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KNOT THEORY AND PARTICLE PHYSICS

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Abstract

In this work (representing also enlarge versión of [18] our preliminary article "Knot Theory and Particle Physics published in the "Proceed.of VIAM",v.44,1992) an attampt is made to do topological interpretation of quantum numbers (s,Q,B,S,J) and discrete symmetries (C,P,CP) of particles. As a result,a topological "diagram" is obtained,which can be "read" as Gell-Mann-Nishijima rule.In the last part a séquense of cohomological groups describing hierarchy of anomalies well-known in QFT is interprated as a certain sequence of cohomological groups for cut knots.Further the topological singularity model for elementary particles is proposed. A particle is identified with a branching set of the 3-dim space,which is ingeneral a knotted S1-spere.

As application in the last part Knotted singularity model for elementary particles is proposed and hence some consequences for cosmology are obtained which are in good agreement with the recent astronomical data.

Keywords: *noninvertible knots, neutral kaons,particle, handle, integral curvature, spin, meson, Mewbius sheet, Weil anomaly, cut knot, chain, homological group, manifold, cosmological problems*

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Part I. Introduction

1.1. *Noninvertible knots; analogy to* K^0*-mesons*

For space containing a knotted S^1 sphere, there are two operations of conversion of orientation: oriented conversion of the whole space ρ, that is a equivalent to taking a mirror image of the knot, and oriented conversion of knot σ – transformation of direction of circumvention along the knot. Knots ρl and σl are considered to belong to l type knot.

Knots are divided into mirror and nonmirror ones: if a knot can be deformed into its mirror image, it is called a mirror, otherwise – nonmirror.

In 1963 Trotter found out endless variety of noninvertible knots: for such knots transformed direction of circumvention is given by means of nonisotopic towards original knot, i.e, four $\{l,\rho l,\sigma l,\rho\sigma l\}$ belonging to the same type of knots, (i.e., having one and the same fundamental group), but all four nonisotopic to one another [17].

Let's consider linkage $(l\otimes\rho l)_m$ with the coefficient $+m$: mirror image of such a knot will differ from initial only by linkage coefficient:

$$\rho\left[(l\otimes\rho l)_{+m}\right]=(l\otimes\rho l)_{-m}.$$

From $\{l,\ldots\}$ we can construct "$\pm$ mirror" knots: (a knot is + mirror, if $\rho l=l$ and - mirror, if $\rho l=\sigma l$).

Knots I: $l\#\rho l,\quad \sigma l\#\rho\sigma l,$ will be "+ mirror", and

Knots II: $l\#\sigma l,\quad \rho l\#\sigma l,$ - "- mirror".

i.e., to these knots we can ascribe their own meaning in relation to the operations $\rho,\sigma,\rho\sigma$:

I $\quad[\rho]=+1,\quad[\sigma]=-1,\quad[\rho\sigma]=-1,$

II $\quad[\rho]=-1,\quad[\sigma]=-1,\quad[\rho\sigma]=+1.$

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Let's recall now some features of neutral K^0-mesons: in strong and electro-magnetic interactions there take part pseudo scalar K^0 and $\overline{K^0}$-mesons, having definite strangeness (+ I and – I respectively), but having no definite charged and combined parity:

$$\hat{P} \mid K_0 \rangle = - \mid K_0 \rangle,$$
$$\hat{C} \mid K_0 \rangle = \mid \overline{K_0} \rangle,$$
$$\hat{C}\hat{P} \mid K_0 \rangle = - \mid \overline{K_0} \rangle,$$

and in weak interactions only really neutral K_1^0 and K_2^0 –mesons take part, having no strangeness, but characterized by definite charged and combined parity:

$$\mid K_1^0 \rangle = \frac{1}{\sqrt{2}} \left(\mid K^0 \rangle - \mid \overline{K^0} \rangle \right), \quad \hat{P} \mid K_1^0 \rangle = - \mid K_1^0 \rangle, \quad \hat{C} \mid K_1^0 \rangle = - \mid K_1^0 \rangle, \quad \hat{C}\hat{P} \mid K_1^0 \rangle = \mid K_1^0 \rangle,$$

$$\mid K_2^0 \rangle = \frac{1}{\sqrt{2}} \left(\mid K^0 \rangle + \mid \overline{K^0} \rangle \right), \quad \hat{P} \mid K_2^0 \rangle = - \mid K_2^0 \rangle, \quad \hat{C} \mid K_2^0 \rangle = - \mid K_2^0 \rangle, \quad \hat{C}\hat{P} \mid K_2^0 \rangle = - \mid K_2^0 \rangle.$$

As is known, neutral kaons may exists as K^0 and as $\overline{K^0}$ – mesons and as K_1^0 and K_2^0 – mesons, each pair being superposition of the other. Then, we watch appearance of $\overline{K^0}$ – mesons in K^0 bundle (disturbance $\hat{P}$ – invariance), and also self-arbitrary transformation of K_1^0 and K_2^0 into each other (disturbance $\hat{C}\hat{P}$ - invariance).

All this gives us possibility to detect some analogy between the properties of neutral kaons and above-mentioned knots, if we compare:

$$\hat{P} \to \sigma,$$
$$\hat{C} \to \rho,$$
$$\hat{C}\hat{P} \to \rho\sigma,$$

then to transformation of various neutral kaons into one another will correspond various combinations (compositions and linkage) of initial four knots:

$$\left((\sigma) l \otimes (\sigma) \rho l \right)_{+m} \to \mid K^0 \rangle, \qquad \left((\sigma) l \otimes (\sigma) \rho l \right)_{-m} \to \mid \overline{K^0} \rangle,$$

$$\begin{Bmatrix} l \# \rho l \\ \sigma l \# \sigma \rho l \end{Bmatrix} \to \mid K_2^0 \rangle, \qquad \begin{Bmatrix} l \# \sigma \rho l \\ \sigma l \# \rho l \end{Bmatrix} \to \mid K_1^0 \rangle.$$

In case we find $\hat{C}$ and $\hat{P}$ – parity K_1^0 – meson out of wave function $\mid K_1^0 \rangle = 1/\sqrt{2} \left[\mid K^0 \rangle - \mid \overline{K^0} \rangle \right]$, then we would have got $\left[\hat{P} \right] = \left[\hat{C} \right] = -1$, but decomposition $K_1^0 \to \pi^0 + \pi^0$ induces us to think $\left[\hat{P} \right] = \left[\hat{C} \right] = +1$.

