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A Multiple-Attributes Decision Making in Hesitant Fuzzy Environment: Application to Evaluation of Investment Projects

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Abstract

The work proposes an approach for solving a multiple-attributes decision making (MADM) problems in hesitant fuzzy environment based on TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) method. The case when the information on the attributes weights is completely unknown is considered. The attributes weights identification based on De Luca-Termini information entropy is offered in context of hesitant fuzzy sets. In the TOPSIS method the ranking of alternatives is made in accordance with the proximity of their distance to the positive and negative ideal solutions. The developed approach is applied to evaluation of Investment Projects with the aim of their ranking and identification of high-quality projects for investment.

Keywords: *Multiple-attribute decision making, hesitant fuzzy set, information entropy, TOPSIS method, ranking of Investment Projects.*

1. Introduction

A multi-attribute decision making (MADM) problem deals with a selection of one alternative (decision) or several ranked alternatives involving multiple attributes. From this perspective, the investment decision-making is a MADM problem.

Investment decision making is based on various special methods. The further development in the field received the probabilistic approach to the evaluation of risks of investment decisions. Along with that, many other methods were developed based on possibility analysis [1] and fuzzy-set approach [2-4].

If there are not enough objective data, or they aren't present to make the investment decision, application of traditional statistical methods becomes impossible. Then experienced experts (decision makers - DMs) are invited to solve a problem. In this case, knowledge and intellectual work of the experts produces expert data. Thus, the analysis of investment projects involves experts' evaluations that may become dominant in decision making process.

Because of the inherent uncertainty of expert preferences, as well as due to the fact that objects can be fuzzy and uncertain, evaluations of attributes involved in the decision making problems most often are expressed in fuzzy numbers or triangular fuzzy numbers, confidence intervals, linguistic variables, intuitionistic fuzzy values, hesitant fuzzy elements, interval-valued hesitant fuzzy elements and so on. In this connection, many well-known MADM methods have been extended to take into account fuzzy types of attributes values [5,6].

Nowadays there exists a large amount of literature for the theory of hesitant fuzzy sets (HFS) and their application in MADM. Different from other studies, in this paper the novel approach based on hesitant fuzzy TOPSIS decision making model with entropy weights is developed. The case when the information on the attributes weights is completely unknown is considered. The attributes weights are obtained by applying De Luca-Termini non-probabilistic entropy concept [7],

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which is offered in context of hesitant fuzzy sets. After that, a fuzzy hesitant TOPSIS method is employed to ranking the alternatives. While using this method both attributes types are under consideration: as attributes of benefit type, as well as attributes of cost type.

The developed approach is applied to evaluation of investment projects with the aim of their ranking and identification of high-quality projects for investment.

2. Preliminaries

Hesitant fuzzy set (HFS) was introduced by Torra and Narukawa in [8] and Torra in [9] as a generalization of a fuzzy set. In HFS the degree of membership of an element to a reference set is presented by several possible fuzzy values. This allows describing situations when DMs have hesitancy in providing their preferences over alternatives. The HFS is defined as follows:

Definition 1.[8,9]. Let X be a reference set, a hesitant fuzzy set E on X is defined in terms of a function $h_E(x)$ when applied to X returns a subset of [0,1]:

$$E = \{ < x, h_E(x) > \mid x \in X \}, \tag{1}$$

where $h_E(x)$ is a set of some different values in [0,1], representing the possible membership degrees of the element $x \in X$ to E; $h_E(x)$ is called a hesitant fuzzy element (HFE).

Definition 2.[10]. Let M and N be two HFSs on $X = \{x_1, x_2, ..., x_n\}$, then the distance measure between M and N is defined as $d(M,N)$, which satisfies the following properties:

1). $0 \le d(M,N) \le 1$; 2). $d(M,N) = 0$ if and only if $M = N$; 3). $d(M,N) = d(N,M)$.

It is clear that the number of values (length) for different HFEs may be different. Let $l(h_E(x))$ be the length of $h_E(x)$. After arranging the elements of $h_E(x)$ in a decreasing order, let $h_E^{\sigma(j)}(x)$ be the *j*th largest value in $h_E(x)$. To calculate the distance between M and N when $l(h_M(x_i)) \neq l(h_N(x_i))$, it is necessary extend the shorter one by adding any value in it, until both will have the same length. The choice of this value depends on the DMs' risk preferences. Optimists DMs' may add the maximum value from HFE, while pessimists may add the minimal value.

