^{4k}_{\ \ n}.
\end{gathered} \tag{10.3}$$

As different from (7.3) [2], in (10.3) we have introduced the constant H_0 having the dimension of the length square.

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(2) In the preceding chapters we have investigated the geometry arising on the basis of process (10.1). In constructing the geometry we used the calculus in the terms of which the equation itself was represented.

Using (10.2) we find the metric

$$\widehat{d}\widehat{s}^2 = \overset{\circ}{g}_{\widehat{\nu}\widehat{\tau}}\widehat{d}\widehat{x}^{\widehat{\nu}}\widehat{d}\widehat{x}^{\widehat{\tau}} \ddot{+} \frac{\widehat{H}}{\rho}\widehat{d}\widehat{w}^{+}_{\widehat{k}}\widehat{d}\widehat{w}^{\widehat{k}}. \tag{10.4}$$

Taking into account (10.3), from [2] it follows that (10.4) and (1.21)[1] are interrelated as follows:

$$\begin{aligned} \widehat{d}\widehat{x}^{\nu} &= H_0/dx^{\nu}, \\ H\,\widehat{H} &= H_0^2, \\ \widehat{w}^{k} &= \widehat{\gamma}^{2k}_{\ \widehat{n}}\frac{1}{w^{\widehat{n}}}, \\ \widehat{w}^{+}_{k} &= \frac{1}{w^{+}_{\widehat{n}}}\widehat{\gamma}^{2\widehat{n}}_{\ \ k}, \\ \widehat{d}\widehat{w} &= \widehat{\gamma}^{2k}_{\ \widehat{n}}\frac{1}{w^{\widehat{n}}}, \\ \widehat{d}\widehat{w}^{+} &= \frac{1}{dw^{+}_{\widehat{n}}}\widehat{\gamma}^{2\widehat{n}}_{\ \ k}, \\ \widehat{\rho} &= 1/\rho, \end{aligned} \tag{10.5}$$

where $\widehat{\rho} = \widehat{w}^{+}_{\widehat{k}}\widehat{w}^{\widehat{k}}$. One can easily show the validity of

$$ds\widehat{d}\widehat{s} = H_0. \tag{10.6}$$

(3) As is known, equation (10.1) is invariant with respect to group (2.3)[1]. Using the mathematical methods presented in [2] and the reasoning of Sections *1–2* from [1], we find the group preserving the invariance of equation (10.2). These transformations can be written in the infinitesimal form

$$\begin{aligned} \overline{\widehat{x}}^{\nu} &= \widehat{x}^{\nu} \ddot{+} \widehat{\xi}^{\nu}_{\widehat{\sigma}}\widehat{x}^{\widehat{\sigma}} \ddot{+} \widehat{\xi}^{\nu}, \\ \overline{\widehat{w}} &= \widehat{w} \ddot{+} 8\widehat{\xi}^{\widehat{\nu}}_{\widehat{\sigma}}[\widehat{\gamma}_{\widehat{\nu}} \widehat{,} \widehat{\gamma}^{\widehat{\sigma}}]\widehat{\cdot}\widehat{w} \ddot{-} i\widehat{\xi}\widehat{w}, \\ \overline{\widehat{w}}^{+} &= \widehat{w}^{+} \ddot{-} 8\widehat{\xi}^{\widehat{\nu}}_{\widehat{\sigma}}\widehat{w}^{+}\widehat{\cdot}[\widehat{\gamma}_{\widehat{\nu}} \widehat{,} \widehat{\gamma}^{\widehat{\sigma}}] \ddot{+} i\widehat{\xi}\,\widehat{w}^{+}, \end{aligned} \tag{10.7}$$

where

$$[\widehat{\gamma}_{\nu} \widehat{;} \widehat{\gamma}^{\sigma}] = \widehat{\gamma}_{\nu}\widehat{\cdot}\widehat{\gamma}^{\sigma} \ddot{-} \widehat{\gamma}^{\sigma}\widehat{\cdot}\widehat{\gamma}_{\nu}$$

Here we have used the notation introduced in [2]. Note that, as different from (2.3)[1], in (10.7) the group parameters are infinitely large values.