This fact can easily be explained in terms of knots.

The knot $l \# \sigma \rho l$ is a section of knotty 2-sphere, lying in R^4 (with this components are already rightly oriented).

But as is pointed in [5], it is not known whether there exist locally flat 2-discs in R^3 and R^4, that are not mirror and invertible, i.e., we have all the reasons to suppose that a disc Δ_1^3 defined by a knot $l \# \sigma \rho l$ in four-dimensional semispace is isotopic towards the disc Δ_2^3, defined by a knot $\rho l \# \sigma l$; then this pair of knots represents different sections of the same disc $\Delta^3 = R^4$.

Then, for the disc corresponding to our $\mid K_1^0 \rangle$.

$$\left[\hat{\rho} \right] = \left[\hat{\sigma} \right] = \left[\hat{\rho}\hat{\sigma} \right] = +1 \qquad \left(\text{in } \; R^4 \right).$$

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1.2. *Type of knot. Analogy with spin*

On any knot one can put various surfaces – each having its own type g_i. Let $g = \min(g_i)$. Then g is a type of knot.

The surface of a minimal type can be oriented and nonoriented as well. (Orientation can be defined with the help of so-called Frenkle criterion).

One-sided surfaces are then homeomorphic to several times twisted Mewbius sheet with some handles; and a normal form corresponds to two-sided surfaces and is a sphere with P handles and I hole.

If we consider the doubling of these surfaces, realized by sticking of 2 surfaces along border curve of a knot, then in case of one-sided surface we receive $\square P^2$ with $(2n+1)$ handles, and in case of two-sided – sphere with $2n$ handles.

Integral curvature is simply defined by a number of handles and is not dependent on one-sidedness).

Nonoriented surface has a period 4π, i.e., small oriented circumference, moving along the surface (which we consider having no thickness), comes back to the initial position after two full rounds along the closed circle.

Wave function of fermion has also a period 4π (realizes spinoral presentation of Lorenz group).

If we take on the surface of type g oriented loop which divides surface into two parts and consider the difference of integral curvature of right and left parts, received while drawing up the loop into a point,

$$\Delta J = J_1 - J_2 = 4\pi(1-g_1) - 4\pi(1-g_2) = 4\pi(g_2 - g_1),$$

then magnitude $\Delta J / 8\pi$ will have the meaning $\pm\frac{1}{2}; \pm\frac{3}{2}; \cdots$ for the surfaces with odd handles and $0; \pm 1; \pm 2; \cdots$ for the surfaces of even type; in all $g+1$ meanings.

It is known that if we put close from infinity 2 knots of p_1 and p_2 type, bound on one contour, then on their composition we can stretch surface of $p_1 + p_2$ type, but in common surface can be found to be linked. Then linking can be characterized by integral curvature of surface, obtained when each linked handle completely fills corresponding hole of the second surface. With this surfaces of type p, and p_2 can give surface of types $p_1 + p_2, p_1 + p_2 - 2, \ldots, p_1 - p_2 + 2, p_1 - p_2$ – in all $(p_2 + 1)$ types of surfaces.

As in known, projection of spin of elementary particle on arbitrary direction can acquire the meaning: $-S, -S+1, \ldots, +S$, in all $2S+1$ meanings – in the units in case of bosons an $\pm\frac{1}{2}; \pm\frac{3}{2}; \cdots$ in case of fermions. In composition of spins of two particles S_1 and S_2 summary spin can turn out to be equal to numbers from $S_1 + S_2$, $S_1 + S_2 + 1, \ldots,$ to $S_1 - S_2$, $2S_2 + 1$ meanings in all.

All above said with the fact that particles with half-integer spin are subject to Fermi statistics, and particles with complete spin are bosons and give us a possibility to find an analogy between the algebra of spin and description of surfaces, which determine the type of knots, if we compare a particle with spin S to the knot, whose surface doubling has a type $g = 2S$.

Part 2. Gell-Manu-Nishijima Rule, Yanss-Bonne Formula

2.1. *Branched cyclic coverings*

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Let's consider layers obtained on the doubled and interlinked knots for g – fold cyclic covering with the branching on the knot, performed with S^3 sphere, which can be described as following: consider the knot k as the edge oriented surface F, and let's cut S^3 sphere along this surface. We obtain manifold T, with the edge formed by 2 surfaces F' and F'', homeomorphic surface to F, which are stuck along the line k. Let's further take g examples of manifold T, namely $T_1, T_2, \ldots, T_g$ with the edges $F_1' \cup F_1''$, $F_2' \cup F_2'', \ldots, F_g' \cup F_g''$ and stuck them so that, surface F'' is pasted to F_2', F_2'' - to $F_3', \ldots, F_g''$ - to F_1'. The obtained manifold is g-fold branched cyclic covering conformable to knot k.

Thus, for any knot k with corresponding Zeifert surface we have $2g$ stuck "hemispheres".

Let's take a doubled knot and consider given Zeifert surface, as 2-fold covering of one-sided surface, corresponding to doubling knot. Then for a doubling knot we receive the layer of covering, consisting of g examples of stuck hemispheres.

Now let's consider covering conformable to two-component linkage. Instead of cutting S^3 along 2 intersecting surfaces we can take $1/2g$ examples of S^3 and for each component of linkage we can build branched covering. We get examples of stuck hemispheres within the covering layer.

So, with branched covering of a certain complex knot K with a fixed covering space, for doubling or linking we think we get twice more-valent cover then for é "enternal" knot K.

