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Fuzzy Aggregations in the Decision Making Problem regarding the Choosing of the best Version of the Students' Group Project for Implementation

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Abstract

In this work a new fuzzy generalization of the OWA aggregation operator is presented. A fuzzy measure and its associated probability class are used for the generalization. For the illustration of the applicability of the new aggregation operator - AsPOWA an example of the fuzzy decision making problem regarding the choosing of the students' group project for implementation is presented. Several variants of the new aggregation operator are used for the comparing of decision making results. In the role of a fuzzy measure the plausibility and believe measures associated with the Dempster-Shafer believe structure are used.

Keywords: *mean aggregation operator, OWA operator, fuzzy measure, associated probabilities, multiple criteria decision making, project management.*

1. New fuzzy Aggregations in the OWA operator

It is well recognized that intelligent decision making systems (IDMS) and technologies have been playing an important role in improving almost every aspect of human society. In this type of problem the decision making person (DMP) has a collection $D=\{d_1,d_2,...,d_n\}$ of possible uncertain alternatives from which he/her must select one or some rank decisions by some expert's preference relation values. Associated with this problem as a result is a variable of characteristics, activities, symptoms and so on, acts on the decision procedure. This variable normally called the state of nature, which affects the payoff, utilities, valuations and others to the DMP's preferences or subjective activities. This variable is assumed to take its values (states of nature) in the some set $S=\{s_1,s_2,...,s_m\}$. As a result the DMP knows that if he/she selects d_i and the state of nature assumes the value s_j then his/her payoff (valuation, utility and soon) is a_{ij}. The objective of the decision is to select the "best" alternative, get the biggest payoff. But in IDMS the selection procedure becomes more difficult. In this case each alternative can be seen as corresponding to a row vector of possible payoffs. To make a choice the DMP must compare these vectors, a problem which generally doesn't lead to a compelling solution. Assume d_i and d_k are two alternatives such that for all $j, j=1,2,....,m$ $a_{ij} \geq a_{kj}$. In this case there is no reason to select d_i. In this situation we shall say d_i dominates d_k $(d_i \succeq d_k)$. Furthermore if there exists one alternative (optimal decision) that dominates all the alternatives then it will be optimal solution. Faced with the general difficulty of comparing vector payoffs we must provide some means of comparing these vectors. Our focus in this work is on the construction of fuzzy aggregation operator F that can take a collection of m values and convert it into a single value, $F:R^m \Rightarrow R^1$.In [5] R.R. Yager introduced a class of mean aggregation operators called Ordered Weighed Averaging (OWA) operator.

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Definition 1: *An OWA operator of dimension* m *is mapping* $OWA: R^m \Rightarrow R^1$ *that has an associated weighting vector* W *of dimension* m *with* $w_j \in [0;1]$ *and* $\sum_{j=1}^{m} w_j = 1,$ *such that*

$$OWA(a_1,....,a_m) = \sum_{j=1}^{m} w_j b_j \;, \tag{1}$$

where b_j *is the* j*-th largest of the* $\{a_i\}, i = 1,2,...,m$.

Dempster–Shafer Belief Structure(the Theory of Evidence ([1-4] and others)) is a powerful tool which enables one to build: 1. models of decisions and their risks' measures; 2. Aggregation operators in an uncertain environment and so on.

In this work we consider Dempster–Shafer Belief Structure (DSBS) [1-3] aggregations based on the OWA operator. Therefore we introduce the definition of a fuzzy measure [2-4] and the DSBS.

We introduce the definition of a fuzzy measure (monotone measure) [2] adapted to the case of a finite referential.

Definition 2: *Let* $S = \{s_1, s_2,..., s_m\}$ *be a finite set and* g *be a set function* $g: 2^S \to [0,1]$. *We say* g *is a fuzzy measure on* S *if it satisfies*

$$(i) g(\varnothing)=0;\; g(S)=1;\;\; (ii) \forall A,B \subseteq S,\;\; if\; A \subseteq B,\; then\, g(A) \le g(B). \tag{2}$$

A fuzzy measure is a normalized and monotone set function. It can be considered as an extension of the probability concept, where additivity is replaced by the weaker condition of monotonicity. Non-additive but monotone (fuzzy) measures were first used in the fuzzy analysis in the 1980s and is well investigated ([1-3] and others).

In general, the possible orderings of the elements of S are given by the permutations of a set with m elements, which form the group S_m .

Definition 3[2]: *The probability functions* P_σ *defined by*

$$\begin{aligned} &P_\sigma\left(s_{\sigma(1)}\right) = g\left(\{s_{\sigma(1)}\}\right),...., \\ &P_\sigma\left(s_{\sigma(i)}\right) = g\left(\{s_{\sigma(1)},....,s_{\sigma(i)}\}\right) - g\left(\{s_{\sigma(1)},....,s_{\sigma(i-1)}\}\right),...., \\ &P_\sigma\left(s_{\sigma(m)}\right) = 1 - g\left(\{s_{\sigma(1)},....,s_{\sigma(m-1)}\}\right), \end{aligned} \tag{3}$$

for each $\sigma = (\sigma(1),\sigma(2),...,\sigma(m)) \in S_m$, *are called the associated probabilities and the Associated Probability Class (APC)* - $\{P_\sigma\}_{\sigma \in S}$ *of a fuzzy measure* g .

