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Mathematical modeling of nonlinear processes of two-level assimilation

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Abstract

In the real model it is supposed that the powerful state with a widespread state language carries out assimilation of the population of less powerful state and the third population talking in two languages, different in prevalence. Carries out assimilation of the population of the state formation with the least widespread language to the turn, less powerful state.

Not triviality of model assumes negative demographic factor of the powerful state-assimilating and positive demographic factor of the state formation which is under bilateral assimilation. For some ratios between demographic factors of the sides and coefficients of assimilations, for nonlinear system of three differential equations with the corresponding conditions of Cauchy the first integrals are found.

In particular, in the first case the first integral in space of required functions represents a hyperbolic paraboloid, and in the second case – a cone. In these cases, the nonlinear system of three differential equations is reduced to nonlinear system of two differential equations for which the second first integrals are found and in the phase plane of decisions are investigated behavior of integrated curves.

In more general case with application of a criteria of Bendikson the possibility of existence of the closed integrated curves is proved that indicates a possibility of a survival of the population finding under double assimilation.

***Keywords**: Nonlinear mathematical model; two-level assimilation; demographic factor; first integrals; hyperbolic paraboloid; cone.*

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Introduction

Mathematical and computer modeling has been widely recognized in such disciplines as sociology, history, political science, and others [1, 2]. There is an interest in creation of a mathematical model, which would give the opportunity to determine the dynamics of changes in the number of voters of political subjects during the election period. Elections can be divided into two parts: the two-party and multi-party elections.

In [3 - 5] quantities of information streams by means of new mathematical models of information warfare are studied. By information warfare the authors mean an antagonism by means of mass media (an electronic and printing press, the Internet) between the two states or the two associations of states, or the economic structures (consortiums) conducting purposeful misinformation, propagation against each other.It was shown that in case of high aggression of the contradictory countries, not preventive image the operating peacekeeping organizations won't be able to extinguish the expanding information warfare.

In works [6,7] linear and nonlinear mathematical models of information warfare, and also optimizing problems are considered.

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In [8] the new nonlinear mathematical and computer model of information warfare with participation of interstate authoritative institutes is offered. The model is described by Cauchy's problem for nonlinear non-homogeneous system of the differential equations. Confronting sides in extend of provocative statements, the third side (the peacekeeping international organizations) extends of soothing statements, interstate authoritative institutes the peacekeeping statements call the sides for the termination of information warfare. In that specific case, modes of information warfare "aggressor- victim", for the third peacekeeping side are received exact analytical solutions, and functions defining number of the provocative statements distributed by the antagonistic sides satisfy to Cauchy's problems for Riccati certain equations which are solved by a numerical method. For the general model computer modeling is carried out and shown that irrespective of high aggression of confronting sides, interstate authoritative institutes will be able to extinguish information warfare and when for this purpose efforts of only the international organizations insufficiently.

The article [9] concerns of Chilker task is entered - refers to the boundary value problem for a system of ordinary differential equations and optimal control problem. In Chilker tasks right boundary conditions are set in different, uncommitted time points for different coordinates of the unknown vector — functions. Proposed methods solutions of Chilker tasks.

In works [10 - 12] the mathematical model of political rivalry devoted to the description of fight occurring in imperious elite competing (but not necessarily antagonistic) political forces, for example, power branches is considered. It is supposed that each of the sides has ideas of "number" of the power which this side would like to have itself, and about "number" of the power which she would like to have for the partner.

These papers [13 - 17] present the nonlinear mathematical model of the public or the administrative management (or the macro and micro model). The cases of both constant and variable pressure forces on freethinking people were analyzed. Exact analytical decisions which determine dynamics of a spirit both free-thinking people, and operated (conformists) of people by time are received. During this analyses various governance systems were considered: a liberal, democratic, semi dictatorial and dictatorial.

These works [18 - 24] considered a two or three-party (one pro-government and two opposition parties) nonlinear mathematical model of elections when coefficients are constant. The assumption was made that the number of voters remain the same between two consecutive elections (zero demographic factor of voters). The exact analytical solutions were received. The conditions under which opposition party can win the upcoming elections were established.

These works [25 - 28] considered a two-party (pro-government and opposition parties) nonlinear mathematical model of elections with variable coefficients.

In work [29] proposed the nonlinear mathematical model with variable coefficients in the case of three-party elections, that describes the dynamics of the quantitative change of the votes of the pro-government and two opposition parties from election to election. The model takes into account the change in the total number of voters in the period from election to election, i.e. the so-called demographic factor during the elections is taken into account. The model considered the cases with variable coefficients. In the particular case obtained exact analytical solutions. The conditions have been identified under which the opposition can win the forthcoming elections, and in some cases, the pro-government party can stay in power. In general Cauchy problem was solved numerically using the MATLAB software package.