As is known, Euler characteristic of n-fold covering of space with χ_0 is $n\chi_0$. Hence for n-fold covering of space we have: $\chi_{\text{cov}} = n \cdot \chi(K) = 2n \cdot \chi$ (double, link).

Therefore, Euler characteristic of covered space (basic space) in case of doubled knots and linkage will be twice less than Euler characteristic of basic space in case of complex knot.

2.2. H_2^4 handles, equipped linkages

Let's consider handles of index 2 in four-dimensional space. Handle of dimension and of index λ is called direct product of two discs $H_\lambda^n = D^\lambda \times D^{n-\lambda}$. Handle H_λ^n is smooth manifold with the edges [10]:

$$\partial H_\lambda^n = \partial\left(D^\lambda \times D^{n-\lambda}\right) = \left(\partial D^\lambda \times \partial D^{n-\lambda}\right) \cup \left(D^\lambda \times \partial D^{n-\lambda}\right) =$$
$$= \left(S^{\lambda-1} \times D^{n-\lambda}\right) \cup \left(D^\lambda \times D^{n-\lambda-1}\right).$$

Sticking reflection $f : S^1 \times D^2 \to \partial W$ corresponding to 2-handles, can be considered as so-called equipment imbedded in circles.

Equipped linkage (in S^3) is called finite set of intersecting smooth imbedded circles $\gamma_1, \ldots, \gamma_n$ (knotted or not), each of them having the whole number n_i.

Geometric sense of number is that sticking reflection $f : S^1 \times D^2 \to \partial W$ with $f\left(S^1 \times 0\right) = \gamma_i$, associated with the pair $\left(\gamma_i, n_i\right)$ is such that for any $x \in D^2 \setminus 0$ circle $f\left(S^1 \times \{x\}\right)$ has linkage coefficient n_i with γ_i. This means that circle D^2 makes n_i revolutions in positive direction during one round of circle γ_i. (In such case they say that $f\left(S^1 \times \{x\}\right)$ is n_i – parallel curve for γ_i).

If M is one edge of oriented four-dimensional manifold V with induced orientation, then let V_L mean manifold that is obtained by sticking V along M handles according to equipped linkage L, edge ∂V_L is across $\chi_L(M)$. Construction of manifold V_L depends on orientation V and is independent of orientation of linkage component [12].

Let's take any knot k.

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Zeifert surface N corresponding to any knot k isotopic may be transformed into a square with stuck bands with curls [5]. $N \cup \tilde{N}$ is a joining of N surface with its image $\tilde{N}$, symmetrical in relation to a straight line on which connected part of limit ∂N is situated (we men rotation on 180°, but not reflection), n_i – a number of half-revolutions of each bend, $\tilde{a}_i$ – curves on $N \cup \tilde{N}$ which are obtained from a_i, realizing a basis of group $H_1(N)$ in the same way as $N \cup \tilde{N}$ from N.

Thus, if L – equipped linkage in with components $\tilde{a}_i$ with equipment n_i, then corresponding to this L manifold B_L and $\chi_L(S^3)$ are two-sheet branching covering of bell B^4 and sphere S^3 with branching over k and N.

If γ_i, γ_j – components of equipped linkage L, then γ_i is added to γ_j in the following way. Let $\tilde{\gamma}_i$ – be equipped component (n_i), and $\tilde{\gamma}_i$ be n_i – parallel curve for γ_i. Let's change L, replacing γ_j for $\tilde{\gamma}_i \# \gamma_j$ such transformation does not change either B_L or $\chi_L(M)$. It simply gives their different decomposition into handles.

2.3. *'"Senior" handle of a knot*

Let's apply described transformation in turn to all generators $\tilde{a}_i$ on the surface $N \cup \tilde{N}$ of a knot k and characterize the knot by "senior handle", which is obtained as a result of summing up of all the handles: $n' = \Sigma n_i + \Sigma a_{i,i+1}$.

Let's consider $k \# k$, $\rho k \# \rho k$, $k \# \rho k$ with equipments of "senior handle". $+n$, $-n$, 0.

Noting that addition to Zeifert surface of a hand with q half-revolutions leads to joining of 2-handles with equipment q to two-sheet branched covering manifold let's ascribe to handles with equipment $(+n)$ and $(-n)$ Euler characteristic corresponding to twisted rings, but with the respect to sign of equipment, i.e., ± 1 and 0 – to a handle with zero equipment.

We'll see below, that for a knot of $k \# \rho k$ type 2-handlecan be removed in fact.

So, for knot of k in B^4 we have 3 "degrees of freedom", corresponding to right, left and zero handles. If we compare each of them, as is the case with spin, to quantities $\Delta J / 2\pi$ taken for regions separated by holes, and by χ assign Euler characteristic of twisted ring covered by handle, we get a "diagram" (I).

If illustrates the behavior of mesons with isospin equal to I.

Let's consider now the equipment of the pairs of linked noninvertible knots. The transformation of more general type than discussed above, which leavers $\chi_L(M)$ unchanged, is shown in Fig. (2).

This transformation allows us to conclude that positive linking coefficients, which may appear to linked noninvertible knots of type (p,q,r), having all the parameters of the same sign, has the same sign as equipment of linking component.

Therefore, linking of this family of knots may have the following type:

$$\begin{array}{ccccc} & k \otimes k & k \otimes \rho k & k \otimes \rho k & \rho k \otimes \rho k \\ & +n\ +n & +n\ -n & +n\ -n & -n\ -n \\ lk: & +a & +a & -a & -a \\ n' & 2n+2a & 2a & -2a & -2n-2a \end{array}$$

We had taken Euler characteristic of arising basic space for 2-fold covering with equipped handle, i.e., basic space – twisted hand – was substituting Zeifert surface of s knot. In case of linked knots with fixed coefficient of linkage we have two "degrees of freedom", corresponding to the handles with equipment $2a$ and $(2n+2a)$.

Availability of a handle equipped with $(2n+2a)$ is equivalent to the existence of two noninterlinked knots, with 2-fold covering equipped with $(2n)$ and $(2a)$. In this case a handle

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equipped with exists due to linking and a handle equipped with $(2n)$ – entirely at the expense of knots.