It is not difficult to verify that transformations (2.3)[1] and (10.7) are a-conjugate to each other. Therefore these two groups are isomorphic.

(4) Suppose the observer passes from process (10.1) to antiprocess (10.2). He begins to investigate process (10.2). According to (1) of Section *9*, the observer finds calculi and the frame of reference used by the process under investigation. In these systems he describes the motion equation of this process which is written in form (10.2). Without experiencing any inconvenience he can apply the same methods of geometry investigation which he used in process (10.1).

Along with group (10.7), the observer discovers the dynamic group with its representation. He finds the rule of differentiation of geometric objects.

(5) After finding the group (10.7) and the dynamic group, the observer, being in process (10.2),

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begins to localize the group parameters with respect to the points of his total space. By analogy with (1.1) he writes

$$\begin{aligned} \widehat{\xi}^{\nu}_{\sigma} &\to \widehat{\Theta}^{\nu}_{\sigma}(\widehat{x}), \\ \widehat{\xi}^{\nu} &\to \widehat{\Theta}^{\nu}(\widehat{x}), \\ \widehat{\xi}^{a} &\to \widehat{\Theta}^{a}(\widehat{x}, \widehat{w}, \widehat{w}^{+}), \\ \widehat{\xi} &\to \widehat{\Theta}^{a}(\widehat{x}, \widehat{w}, \widehat{w}^{+}). \end{aligned} \tag{10.8}$$

Using his alternative mathematical methods [2–3], the observer constructs the geometry under condition (10.8). He defines the scalar curvature and writes the alternative integral of action

$$\widehat{S} = \widehat{\int} \widehat{L}(\widehat{x}) \widehat{\det}\, \widehat{e}^{\nu}_{\tau} \widehat{d}^4 \widehat{x}, \tag{10.9}$$

where

$$\widehat{L}(\widehat{x}) = \frac{1}{\widehat{V}_0} \widehat{\int_{\widehat{\rho} \geq \widehat{\ell}^2_0}} \widehat{R}(\widehat{x}, \widehat{w}, \widehat{w}^{+}) \widehat{\det}\, \widehat{e}^{\xi}_{\eta}\, \widehat{d}\widehat{V}_{\widehat{w}}. \tag{10.10}$$

Performing the same approximate calculations as when deriving (8.19), the observer finds the Lagrangian from (10.2):

$$\begin{aligned} \widehat{L}(\widehat{x}) = {} & \frac{1}{\widehat{H}} \overset{\widehat{\circ\circ}}{R} \ddot{+} \widehat{\mu}_2\, i \sqrt{\widehat{H}}\, \overset{\widehat{\circ}}{F}_{\widehat{\nu}\widehat{\tau}}\, \widehat{\Psi} \widehat{\cdot} [\widehat{\gamma}^{\widehat{\tau}} \widehat{,} \widehat{\gamma}^{\widehat{\nu}}] \widehat{\cdot} \widehat{\Psi} \ddot{+} \widehat{\mu}_4\, \overset{\widehat{\circ}}{F}{}^{\widehat{\nu}}_{\widehat{\tau}}\, \overset{\widehat{\circ}}{F}{}^{\widehat{\tau}}_{\widehat{\nu}} \\ & \ddot{+} \mu_5 i \left(\widehat{\nabla}_{\widehat{\mu}} \overline{\widehat{\Psi}} \widehat{\cdot} \widehat{\gamma}^{\widehat{\nu}} \widehat{\cdot} \widehat{\Psi} \ddot{-} \overline{\widehat{\Psi}} \widehat{\cdot} \widehat{\gamma}^{\widehat{\mu}}_{\widehat{\cdot}}\, \widehat{\nabla}_{\widehat{\mu}}\, \widehat{\Psi} \right) \ddot{+} \widehat{\mu}_6 \frac{1}{\sqrt{\widehat{H}}} \overline{\widehat{\Psi}} \widehat{\cdot} \widehat{\Psi}. \end{aligned} \tag{10.11}$$

where

$$\widehat{\nabla}_{\nu} \widehat{\Psi} = \widehat{\widehat{D}}_{\nu} \widehat{\Psi} \ddot{+} 8 \widehat{\Gamma}_{\nu}{}^{\widehat{\sigma}}_{\widehat{\mu}} [\widehat{\gamma}_{\widehat{\sigma}} \widehat{,} \widehat{\gamma}^{\widehat{\mu}}] \widehat{\cdot} \widehat{\Psi} \ddot{+} i \widehat{\varphi}_{\nu} \widehat{\Psi}.$$

The a-conjugancy of Lagrangians (8.19) and (10.11) can be proved by using the mathematical tools presented in [2–3] and equalities (10.5). This means that each compensating field has its own compensating antifield.