In work [30] computer research of a trajectory of development of three ethnos living in one territory is conducted. Thus assimilation is supposed as a result of mixed marriages.

In [31] consider the nonlinear mathematical model of bilateral assimilation without demographic factor. It was shown that the most part of the population talking in the third language is assimilated by that widespread language which speaks bigger number of people (linear assimilation). Also it was shown that in case of zero demographic factor of all three subjects, the population with less widespread language completely assimilates the states with two various widespread languages, and the result of assimilation (redistribution of the assimilated population) is connected with initial quantities, technological and economic capabilities of the assimilating states.

In [32] mathematical modeling of nonlinear process of assimilation taking into account demographic factor is offered. In considered model taking into account demographic factor natural decrease in the population of the assimilating states and a natural increase of the population which has undergone bilateral assimilation is supposed. At some ratios between coefficients of natural change of the population of the assimilating states, and also assimilation coefficients, for nonlinear system of three differential equations are received the two first integral. Cases of two powerful states assimilating the population of small state formation (autonomy), with different number of the population, both with identical and with various economic and technological capabilities are considered. It is shown that in the first case the problem is actually reduced to nonlinear system of two differential equations describing the classical model "predator - the victim", thus, naturally a role of the victim plays the population which has undergone assimilation, and a predator role the population of one of the assimilating states. The population of the second assimilating state in the first case changes in proportion (the coefficient of proportionality is equal to the relation of the population of assimilators in an initial time point) to the population of the first assimilating side. In the second case the problem is actually reduced to nonlinear system of two differential equations describing type model "a predator – the victim", with the closed integrated curves on the phase plane. In both cases there is no full assimilation of the population to less widespread language. Intervals of change of number of the population of all three objects of model are found. The considered mathematical models which in some approach can model real situations, with the real assimilating countries and the state formations (an autonomy or formation with the unrecognized status), undergone to bilateral assimilation, show that for them the only possibility to avoid from assimilation is the natural demographic increase in population and hope for natural decrease in the population of the assimilating states.

In [33] mathematical modeling of nonlinear process of the assimilation taking into account positive demographic factor which underwent bilateral assimilation of the side and zero demographic factor of the assimilating sides is considered. In model three objects are considered: the population and government institutions with widespread first language, influencing by means of state and administrative resources on the population of the third state formation for the purpose of their assimilation; the population and government institutions with widespread second language, influencing by means of state and administrative resources on the population of the third state formation for the purpose of their assimilation; population of the third state formation which is exposed to bilateral assimilation from two powerful states or the coalitions.

For nonlinear system of three differential equations of the first order are received the two first integral. Special cases of two powerful states assimilating the population of small state formation (autonomy), with different initial number of the population, both with identical and with various economic and technological capabilities are considered. It is shown that in all cases there is a full assimilation of the population to less widespread language. Thus, proportions in which assimilate the powerful states the population of small state formation are found.

In [34] work mathematical and computer modeling of nonlinear process of the assimilation taking into account positive demographic factor which has undergone bilateral assimilation of the party and negative demographic factors of the assimilating parties is considered.Computer modeling of nonlinear system of three differential equations of the first order in case of fixed coefficients of model is carried out. Special cases of two powerful states assimilating the population of small state education with different initial quantities of the population, both with identical and with various economic and technological capabilities are considered. New numerical results which qualitatively differ from results in case of zero demographic factors of the assimilating parties are received.

In [35] work mathematical modeling of nonlinear process of two-level assimilation taking into account demographic factors of three sides is offered.

I. System of the equations and initial conditions

Using analogies to earlier us of the offered mathematical model [35] , and also developing mathematical model of bilateral assimilation [31 – 34] , we will consider the following general nonlinear mathematical model of two-level assimilation [36]

$$\begin{cases} \dfrac{du(t)}{dt} = \alpha_1(t)u(t) + \beta_1(t)u(t)v(t) + \beta_2(t)u(t)w(t) \\ \dfrac{dv(t)}{dt} = \alpha_2(t)v(t) - \beta_3(t)u(t)v(t) + \beta_4(t)v(t)w(t) \\ \dfrac{dw(t)}{dt} = \alpha_3(t)w(t) - \beta_5(t)u(t)w(t) - \beta_6(t)v(t)w(t) \end{cases} \tag{1.1}$$

$$u(0) = u_0, v(0) = v_0, w(0) = w_0, \tag{1.2}$$

$$u(t), v(t), w(t) \in C^1[0,T],\ t \in [0,T].$$

[0, T] - assimilation process consideration period (as a rule some tens years);

$u(t)$-the population and powerful government institutions with very widespread language, influencing by means of the state and administrative resources the population of two states or theautonomy for the purpose of their assimilation;

$v(t)$- the population and government institutions with widespread second language which underwent assimilation from the powerful state, but in the turn, influencing by means of the state and administrative resources the third population with some less widespread language for the purpose of their assimilation;

$w(t)$- the third population (autonomy) which underwent bilateral assimilation from two rather powerful states (look the scenario of process of two-level assimilation, fig. 1).