Analogous decompositions in case of knots $(k\times\rho k)$ gives a handle equipped with $2a$-at the expense of linkage and a handle with zero equipment – at the expense of amphichieralty of knots.

As we put in accordance to 2-fold cover by equipped 2-handle $\pm$ Euler characteristic of arising, i.e., ±1, then, according to point (2), in case of linkage we shall take $\pm1/2\chi$ of a band arising in the basic space.

In terms of considered above analogy, we assign to K^0 – mesons with strangeness + I: in the first case – handles equipped with $2n$ and $2a$ corresponding to them Euler characteristic with coefficients: +1 and +1/2; in the second case – handles equipped with 0 and $2a$ – half the Euler characteristic of twisted hand – with coefficient +1/2/ In case of mirror linkage - $Sa-1$ we shall have for Euler characteristic with coefficients: (-1 and (-1/2)) and (-1/2) (see Fig.).

2.4. *Handles of a cut knot*

Suppose, we construct manifold $\chi_L(M)$ for L consisting of $\{\tilde{a}_i\}$ on $N\cup\tilde{N}$ of a cut knot of a $k\#\rho k$ type. As is shown in [17] , oriented closed four-dimensional manifold is wholly defined by handles $H^0\cup\lambda H^{(1)}\cup\mu H^{(2)}$.

A out knot (type $k\#\rho k$) in (semi) R^4 limits locally a flat disc Δ^3 and defines knotted 2-pshere which is the edge of Δ^3 in R^4.

We'll imply that disc Δ^3 lies in limiting M^4 3-dimensional manifold $\chi_L(M)$.

Then we can juxtapose to a cut knot, extra 3-handle in comparison with a non-cut one: $\Delta^3\times D^1$; $\partial\left(\Delta^3\times D^1\right)=\left(S^2\times D^1\right)\cup\left(\Delta^3\times S^0\right)$ with sticking reflection $f:S^2\times D^1\to\partial H_3^4$. The remaining part of the border $S^0\times D^3$ can be intersected with initial (before constructing H_3^4) manifold V^4 by two-dimensional knotty sphere $\partial\left(\Delta^3\times S^0\right)=S^2\times S^0$.

(Analogous to a described operation in R^3 will be sticking of a handle of index 2 to a full torus, if for a sticking circle we take any toroidal knot, for there may exist any number of non-intersecting toroidal knots of the same type on the torus. Four such knots oermit us to construct H_2^3 (see Fig.).

Let's see now 2-handles connected with the family of knots $\{k_i\}$ which we can compare with the family of K^0-mesons. Let's distinguish handles H_2^4, which exist because of linkage (not concerning equipments caused by separate simple knots of (p,q,r) type).

$$lk=\pm m,\qquad n\left(H_2^4(lk)\right)\neq 0,$$

$$lk=0,\qquad n\left(H_2^4(lk)\right)=0.$$

Presence of a cut knot, as was mentioned, causes an additional 3-handle. It is known, however, that if manifold M_2^4 is obtained from manifold M_1^n by successive sticking of handle $H_\lambda^{(i)}$ and $H_{\lambda+1}^{(i1)}$ so, that stacking sphere of handle $H^{(i1)}$ transversally intersects secant of handle $H^{(1)}$ exactly in one point, then this pair of handles can be removed. This means, that 3-handle of cut knot may be removed be (equipped) 2-handle, due to linkage.

If there is no cut knot (e.g., $k\#k$), then handle $H_2^4(lk)$ cannot be removed.

All this said above permits us to illustrate in terms of knots the fact of non-existence of non-strange K – mesons, (knots $k\#k$), with availability of strange and unstrange K^0 – mesons $\left(k\otimes\rho k,\ \ k\#k\right)$, with $(S=1,\ \ S=0)$ if we demand for this family of knots "conservation of handles" $H_\lambda^4(lk)$.

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2.5. *Isospin diagram*

Facts stated above can serve as some quality illustration of quantum numbers Q, S, B, I and the fact that S and B appear with coefficient 1/2, appearing next to Q on the rule of Gell-Mann-Nishijima.

But let's try to unite these topological invariants, compared to quantum numbers, in one comparatively simple object.

Consider surface of the type n. Let's assign as before, to topologically different points,

$$\frac{\Delta J}{8\pi} = \frac{1}{2}\{(1-g_1)-(1-g_2)\}.$$

Assign also to any point Euler characteristic of submanifold of the given surface, lying from "positive" side, (i.e., side, corresponding to positive J) in relation to a separating loop drawn into a point. Then $\chi(g_1)$ will sun through the values 2, 0, -1, -4 and so on.

If we compare to electricity charge $Q \to \frac{1}{2}\chi(g_1)$ third projection of isospin $I_3 \to \Delta J / 8\pi$ strangeness $S \to \frac{1}{2}\chi(n-1)$, i.e., half the Euler characteristic of a surface of type $(n-1)$ (yet without beryon number), then Gell-Mann-Nishijima rule (with $B = 0$) will be this: $\chi(g_1)/2 = \Delta J / 8\pi + 1/2\,\chi(n-1)/2$. Noticing that with

$$n = g_1 + g_2, \quad \chi(n-1) = \chi(g_1 + g_2 - 1) = \chi(g_1) + \chi(g_2),$$

$$2 \cdot \frac{\Delta J}{8\pi} = \chi(g_1) - \frac{1}{2}\chi(n-1) = \frac{1}{2}\left[\chi(g_1) - \chi(g_2)\right].$$

we come to Gauss-Bonne formula $J = 2\pi\chi$.

Now let's add to the surface of type n one handle of "doubling" (that corresponds to $B = 1$) from the side of negative J. The full number of handles: $n' = n+1$. We'll assume that spectre Q and I_3 hasn't changed (g_2 doesn't influence Q), $S \to \frac{1}{2}\chi(n'-1)$, i.e., S is the same as before, but for the full number of handles, a doubling handle will be considered.