In this work the hesitant weighted Hamming distance is used that is defined by following formula

$$d_{hwh}(M,N) = \sum_{i=1}^{n} w_i \left[\frac{1}{l_{x_i}} \sum_{j=1}^{l_{x_i}} \left| h_M^{\sigma(j)}(x_i) - h_N^{\sigma(j)}(x_i) \right| \right], \tag{2}$$

where $h_M^{\sigma(j)}(x_i)$ and $h_N^{\sigma(j)}(x_i)$ are the *j*th largest values in $h_M(x_i)$ and $h_N(x_i)$ respectively;
$l_{x_i} = \max\{l(h_M(x_i)), l(h_N(x_i))\}$ for each $x_i \in X$;
w_i $(i = 1,2,...,n)$ is the weight of the element $x_i \in X$ such that $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$.

Definition 3. [11] For a HFE $h_E(x)$, the score function $s(h_E(x))$ is defined as follows:

$$s(h_E(x)) = \sum_{j=1}^{l(h_E(x))} h_E^{\sigma(j)}(x) \Big/ l(h_E(x)), \text{ where } s(h_E(x)) \in [0,1]. \tag{3}$$

Let h_1 and h_2 are two HFEs. Based on score function it is possible to make ranking of HFEs according to the following rules: $h_1 > h_2$, if $s(h_1) > s(h_2)$; $h_1 < h_2$, if $s(h_1) < s(h_2)$ and $h_1 = h_2$, if $s(h_1) = s(h_2)$.

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3. Investment MADM problem in hesitant fuzzy environment

Consider a MADM problem for investment decision making.

Assume that there are m investment projects – decision making alternatives – $A = \{A_1, A_2, \ldots, A_m\}$, and the group of DMs evaluates them with respect to an n attributes $X = \{x_1, x_2, \ldots, x_n\}$. DMs give the evaluations over attributes in form of hesitant fuzzy numbers. Therefore, their joint assessments concerning each alternative represent HFSs. A HFS A_i of the ith alternative on X is given by $A_i = \left\{ \left\langle x_j, h_{A_i}(x_j) \right\rangle | x_j \in X \right\}$, where $h_{A_i}(x_j) = \left\{ \gamma \,|\, \gamma \in h_{A_i}(x_j), 0 \le \gamma \le 1 \right\}$, $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$. $h_{A_i}(x_j)$ indicates the possible membership degrees of the ith alternative A_i under the jth attribute x_j, and it can be expressed as a HFE h_{ij}.

Considering that the attributes have different importance degrees, the weighting vector of all attributes, given by the DMs, is defined by $w = (w_1, w_2, \ldots, w_n)^T$, where $0 \le w_j \le 1$, $\sum_{j=1}^{n} w_j = 1$, and w_j is the importance degree of jth attribute.

Then a hesitant MADM problem can be expressed in matrix format as follows

$$H = \begin{array}{c} \\ A_1 \\ A_2 \\ \vdots \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} X_1 & X_2 & \cdots & X_n \end{array} \\ \begin{bmatrix} h_{11} & h_{12} & \cdots & h_{1n} \\ h_{21} & h_{22} & \cdots & h_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ h_{m1} & h_{m2} & \cdots & h_{mn} \end{bmatrix} \end{array}, \quad w = (w_1, w_2, \ldots, w_n)^T,$$

where H is the hesitant decision matrix, each element of which represents a HFE h_{ij}.

3.1 Determination of the attributes weights using De Luca-Termini entropy

Complexity and uncertainty of investment decision making problems leads to the fact that the information about attributes weights is usually incomplete or completely unknown. Here the case when the attributes weights are unknown is considered.

De Luca and Termini [7] defined a non-probabilistic entropy formula of a fuzzy set based on Shannon's function on a finite universal set $Y = \{y_1, \ldots, y_l\}$ as:

$$E_{LT} = -k \sum_{i=1}^{l} \left[\mu_A(y_i) \ln \mu_A(y_i) + (1 - \mu_A(y_i)) \ln (1 - \mu_A(y_i)) \right], \; k > 0,$$

where $\mu_A : Y \to [0,1]$ is a membership function of some fuzzy set A on Y; k is a positive constant.

The attributes weights definition method based on the De Luca-Termini entropy can be described as follows:

Step1: Calculate the score matrix $S = (s_{ij})_{m \times n}$ of hesitant decision matrix H, where $s_{ij} = s(h_{ij})$ is the score value of h_{ij} (see (3)).

Step2: Calculate the normalized score matrix $S' = (s'_{ij})_{m \times n}$, where

$$s'_{ij} = s_{ij} \Big/ \sum_{i=1}^{m} s_{ij} \,. \quad (4)$$

Step3: Determine the attributes weights.