The Theory of Evidence is based on two dual fuzzy measures: Belief measures and Plausibility measures. Belief and Plausibility measures can be characterized by the set function: $m: 2^S \to [0,1]$,which is required to satisfy two conditions:

$$\begin{aligned} &\text{(a)}\; m(\varnothing) = 0, \\ &\text{(b)} \sum_{B \in 2^S} m(B) = 1. \end{aligned} \tag{4}$$

This function is called a Basic Probability Assignment (BPA). For each set $B \in 2^S$, the value $m(B)$ expresses the proportion that all available and relevant evidence supporting the claim that a particular element of S , whose characterization in terms of relevant attributes is deficient, belongs to the set B . This value $m(B)$, pertains solely to one set $-B$; it does not imply any additional claims regarding subsets of B . If there is some additional evidence supporting the claim that the element belongs to a subset of B , say $B_1 \subseteq B$, it must be expressed by another value $m(B_1)$.

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If $m(B) > 0, B \subset S$ then B is called a focal element. Left $\mathcal{F} = \{B_1, \ldots, B_q\}$ be the set of all focal elements. The pair $< F, m >$ is called a body of Evidence (or Dempster–Shafer Belief believe structure)

Definition 4 [2]:*Let* m *be a PBA on* S . *The plausibility measure* Pl *associated to* m *is given by*

$$Pl(A) = \sum_{B \in F:\, A \cap B \neq \emptyset} m(B), \ \forall A \in 2^S \tag{5}$$

and the Belief measure Bel *associated to* m *is given by*

$$Bel(A) = \sum_{B \in F:\, B \subset A} m(B), \ \forall A \in 2^S \tag{6}$$

Note, that Believe and Plausibility measures are fuzzy measures.

Let $M : R^m \Rightarrow R^+ \quad (k = m!)$ be some deterministic mean aggregation function with properties of idempotency, symmetry, monotonicity and boundedness. [5].

Definition 5: *An associated fuzzy-probabilistic OWA operator AsPOWAof dimension m is mapping* $AsPOWA : R^m \Rightarrow R^+$, *that has an associated objective weighted vector* W *of dimension* m *such that* $w_j \in (0,1)$ *and* $\sum_{j=1}^{m} w_j = 1$ *some fuzzy measure* $g : 2^S \Rightarrow [0,1]$, *according the following formula:*

$$\begin{aligned} AsPOWA(a_1, a_2, ..., a_m) = \beta \sum_{j=1}^{m} w_j b_j + \\ + (1-\beta) M\left(E_{P_{\sigma_1}}(a), E_{P_{\sigma_2}}(a), ..., E_{P_{\sigma_k}}(a)\right) \end{aligned}, \tag{7}$$

where b_j *is the j-th largest of the* $\{a_i\}, i = 1,...,m$; $E_{P_{\sigma_i}}(a)$ *is a Mathematical Expectation of a variable* $a = (a_1, \ldots, a_m)$ *with respect to associated probability* P_{σ_i} *of a fuzzy measure* g .

We will consider concrete AsPOWA operators for concrete mean function M : AsPOWAmin if M=*Min*, AsPOWAmax, if M=*Max*and AsPOWAmean if M=*Mean*.

2. Decision Making Problem regarding the Choosing of the best Version of the Students' Group Project for Implementation

We analyse an illustrative example of the using of the new AsPOWA operator in a fuzzy decision-making problem regarding the choosing of the students' group project for implementation.The authors of this work has experience working with graduate students pursuing a master's degree in 'intelligent information systems', in which students work on group projects, which involves the evolution, control, engineering and management of simulation models for studied complex systems. The students always create several versions of project for implementation, because usually it is very hard to figure out the role of each student in group and their utility. Also, we have to take into account the fact that each student is working in several groups. After studying the various versions of project, we have the possibility to consider the levels of competency of each student concerning the implementation of the project and evaluate each student by utility levels for each given version of the project.

In one such case, we were dealing with the estimation of the financial state of a certain business organization. The estimation of the linguistic variable is represented by several fuzzy terms, which represent the output of a fuzzy control system. The input information was the objective-statistical data – linguistic variables, which influence the financial state of the

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organization. After analyzingthe problem, we found out that the number of input linguistic variables was 14. Their fuzzification was performed, and the students elaborated three versions of the project of constructing a system for the same input and output information. (d_1) The fuzzy logic rules, corresponding knowledge base, and the decision support system must be built using the MatLab Fuzzy-Logic Toolbox. (d_2) The fuzzy rules, knowledge base, architecture and interface all would be developed using the programming language C#. (d_3) The body of the control system – the transaction between input and output variables – would be developed using fuzzy relations and their compositions, Corresponding software also developed using C#.