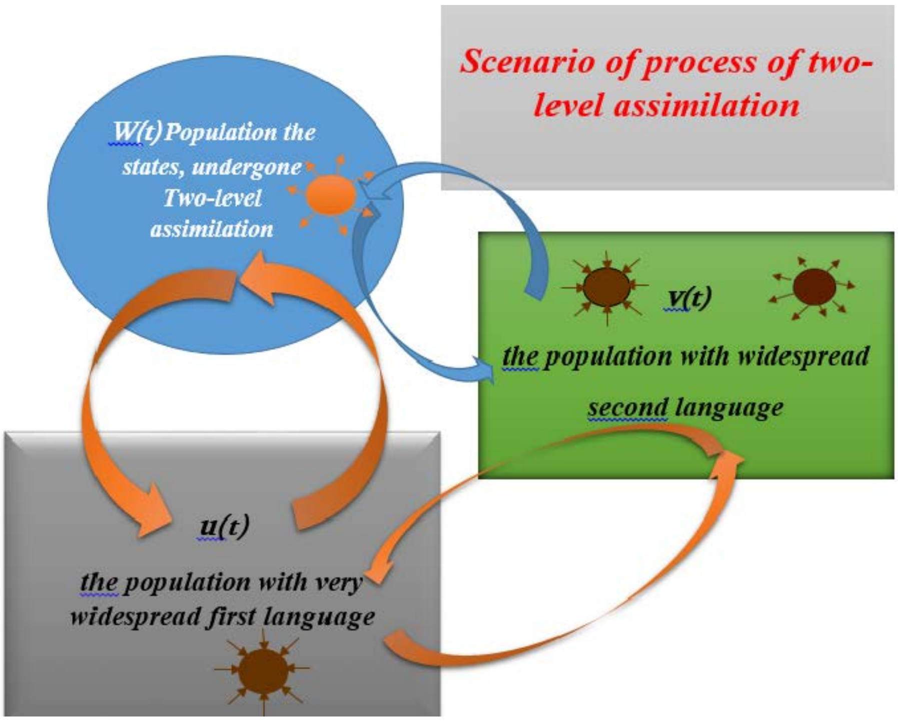

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Fig. 1

The new mathematical model of assimilation offered by us assumes performance of the following natural inequalities

$$\beta_i(t) > 0, i = \overline{1-6}, \qquad t \in [0,T]. \tag{1.3}$$

For the description of nontrivial process (trivial we will call assimilation process when one strong side completely assimilates two other sides) of assimilation it is necessary to make one or the other the following assumptions:

Assumption 1:

$$\begin{cases} \alpha_1(t) < 0 \\ \alpha_2(t) \le 0 \\ \alpha_3(t) > 0 \end{cases} \qquad t \in [0,T] \tag{1.4}$$

Assumption 2:

$$\begin{cases} \alpha_1(t) < 0 \\ \alpha_2(t) \ge 0 \\ \alpha_3(t) > 0 \end{cases} \qquad t \in [0,T] \tag{1.5}$$

II. Some special cases

We will assume that all coefficients of system of the equations (1.1) are constants.

$$\begin{cases} \dfrac{du(t)}{dt} = \alpha_1 u(t) + \beta_1 u(t)v(t) + \beta_2 u(t)w(t) \\ \dfrac{dv(t)}{dt} = \alpha_2 v(t) - \beta_3 u(t)v(t) + \beta_4 v(t)w(t) \\ \dfrac{dw(t)}{dt} = \alpha_3 w(t) - \beta_5 u(t)w(t) - \beta_6 v(t)w(t) \end{cases} \tag{2.1}$$

Assumptions 1, 2 will take a form

$$\begin{cases} \alpha_1 < 0 \\ \alpha_2 \le 0 \\ \alpha_3 > 0 \end{cases} \qquad\qquad \begin{cases} \alpha_1 < 0 \\ \alpha_2 \ge 0 \\ \alpha_3 > 0 \end{cases} \tag{2.2}$$

We will enter transformation

$$\overline{u} = u - u_0, \qquad \overline{v} = v - v_0, \qquad \overline{w} = w - w_0 \tag{2.3}$$