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By using De Luca-Termini normalized entropy in context of hesitant fuzzy sets

$$E_j = -\frac{1}{m\ln 2}\sum_{i=1}^{m} s'_{ij}\ln s'_{ij} + (1-s'_{ij})\ln(1-s'_{ij}),\ \ j = 1, 2, \ldots, n, \tag{5}$$

the definition of the attributes weights is expressed by the formula

$$w_j = \frac{1-E_j}{\sum_{j=1}^{n}(1-E_j)},\ \ j = 1, 2, \ldots, n. \tag{6}$$

where the value of w_j represents the relative intensity of X_j attribute importance.

3.2 Hesitant fuzzy MADM approach based on TOPSIS method

The idea of TOPSIS method as applied to the problem of MADM is to choose an alternative with the nearest distance from the so-called positive ideal solution (PIS) and the farthest distance from the negative ideal solution (NIS).

Here the MADM approach based on the hesitant fuzzy TOPSIS method with entropy weights model proposed in Section 3.1 is present. Different from existing extensions of TOPSIS under hesitant fuzzy environment, here the attributes of both types are considered: as attributes of benefit type, as well as attributes of cost type.

The algorithm of practical solving an investment MADM problem can be formulated as follows:

Step 1: Based on the DMs hesitant evaluations construct the aggregate hesitant decision matrix $H = (h_{ij})_{m\times n}$.

Step 2: Determine the attributes weights $w = (w_1, w_2, \ldots, w_n)^T$ based on the method in Section 3.1.

Step 3: Determine the corresponding hesitant fuzzy PIS A^+ and the hesitant fuzzy NIS A^- by formulas:

$$A^+ = \left\{ \left\langle \max_{i=1,\ldots m}\left[h_{ij}^{\sigma(l)}\right], l = 1,\ldots, l_{x_j} \right\rangle \mid j \in J';\ \left\langle \min_{i=1,\ldots m}\left[h_{ij}^{\sigma(l)}\right], l = 1,\ldots, l_{x_j} \right\rangle \mid j \in J'' \right\} \tag{7}$$

$$A^- = \left\{ \left\langle \min_{i=1,\ldots m}\left[h_{ij}^{\sigma(l)}\right], l = 1,\ldots, l_{x_j} \right\rangle \mid j \in J';\ \left\langle \max_{=1,\ldots m}\left[h_{ij}^{\sigma(l)}\right], l = 1,\ldots, l_{x_j} \right\rangle \mid j \in J'' \right\} \tag{8}$$

where J' is associated with a benefit attributes, and J'' - with a cost attributes.

Step 4: Using (2) calculate the separation measures d_i^+ and d_i^- of each alternative A_i from the hesitant fuzzy PIS A^+ and the hesitant fuzzy NIS A^-, respectively:

$$d_i^+ = \sum_{j=1}^{n} d(h_{ij}, h_j^+)\,w_j = \sum_{j=1}^{n} w_j\left[\frac{1}{l}\sum_{j=1}^{l}\left|h_{ij}^{\sigma(j)} - (h_j^{\sigma(j)})^+\right|\right], \tag{9}$$

$$d_i^- = \sum_{j=1}^{n} d(h_{ij}, h_j^-)\,w_j = \sum_{j=1}^{n} w_j\left[\frac{1}{l}\sum_{j=1}^{l}\left|h_{ij}^{\sigma(j)} - (h_j^{\sigma(j)})^-\right|\right],\ i = 1, 2, \ldots, m.$$

Step 5: Calculate the relative closeness coefficient C_i of each alternative A_i to the hesitant fuzzy PIS A^+:

$$C_i = \frac{d_i^-}{d_i^+ + d_i^-}. \tag{10}$$

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Step 6: Perform the ranking of the alternatives A_i, $i = 1, 2, \ldots, m$ according to the relative closeness coefficients $C_i, i = 1, 2, \ldots, m$ by the rule: for two alternatives A_α and A_β $A_\alpha \succ A_\beta$, if $C_\alpha > C_\beta$, where $\succ$ is a preference relation on A.