It is easy to see that again we can get identity:

$$Q - \frac{S}{2} = I_3 + \frac{B}{2} \quad \Rightarrow \quad \chi(g_1) - \chi(g_2') = \frac{1}{2\pi}\Delta J'.$$

A diagram can be easily generalized for the case of "negative" doubling handle, which destroys one basic handle: $n' = n-1$.

Let's assume that the whole diagram changed its sign. Euler characteristic $\chi_i(Q)$ is multiplied by (-I) and take ΔJ corresponding to different topological points, in inverse order; S is also changed by $-S = -\frac{1}{2}\chi(n'-1)$.

Thus, suggested topological analogies of quantum numbers permit us to write down Cell-Mann-Nishijima rule, as Gauss-Bonne formula for closed oriented surface of a certain type.

Diagram (y) describes isotopic multiplets, containing at least one charged particle and including no quantum numbers following strangeness (mesons with $I = I$, kaons, nuclons, $\Delta^-, \Lambda^-, \Sigma^-, \Xi^-, \Omega$ - hyperongs).

It would allow us to explain assymetric spectre of electricity change in the family Δ of hyperons: $\Delta^{++}, \Delta^{+}, \Delta^{0}, \Delta^{-}$.

With greater isospin assymetry must increase – supposed spectre can be:

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$$B = +1, \quad x^-, x^0, x^+, x^{++}, x^{+++}$$

and so on.

With this, for example, we can have multiplets with the following quantum numbers:

$$B = +1, \quad I = 2, \quad S = 3,$$

$$B = +1, \quad I = \frac{5}{2}, \quad S = 4.$$

Part 3. On Cut Knots and Hierarchy of Anomalies

3.1. On quantum anomalies

On quantizing field theory, one may need introduction of new – regulatory field and, corresponding change of classical action and equations of motion. For each physical field several regulatory fields are introduced. Interaction between regulators and with initial fields has the same type as interaction of physical fields with the only exception have great mass M_{reg}.

The symmetry of resultant action may turn out to be more narrow than the symmetry of initial one.

Symmetries characterizing of massless particles – as conformal, axial, gauge, supersymmetry – may not be generalized for massless regulatory fields. This noninvariance may remain for the limit $M_{reg} \to \infty$ when "removing" regularization. In such a case quantum anomaly is introduced which is responsible for the breaking of classical symmetry.

Anomaly on global symmetry manifests itself in the nonconservation of regularized Noeter current

$$J_\mu^{reg} = J_\mu(\psi) - J_\mu(\Psi),$$
$$\partial_\mu J_\mu^{reg} = -\partial_\mu J_\mu(\Psi) = \hbar A\{\Phi\},$$

(where Ψ are regulatory fields) anomaly in the local symmetry is related to nonzero value of covariant divergence of regularized current of matter interacting with gauge bosons. Obviously, in the selfcoordinated theory contributions of different particles into internal anomaly must reduce each other.

Anomalies breaking global symmetries cause deformations of spectra of masses, dissappearance of degeneration of states, forbidden channels of decay are opened, amplitudes of reaction may change, but no now physical states arise, the unitarity and ultraviolet properties of theory are preserved. So breaking of a global symmetry doesn't change selfcoordination of theory.

Anomalies of fermion currents have the following form:

1. Abelian Dirac anomaly in $2n$ dimensions

$$\langle \partial_\mu J_\mu^5 \rangle = 2S_p M g^5 \frac{1}{i\widehat{b} - iM} = \int W^{(2n)}(A) d^{2n}x.$$

2. Coloured Dirac anomaly in $2n$

$$\langle D_\mu J_\mu^5 \rangle^a = 2S_p M g^5 t^a \frac{1}{i\widehat{b} - iM} = \int W^{a(2n)}(A) d^{2n}x.$$

3. Weil anomaly in the left current in $2n$ dimensions

$$\langle D_\mu J_\mu^L \rangle^a = 2S_p M g^5 t^a \frac{1}{i\widehat{b} - iM} = \int W_1^{a(2n)}(A) d^{2n}x,$$

$$\widehat{b} = \overline{\partial} + \widehat{A}\frac{1-\gamma^5}{2}.$$

4. Anomalous part of fermion determinant in $(2n+1)$ dimensions

$$\left\{ \ln \det(i\widehat{b} - iM) \right\} = \int W_0^{(2n+1)}(A) d^{2n+1}x.$$

In the works [2,9,10, 20,21] the following relations have been found between them

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$$dW_0^{(2n+1)} = \frac{\pi}{2} W^{2n+2} \qquad \text{or} \qquad W^{a(2n)} = 4\frac{\delta W_0^{(2n+1)}}{\delta A^a},$$

$$d\left(u^a W_1^{a(2n)}\right) = 4\delta_u W_0^{2n+2} \qquad \text{or} \qquad u^a W_1^{a(2n)} = 4k_z W_0^{(2n+1)},$$

where operations d and δ have the property: $\Delta^2 = 0$. "δ" acts in the space of A_D^n forms (forms, defined for a space of fixed dimension)

$$d^2 = 0: \quad A_D^0 \xrightarrow{\delta} A_D^1 \xrightarrow{\delta} \cdots \xrightarrow{\delta} 0.$$

The operation of external derivation d transforms $A_D^n \to A_{D+1}^n$ and also satisfies the condition $d^2 = 0$.