(6) The alternativeness inherent in the double numerical field is preserved when passing to the double process (process and antiprocess). This tells us that during the origination of the process there is no antiprocess and, vice versa, if there is the antiprocess, then the process is absent. This conclusion should not be surprising, since the formation of the double numerical field is a consequence of the properties inherent in a double process.

The above reasoning implies that there is no interaction between the process and the antiprocess. Be it otherwise, the alternativeness law would be violated. Therefore the process and the antiprocess are not mutually observable.

(7) Let us consider the space Π^N. To each process from Π^N we assign the antiprocess. We call the obtained set the space of antiprocesses of order N and denote it by $\widehat{\Pi}^N$. If we carry out some algebraic operation in Π^N, then the same operations can be carried out in $\widehat{\Pi}^N$ and they will be isomorphic to each other. Then the mutual observability of processes from Π^N implies the mutual observability of processes within the space $\widehat{\Pi}^N$.

Assume that there is some antiprocess $\widehat{\pi} \in \widehat{\Pi}^N$ that is observable from a process $\pi_1 \in \Pi^N$. $\widehat{\pi}$ is the antiprocess of a process $\pi \in \Pi^N$. If the observer is in π, then he can pass from π to π_1 and then to $\widehat{\pi}$. Using the transitive property of passages of the observable processes that has been discussed in (5) of Section *9*, we come to the conclusion that the processes π and $\widehat{\pi}$ are mutually observable. But this contradicts the conclusion of (6) on the mutual observability of the process and the antiprocess.

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Thus the spaces of processes Π^N and $\widehat{\Pi}^N$ are mutually nonobservable.

(8) Let $\widehat{\pi}_1, \widehat{\pi}_2 \in \widehat{\Pi}^N$ be respectively the antiprocesses of the processes $\pi_1, \pi_2 \in \Pi^N$. As stated in (7) of Section *9*, there exists a transformation changing the process π_1 into π_2. In [2] its has been shown that this transformation converts with no changes at all the antiprocess $\widehat{\pi}_1$ to the antiprocess $\widehat{\pi}_2$. Algebraic objects from antiprocesses are formed by the same rule as the corresponding objects from processes. In particular, if $\widehat{x}_1^\nu$ and $\widehat{x}_2^\nu$ are the coordinates of the respective antiprocesses $\widehat{\pi}_1$ and $\widehat{\pi}_2$, then by (9.2) can write

$$\widehat{x}_2^\nu = f_{12}(\widehat{x}_1{}^\nu). \tag{10.12}$$

11. On gravitation

Let us again turn our attention to the space of processes Π^N. As has been indicated, Π^N consists of mutually observable but not interacting processes of order N.

When one process converts to another process, for frames of reference there arises a nonlinear transformation of form (9.2). As has been said in Section *9* these transformations have quite specific features but at the same time they are of quite a general character. After all Π^N consists of a multitude of arbitrary processes of order N. The only restriction is that in its system each process is invariant with respect to the Lorentz group.

It is appropriate to recall here Einstein's lift. The arguments of the great scientist as to this mental experiment are well applicable for the interpretation of transformation (9.2). Then, being guided by Einstein's theory of gravitation, due to the existence of transformation (9.2) we make the following conclusion: between processes Π^N there must exist gravitational interaction.

It is obvious that the reasoning given above for transformation (9.2) is quite applicable to transformation (10.12) as well. Then we can say that there exists gravitational interaction between processes of the space $\widehat{\Pi}^N$.