Thus three versions $D = \{d_1, d_2, d_3\}$of the project were created in which seven students participated, say $\Omega = \{s_1, s_2, s_3, s_4, s_5, s_6, s_7\}$. All seven of them participated in the development of all three versions, but in different subgroups, as often happens in engineering and management of simulation modelling. They created four groups (hereinafter called focal elements):

1. A_1 – The group for problem analysis, gathering of input data, its initial processing and construction of the conceptual model.
2. A_2– The group for conceptual model validation and software development.
3. A_3 – Group for software verification and testing.
4. A_4 – Management group.

The students were divided into subgroups in the following way:

$$A_1 = \{s_1, s_3, s_4\}, A_2 = \{s_3, s_4, s_5, s_6\}, A_3 = \{s_1, s_2, s_6, s_7\},\ A_4 = \{s_4, s_6, s_7\}.$$

We assigned following weights to subgroups:

$$m(A_1) = 0.2, m(A_2) = 0.4, m(A_3) = 0.1, m(A_4) = 0.3.$$

So we built the body of evidence - $< \mathcal{F}, m >$ where $\mathcal{F}$ - is a set of focal elements – subgroups A_1, A_2, A_3, A_4 and m is a focal probability on the $\mathcal{F}$. Based on the theory of body of evidence [1-2], we create dual fuzzy measures of uncertainty: plausibility measure and believe measure (see formulas 5 and 6).

Student weights are presented in the Table 1. These weights are calculated based on student's knowledge and competency in project implementation. In the role of β value0.3was taken.

Table 1: Students' weights

S	s_1	s_2	s_3	s_4	s_5	s_6	s_7
W	$\frac{2}{14}$	$\frac{4}{14}$	$\frac{1}{14}$	$\frac{3}{14}$	$\frac{1}{14}$	$\frac{2}{14}$	$\frac{1}{14}$

After some time, the students presented all three variants of the projects (d_1, d_2, d_3). We had to choose the best one with the objective of optimal realization of the problem. We had to evaluate the utilities of students concerning each version. So we had to study the projects in detail. So we had to consider students' competence and knowledge in given topics, the quality and reliability of the realization of project, the ability to work in groups, etc.

The results of evaluation process were as follows (results and normalized in the interval [0,1], and normalized utilities present some possibilistic levels (see Table 2).

Table 2: Decision making matrix – evaluation of students

$D\backslash\Omega$	s_1	s_2	s_3	s_4	s_5	s_6	s_7
d_1	0.4	0.6	0.7	0.8	0.5	0.4	0.6
d_2	0.6	0.8	0.5	0.9	0.6	0.8	0.7

d_3	0.3	0.9	0.4	0.7	0.3	0.7	0.6

Obviously, the comparison of projects $d_i, i = 1,2,3$ and selection of the best one by their utilities is impossible. Using the data of this problem and the definition of the AsPOWA operator we calculate the values of the AsPOWA operator for every possible alternative and different mean aggregation function $M = \{mean, max, min\}$ (formula (7)).For these calculations, software has been implemented that is dynamical by all of its input parameters. Aggregation results calculated by the software are presented in the Table 3.

Table 3': Aggregation results for OWA operator and Pl function

$D\backslash Agg.Op$	$g = Pl$ $AsPOWAmin$	$g = Pl$ $AsPOWAmax$	$g = Pl$ $AsPOWAmean$	OWA
d_1	0.401	0.562	0.474	0.357
d_2	0.495	0.670	0.586	0.414
d_3	0.324	0.618	0.470	0.379

Table 3'': Aggregation results for OWA operator and Bel function

$D\backslash Agg.Op$	$g = Bel$ $AsPOWAmin$	$g = Bel$ $AsPOWAmax$	$g = Bel$ $AsPOWAmean$	OWA
d_1	0.429	0.562	0.486	0.357
d_2	0.544	0.691	0.630	0.414
d_3	0.387	0.583	0.526	0.379

In the Table 4, the alternatives ranked by the values of the AsPOWA and OWA operators are presented.

Table 4: Ranking of alternatives

N	Aggregation Operator	Result
1	OWA	$d_2 \succ d_3 \succ d_1$
2	$AsPOWAmin\ (g = Pl)$	$d_2 \succ d_1 \succ d_3$
3	$AsPOWAmax\ (g = Pl)$	$d_2 \succ d_3 \succ d_1$
4	$AsPOWAmean\ (g = Pl)$	$d_2 \succ d_1 \succ d_3$
5	$AsPOWAmin\ (g = Bel)$	$d_2 \succ d_1 \succ d_3$
6	$AsPOWAmax\ (g = Bel)$	$d_2 \succ d_3 \succ d_1$
7	$AsPOWAmean\ (g = Bel)$	$d_2 \succ d_3 \succ d_1$

As seen in Table 4 (Symbol $\succ$ is the binary relation of preferences on the alternatives), the alternative d_2 or the second version of the project is preferable over other versions. As the decision, students were instructed to implement this version of the project.

3. Conclusion

In this work our focus was directed on the construction of a new fuzzy probabilistic generalization of the aggregation OWA operator – AsPOWA in the fuzzy uncertainty environment. For the illustration of the applicability of the new aggregation operator –AsPOWA, an example of the fuzzy decision making problem regarding the choosing of the students' group project for implementation was constructed.

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