There is a commutative diagram:

$$\begin{array}{ccccccc}
d\uparrow & & d\uparrow & & & d\uparrow & \\
A_D^0 & \xrightarrow{\delta} & A_D^1 & \xrightarrow{\delta} \cdots \xrightarrow{\delta} & & A_D^n & \xrightarrow{\delta} \\
d\uparrow & & d\uparrow & & & d\uparrow & \\
\cdots & & \cdots & & & \cdots & \\
d\uparrow & & d\uparrow & & & d\uparrow & \\
A_1^0 & \xrightarrow{\delta} & A_1^1 & \xrightarrow{\delta} \cdots \xrightarrow{\delta} & & A_1^n & \xrightarrow{\delta} \\
d\uparrow & & d\uparrow & & & d\uparrow & \\
A_0^0 & \xrightarrow{\delta} & A_0^1 & \xrightarrow{\delta} \cdots \xrightarrow{\delta} & & A_0^n & \xrightarrow{\delta}
\end{array}$$

from which we conclude that Δ-closed elements L_D^n are transformed into closed elements L_{D-1}^{n+1}.
More than that, Δ-exact elements from are transformed into Δ-exact elements L_{D-1}^{n+1}.

This means, that an operation P and can be constructed:

$$H_D^n(\Delta) \xrightarrow{p} H_{D-1}^{n+1}(\Delta) \xrightarrow{p} \cdots$$

3.2. *Chain of homologies of cut knots. Wild knots. Analogy with the hierarchy of anomalies*

As it has been already mentioned, knots of type $k\#\rho k$ may be represented as "equatorial" section of locally plane knotted 2-sphere in R^4 by hyperplane R^3. Such knots are called cut knots.

According to [13] we will designate by $P\#Q$ a connected sum by edge of manifolds P and Q (with border sets), and $P\#Q$ their connected sum by their interior.

In these designations we may write:

$$\begin{cases} S_\Phi^1 \# \rho S_\Phi^1 = \partial\sigma^2 \\ \sigma_1^2 \# \rho\sigma^{(2)} = S_\Phi^2 \end{cases} \qquad \begin{cases} S_\Phi^2 \# \rho S_\Phi^2 = \partial\Delta^3 \\ \Delta^3 \# \rho\Delta^3 = S_\Phi^3 \end{cases} \qquad \text{£F \# \#} \qquad (*)$$

(For the operation "$\#$" in case S_Φ^2 and ρS_Φ^2 are isotopic we must also invert orientation of $\left(\rho S_\Phi^2\right)$ – operation σ is implied here).

Here S_Φ^n designates knotted $S^n \subset R^{n+2}$ and $\sigma^{(2)}$, Δ^3 – correspondingly 2 and 3 -dimensional discs.

Expressions (*) are determination of cut knots generalized for higher dimensions.

Hence, transition from a cut knot to a knot of following dimension is realized by successive application of operations # and # for a knot and its mirror image. Let's now consider possibility of application of the operation of connected sum # by interior (it is evident, that repeated "$\#$" gas no sense), it means sticking up by their interior the discs Δ^n and $\rho\Delta^n$ lying on different sides from secant hyperplane R^n and representing two halves of the knotted sphere S_Φ^n. This can be noted: if we turn knotted surface inside out we get a surface with mirror knotting. Thus turning of "knotted semisphere" inside out in relation to separating hyperplane R^n we get isotopic discs on the same side of R^n and R^{n+1}. If after this we turn one of the halves by 180° around "vertical" (it means perpendicular to R^n) axis and then stick them up (which is possible because of their isotopy) we get the identification of diametrally opposite points of the border of the ball R^{n+1}, i.e., we get a model of projective space RP^n.

Recall now, that in the projective space RP^n different from ordinary Euclidean space, the existence of oriented one-sided and nonoriented two-sided surfaces in possible [19].

Comparing these properties with the properties of particle according to ch. I we conclude that in this RP^3 we would observe discrepancy of spin and statistics, i.e., existence of fermions with integer spin and bosons with half-integer spin, would be possible, which means the existence of ghosts and antighosts of Fadeev-Popov and, maybe, other nonphysical states.

These ghosts, as is known, differ from their antiparticles – antighosts and have no mass, while the problem of the existence of nonamplicherical 2-spheres has positive answerin the work of Artin and Fox [3].

Now we shall shortly state one of the considerations permitting us to connect wild arcs with massless particles: for a non-amphicheiral wild arc the difference $|J_1 - J_2|$ on integral curvatures for any position of separating loop is ∞, evidently $|J_1| = \infty$ corresponds to submarifold of the knot, containing the point of wildness).

Taking into account two possible orientations for invertible knot, and one for noninvertible we see that situation is analogous to massless particles where the projection of spin on the direction of motion can get either 2 (bosons) or 1 (fermions) value.

Consider an arbitrary one-dimensional knot C^1 lying in the hyperplane R^3 of four-dimensional space R^4 let C^2 be an superstructure over C^1 in R^4, i.e., double cone over C^1, the vortexes p_1 and p_2 of which lie in different complementary domains of R^3 in R^4. C^2 is two-dimensional sphere then in R^4, and points p_1 and p_2 are points of "local knottedness", i.e., points of local wildness [4].

On the other hand, wild sphere with 2 points of local wildness may be obtained in R^3 by means of two wild arcs and intersecting by their ends.

So wild arcs and tame knots may be considered as different sections of some wild 2-sphere $S^2 \subset R^4$. This fact could be used as an illustration to the fact, that annihilation and decay of particles occur with radiation of γ-quantums.

Let's turn now to expressions. The "#" operation transforms some n-dimensional knot into the edge of some $(n+1)$ dimensional object, which is not $(n+1)$-cycle. We must use also operation "$\#$" to obtain $(n+1)$-cycle. After "#" relationship between a knot and element may be expressed by a boundary operator δ. Designating n-dimensional cycle by z^n, and a cycle, which is the edge of $(n+1)$-element by $b^n = \partial C^{n+1}$, where C^{n+1} is $n+1$-dimensional noncyclic element, we get a chain $z^n \xrightarrow{\#} b^n \xrightarrow{\partial^{-1}} C^{n+1} \xrightarrow{\#?} z^{n+1}$. If K is the dimension of knotted sphere $S_\Phi^{n=k}$, and D – the dimension of element space, then the chain $(**)$ is equivalent to the following:

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$$H_k(D) \xrightarrow{\partial^{-1}} \xrightarrow{\#^{-1}?} \xrightarrow{\#} H_{k+1}(D+1),$$

$$\begin{array}{ccccccccc} D & & D+1 & & & & & & \\ \uparrow & & \uparrow & & & & & & \\ Z^k & \xrightarrow{\#} & B^k & \xrightarrow{\partial^{-1}} & C^{k+1} & \xrightarrow{\#?} & Z^{k+1} & \xrightarrow{\#} & B^{k+1}, \\ & & \downarrow & & & & & & \downarrow \\ & & H_k & & & & & & H_{k+1} \end{array}$$

or

$$H_k(D) \xrightarrow{\#^{-1}} \xrightarrow{\#^{-1}?} \xrightarrow{\partial} H_{k-1}(D-1),$$
$$H_k(D) \xrightarrow{\partial'} H_{k-1}(D-1),$$
$$\left(\partial' \equiv \#^{-1}\#^{-1}?\,\partial\right).$$

If the recall now the hierarchy of anomalies expressed in terms of homological groups $H_k(D)$:

$$H_k(D) \xrightarrow{p} H_{k-1}(D-1) \xrightarrow{p},$$

where for Weil anomaly in 2 dimensions we have $H_1(D=2)$. If a cut knot $S^1 = b^1$ is considered in R^4 we get

$$H_1(D=4) = H_3(D=4),$$
$$H_4^3 \xrightarrow{\partial'} H_3^2 \xrightarrow{\partial'} H_2^1,$$

just as in the case of Weil anomaly.

As the hierarchy takes place for $H_k(D)$, transitions between $B_k(D)$ must be considered. $S'_\Phi = \partial\sigma^2$ needed surface σ^2 exists in 4-sphere, while in R^3 S'_Φ gives no surface of type $h=0$ .We can add here, that axial anomaly in chiral theory leads to decay $\pi^0 + 2\gamma$ while to π^0, as to a meson with isospin $J=1$, $I=1$, in ch. II we have put into correspondence a cut knot, which in its turn leads to the homology chain.

Appendix. From “Knotted Particles” to Some Cosmological Problems

Knots play fundamental role in the modern topology, as in the theory of 3- dimension manifolds. The idea of imaging particles as knots was developed among some others in the widely-referenced papers by Faddeev and Niemi, where also a rich bibliographical data in this direction is provided (for example, [11,12]),

We started to look for analogies as early as in 1992 [18], where we considered wild and non-invertible knots, defined knot as a particle, considering the knot as a branching set of some 3-dim. manifold over sphere S_0^3.

Even more precisely – in our scenario a particle is a branching set for the physical space, where the total topology of space is homeomorphic to H3, we follow the action of universal group U [1,7,8] (introduced byThurston [16]) on hyperbolic space for any closed and oriented 3-dim. manifold. M3 there is a finite subgroup G≤U such that M3 is homeomorphic to H3/G.

The fundamental polyhedron corresponding to this universal group is a regular hyperbolic dodecahedron with dihedral right angles, where identification occurs by some group $G \subset IsomE3$, and the orbit space is S^3 with the singular set $\sum$ - Borromean rings. Then any 3-dim. manifold is a branched covering of S^3 with set $\sum$.

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This group is generated by $\frac{\pi}{2}$ rotating about the axes taken for each of the three pairs of opposite edges of any one dodecahedral on-which can serve as a fundamental domain. An important property (for us) is the fact, that for the families of surfaces which are formed by the left-invariant (under the action of group U) faces of dodecahedron each two do not cross, or cross at right angles (see Fig.1)

Generators of group U have the following form as shown in [4]:

$$A=\frac{\sqrt{2}}{2}\begin{bmatrix}1-iR+iR^2 & -iR-iR^2-iR^3\\ 1-2iR+iR^3 & 1+iR-iR^2\end{bmatrix},\ B=\frac{\sqrt{2}}{2}\begin{bmatrix}1-R+R^2 & 1-R+R^2\\ -R-R^2+R^3 & 1+R-R^2\end{bmatrix},$$

$$C=\frac{\sqrt{2}}{2}\begin{bmatrix}1+R-iR^2 & -i-2iR-iR^3\\ -1-R+R^2 & 1-R+iR^2\end{bmatrix}.$$

We will denote by U – quantization the action of U on H3.

We *postulate*, that:

(*) To the propagation of electromagnetic waves in vacuum corresponds topological U – quantization of given set of space.

Now we state some correspondences following from **(*)** :.

1) *The photons can only travel on the U –invarint Euclidean submanifold of H3.*
2) *The Relic Background Radiation is interpreted as a consequence of global topological quantization of the H3 Topological Bang, which occurred to a non-quantized space.*
3) *Considering the volume of a single quantum of this space – the hyperbolic volume of a hyperbolic dodecahedron where we take it approximately to be equal to a sphere of radius* $r=\alpha$, *where* α *is the half-axis of the dodecahedron* $R/\alpha=1.27sm$ (see [5,6]):

$$V_{hyper}=4\pi\left(\frac{r(1+r^2)}{(1-r^2)^2}-\frac{1}{2}\ln\frac{1+r}{1-r}\right).$$

One can calculate that respect to a Euclidean sphere the hyperbolic volume will be about 44 times more so, the volume of observed space is only 3% of the whole hyperbolic space H3. As known, in order to obtain the critical value of the density of matter in the universe (needed to explain its flatness) it became necessary to introduce dark energy together with the dark mass both of which provide for the missing 95% of the whole mass of the universe.

But according to **(*)** the optically observed Universe will always result Euclidean, while the non-zero vacuum energy can be interpreted as hidden hyperbolicity of the topologically quantized space.

The trajectory of a photon-localized in space (as in atom) will take form of Borromean Rings as the Euclidean lines – the axes of the rotations generation of the universal group for the covering $n^3/u\to S^3\setminus B$ will cover the Borromean rings infinite number of times.