Under the a-conjugation, processes $\pi_1, \pi_2 \in \Pi^N$ transform respectively to antiprocesses $\widehat{\pi}_1, \widehat{\pi}_2 \in \widehat{\Pi}^N$. From (9.2) and (10.12) we see that the a-conjugation does not change the function f_{12}. Therefore for the a-conjugation the gravitational interaction does not change, i.e. for gravitation there exists no antigravitation.

We can evidently say that, while acting, gravitation does not distinguish between processes and antiprocesses. For gravitation they are the same objects. But then using conclusions of (7) of Section *10*, one can explain the existence of dark matter [9].

In this stage we limit our discussion to the above-given argumentation, since the construction of a comprehensive theory of gravitation from the position of the algebraic properties of differential equations requires meticulous investigation.

12. Double algebra and quantization

(1) Let us consider the space of processes Π^N. As has been said, processes from Π^N are mutually observable. Moreover, there always exists a transformation enabling the observer to pass from one process to any other process. In this context, processes from Π^N can be called equivalent processes.

As has been stated in (6) of Section *9*, among other processes of order N there exist, in Π^N, processes consisting of N elementary processes which interact with each other. (9.1) is just a such-like process. It is obvious that in its system of calculus each of these equations takes a linear form.

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Therefore any process from Π^N can be represented in its system of calculus as a set of N elementary processes of which each one is described by its linear differential equation of first order.

(2) In view of the results obtained in Section *10* from [2] and in Section *1* from [1], we think that to construct the consistent and complete algebro-geometric theory of differential equations their solutions must be represented in the matrix form. Though the mechanism of realization of this representation for a given equation needs further detailed investigation, we nevertheless can already make some conclusions.

As is shown in Section *10* from [2], if solutions are represented in the matrix form, an algebraic field expands to an algebraic body [6]. The group properties of a differential equation and the availability of an algebraic body must form a certain algebraic ensemble. It is this ensemble that controls a given process and its changes, defines the rule of interaction with other processes and establishes a possible splitting into other processes. This object forms its elementary process and it can hardly be split into more elementary processes without violating the algebraic body. Perhaps the existence of elementary particles can be explained by these irreducible elementary processes.

(3) Along with the above-said, if we take into account equalities (10.37–38) [2] which are fulfilled for alternative processes, we will not be evidently far from the truth if we say that the quantization of a field is the joint manifestation of the algebraic properties of a process and its antiprocess. Thus we come to a conclusion that, speaking in general, any field described by a differential equation must be quantized.

All this taken together gives us an analogy with photons passing through a plate with two closely lying holes. We assume that the path of a photon from the source to the screen through the first hole is one state of the photon, and through the second hole the second state. For sufficiently large energy of a photon flux two light spots are formed on the screen, i.e. photons behave like classical particles. In that case we say that if a photon is in one state, then the second state is excluded, i.e. these states have an alternative character. When the flux energy decreases, the screen shows the interference picture. Then we say that alternative states of a photon interfere with each.

It would be appropriate to mention here that the probabilistic aspects of quantum theory play an important role in the questions discussed. Therefore, because of the emergence of a numerical double field, alternative analysis and, as a consequence, of the anti process for every process we are faced with a necessity to understand the probability theory in a new light. By taking a course for interpreting the probability theory from the standpoint of the algebraic theory of processes we acquire the right to expect nontrivial results and conclusions. A possibility should not be ruled out that the existence of living matter can be explained in terms of anti-process of a stochastic process.

(4) In Section *11* we have come to the conclusion that gravitational interaction involves all processes without making any distinction between processes and antiprocesses. Simultaneously, we have seen that gravitation and antigravitation identically coincide. Then because of the absence of alternativeness, quantization for a gravitational field becomes meaningless.

Conclusion

In the investigation that we have carried out we wanted first and foremost to reveal those mathematical structures and regularities of the field theory that manifest themselves through the algebro-geometric properties of differential equations. We tried not to enforce any postulates or conjectures from outside.

It is astonishing how much a differential equation can tell us. Evidently, it is not a big exaggeration

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to say that answers to many if not to all fundamental questions concerning the construction of space and the properties of matter are hidden in the algebro-geometric properties of differential equations.

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