4. Application to evaluation of investment projects

Suppose that in the competition for investment five construction companies are involved. The group of DMs evaluates the investment projects taking into account the four attributes that are important for granting investment: x_1 - the credit risk level; x_2 - business profitability; x_3 - location of construction object and x_4 - workmanship. Herewith the first attribute is of a cost type, and the others three - of a benefit type. DMs give evaluations in form of hesitant values. If the evaluation values of any attribute given by experts are coincident, then such values are included in HFE only once. Assume the hesitant fuzzy decision matrix H looks like

Table 1: The hesitant fuzzy decision matrix H

	x_1	x_2	x_3	x_4
A_1	(0.4,0.3,0.1)	(0.9,0.8,0.7,0.1)	(0.9,0.6,0.5,0.3)	(0.5,0.4,0.3)
A_2	(0.5,0.4)	(0.9,0.7,0.6,0.3)	(0.7,0.4,0.3)	(0.6,0.5)
A_3	(0.3,0.2,0.1)	(0.9,0.6)	(0.8,0.7)	(0.7,0.4,0.1)
A_4	(0.2,0.1)	(0.8,0.7,0.5,0.3)	(0.9,0.8,0.6)	(0.8,0.5,0.4)
A_5	(0.7,0.5,0.3)	(0.7,0.4,0.2)	(0.9,0.7,0.6,0.4)	(0.9,0.7,0.6,0.2)

We presume that the DMs are pessimistic, and the hesitant fuzzy data in HFEs are changed by adding the minimal values.

According to the method of determining the attributes weights given in Section 3.1, we first calculate the score matrix S of hesitant decision matrix H based on (3):

$$S = \begin{bmatrix} 0.267 & 0.625 & 0.575 & 0.4 \\ 0.45 & 0.625 & 0.467 & 0.55 \\ 0.2 & 0.75 & 0.75 & 0.4 \\ 0.15 & 0.575 & 0.575 & 0.567 \\ 0.5 & 0.433 & 0.65 & 0.5 \end{bmatrix}.$$

Secondly, we obtain the normalized score matrix S' using (4):

$$S' = \begin{bmatrix} 0.1702 & 0.2078 & 0.1906 & 0.1589 \\ 0.2872 & 0.2078 & 0.1547 & 0.2185 \\ 0.1277 & 0.2493 & 0.2486 & 0.1589 \\ 0.0957 & 0.1911 & 0.1906 & 0.2252 \\ 0.3191 & 0.144 & 0.2155 & 0.2384 \end{bmatrix}.$$

Then the weighting vector of attributes is determined using (5) and (6):

$$w = (0.2695, 0.2437, 0.2428, 0.244)^T .$$

Following the hesitant fuzzy TOPSIS method, we determine the hesitant fuzzy PIS A^+ and the hesitant fuzzy NIS A^- by (7) and (8), respectively:

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$$A^{+} = \{(0.2,0.1,0.1,0.1),(0.9,0.8,0.7,0.6),(0.9,0.8,0.7,0.7),$$
$$(0.9,0.7,0.6,0.5)\};$$
$$A^{-} = \{(0.7,0.5,0.4,0.4),(0.7,0.4,0.2,0.1),(0.7,0.4,0.3,0.3),$$
$$(0.5,0.4,0.1,0.1)\}.$$

Then we calculate the distances d_i^+ and d_i^- of each alternative A_i from the hesitant fuzzy PIS A^+ and the hesitant fuzzy NIS A^- by (9), respectively:

$$d_1^+ = 0.17917,\ d_2^+ = 0.23289,\ d_3^+ = 0.12879,\ d_4^+ = 0.09139,\ d_5^+ = 0.22763;$$
$$d_1^- = 0.25682,\ d_2^- = 0.1543,\ d_3^- = 0.25183,\ d_4^- = 0.28367,\ d_5^- = 0.14133.$$

Using (10) to calculate the relative closeness coefficient C_i of each alternative A_i to the hesitant fuzzy PIS A^+ we obtain:

$$C_1 = 0.58905,\ C_2 = 0.39851,\ C_3 = 0.66163,\ C_4 = 0.75633,\ C_5 = 0.38305.$$

Finally, we perform the ranking of the alternatives A_i, $i = 1, 2, \ldots, 5$ according to the relative closeness coefficients C_i, $i = 1, 2, \ldots, 5$ and obtain:

$$A_4 \succ A_3 \succ A_1 \succ A_2 \succ A_5.$$

That means that when investing the capital only in one project, DMs prefer to the investment project A_4, i.e. the project A_4 receive investment.

5. Conclusions

In this paper the novel approach for solving MADM problem based on hesitant fuzzy TOPSIS method with entropy weights is developed.

The new aspects in the TOPSIS approach have been used:

1. For the determination of the attributes weights the De Luca-Termini information entropy was applied.
2. There are many methods of the applicability of the TOPSIS approach under hesitant environment. The novelty in our work is that both types of attributes - cost and benefit - are considered.
3. The developed approach was applied in the problem of investment decision making.

Based on proposed approach we have developed software package which is used in real investment decision making problem. The application and testing of the software was carried out based on the data provided by the “Bank of Georgia”. The results are illustrated in the example.

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