The Field particles such as W^+,W^-,Z^0, intermediate, Bosons of the Electro-weak theory, are described by Universal knots. So weak decays of the type $A\to BF$ (for instance, $n^0\to p^+w^-$) are viewed as a transformation of a manifold $M_0^3=S^3\setminus k_\Delta$ into a manifold $M_0^3=S^3\setminus k_\beta$ because every closed oriented 3-dim. manifold can be obtained from a finite set of dodecahedrons by pasting along pentagonal faces in pairs.

We conjecture the existence of the following group-type structure which includes leptons and gauge vector Bosons (some unknown definitions are from [3]):

$$\{G^*\}=\{e,\nu_e,\bar{e},\bar{\nu}_e,\bar{\mu},\bar{\nu}_\mu,\tau,\nu_\tau,\bar{\tau},\bar{\nu}_\tau,W^-,W^+,Z^0\}.$$

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Then the expressions for the processes of the type $[g(i)][g(k)(-1)] = K$, where whole K is a subgroup of G*, will divide G* in the equivalence classes $\{\{\alpha\},\{\beta\},ang\{g\}\}$ respect to the semi-group $F = \{W^-, W^+, Z^0\}$. According to E. Noether theorem this division induces a natural homomorphism of the G* on to a group Γ-where the non-identity elements of the group Γ are homeomorphic images of the lepton generations.

Suppose that the complementary spaces for proton and neutron $S^3 \setminus k_p$ and $S^3 \setminus k_n$ are covering spaces for the same M_0^3 [Fig. 5] [8], but there is a perturbation in the covering group of the neutron, which means, that for an ensemble of N neutrons, N>>1, a single branching of the universal covering space for the N neutron wave function includes the leaves of different copies of $S^3 / k_{(n^0)}$, as can be illustrated by the analogy with the Kantor type Wild Knot [Fig. 3] [15], then after the first period of semi-decay, the decay will include 2^k [Fig. 4] neutrons, and the number of the remaining neutrons will be $1 + 2 + ... + 2^{k-1} \approx 2^k$. Here we note that in 1957 H. Everett introduced the concept of a branched wave-function, to explain the probability nature of the particle decay introducing a many world interpretation of quantum mechanics, when for all moments when the neutron can decay there is one copy of the universe, where this really happens [Fig. 2]. We also underline the affinity of this concept to the theoretical bases of which is at the origin of very interesting for us experiments by M. Berry. Here we quote the branching of the dislocation loop of the wave-function. The phase-surface of monochromatic light see for instance Fig.6 .

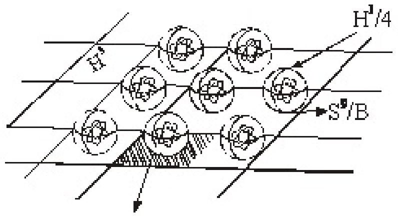

Fig. 1. Acting of group U on space H(3)

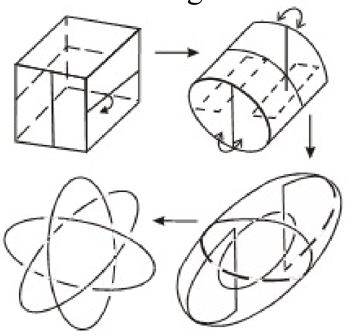

Fig . 2. The image of the axes of rotation is the Borromean rings.

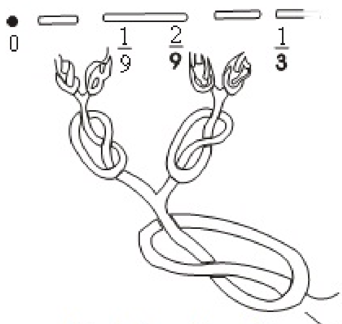

Fig.3. Branching of neutrons wave -function

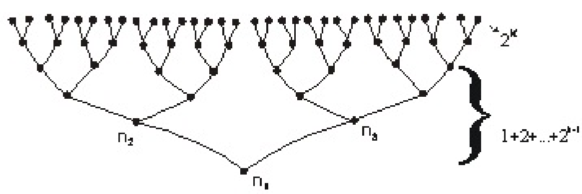

Fig.4. N-particle wave-function of neutrons

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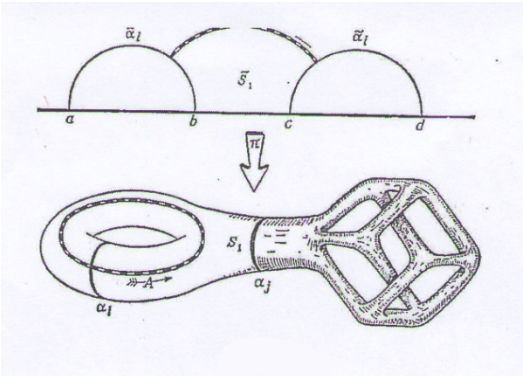

Fig. 5. Perturbation in the covering group. Picture by George Francis .

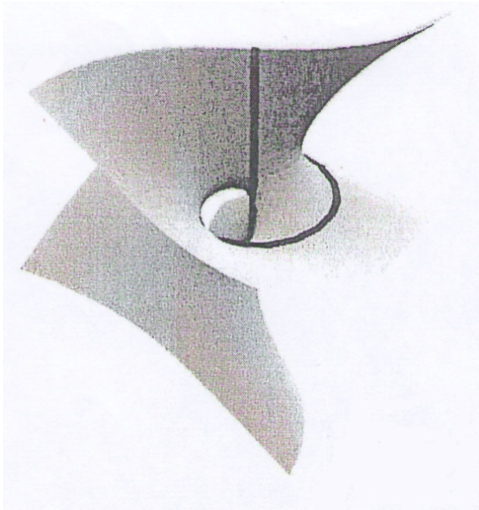

Fig. 6. Phase surface M. Berry

It is natural to ask does this topological bang scenario have any advantages respect to traditional big bang theory.

One can easily seen that for the topologically quantized universe the problems of homogeneity and horizon can be solved without the help of the inflationary theory if one desires of course.

Secondly the large-scale structure of the universe such us filamentary distribution of ordinary meter and structures like Sloan-Great Wall follow naturally from *postulate* **(*)**

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