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Then from (2.1) - (2.3) it is easy to receive

$$\begin{cases} \dfrac{\dfrac{d\bar{u}(t)}{dt}}{\bar{u}+u_0} = \alpha_1 + \beta_1 v_0 + \beta_2 w_0 + \beta_1 \bar{v} + \beta_2 \bar{w} \\ \dfrac{\dfrac{d\bar{v}(t)}{dt}}{\bar{v}+v_0} = \alpha_2 - \beta_3 u_0 + \beta_4 w_0 - \beta_3 \bar{u} + \beta_4 \bar{w} \\ \dfrac{\dfrac{d\bar{w}(t)}{dt}}{\bar{w}+w_0} = \alpha_3 - \beta_5 u_0 - \beta_6 v_0 - \beta_5 \bar{u} - \beta_6 \bar{v} \end{cases} \tag{2.4}$$

We will pick up parameters of model so that constants in the right part of system of the equations (2.4) have become equalzero

$$\begin{cases} \alpha_1 + \beta_1 v_0 + \beta_2 w_0 = 0 \\ \alpha_2 - \beta_3 u_0 + \beta_4 w_0 = 0 \\ \alpha_3 - \beta_5 u_0 - \beta_6 v_0 = 0 \end{cases} \tag{2.5}$$

Then

$$\begin{cases} \beta_1 v_0 + \beta_2 w_0 = -\alpha_1 \\ \beta_3 u_0 - \beta_4 w_0 = \alpha_2 \\ \beta_5 u_0 + \beta_6 v_0 = \alpha_3 \end{cases} \tag{2.6}$$

(2.6) represents linear algebraic system of the non-homogeneous equations for u_0, v_0, w_0.

Existence of a set of decisions requires also enough become equal to zero four determinants

$$\det\begin{pmatrix} 0 & \beta_1 & \beta_2 \\ \beta_3 & 0 & -\beta_4 \\ \beta_5 & \beta_6 & 0 \end{pmatrix} = 0, \tag{2.7}$$

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$$\det\begin{pmatrix} -\alpha_1 & \beta_1 & \beta_2 \\ \alpha_2 & 0 & -\beta_4 \\ \alpha_3 & \beta_6 & 0 \end{pmatrix} = 0, \tag{2.8}$$

$$\det\begin{pmatrix} 0 & -\alpha_1 & \beta_2 \\ \beta_3 & \alpha_2 & -\beta_4 \\ \beta_5 & \alpha_3 & 0 \end{pmatrix} = 0, \tag{2.9}$$

$$\det\begin{pmatrix} 0 & \beta_1 & -\alpha_1 \\ \beta_3 & 0 & \alpha_2 \\ \beta_5 & \beta_6 & \alpha_3 \end{pmatrix} = 0 \tag{2.10}$$

From (2.7) it is easy to receive a condition

$$\beta_2\beta_3\beta_6 = \beta_1\beta_4\beta_5 \tag{2.11}$$

From (2.8) – (2.10) taking into account (2.11), we will receive

$$\alpha_3 = \frac{\beta_5}{\beta_3}\alpha_2 - \frac{\beta_6}{\beta_1}\alpha_1 \tag{2.12}$$

According (2.4), (2.5) we will receive

$$\begin{cases} \dfrac{\frac{d\overline{u}(t)}{dt}}{\overline{u}+u_0} = \beta_1\overline{v} + \beta_2\overline{w} \\ \dfrac{\frac{d\overline{v}(t)}{dt}}{\overline{v}+v_0} = -\beta_3\overline{u} + \beta_4\overline{w} \\ \dfrac{\frac{d\overline{w}(t)}{dt}}{\overline{w}+w_0} = -\beta_5\overline{u} - \beta_6\overline{v} \end{cases} \tag{2.13}$$

We will multiplay the equations of system (2.13) respectively on $\gamma_1, \gamma_2, \gamma_3$ also we will pick up

$$\begin{cases} \beta_2\gamma_1 + \beta_4\gamma_2 = 0 \\ \beta_1\gamma_1 - \beta_6\gamma_3 = 0 \\ \beta_5\gamma_3 + \beta_3\gamma_2 = 0 \end{cases} \tag{2.14}$$

It is easy to receive

$$\frac{d}{dt}\ln(u^{\gamma_1}v^{\gamma_2}w^{\gamma_3}) = 0\,. \tag{2.15}$$

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Then taking into account initial conditions (1.2), from (2.15) we will receive the first integral of system (2.1)

$$u^{\gamma_1} v^{\gamma_2} w^{\gamma_3} = u_0^{\gamma_1} v_0^{\gamma_2} w_0^{\gamma_3} \tag{2.16}$$

The determinant of a matrix of system of the equations (2.14), according to (2.7) is equal to zero, therefore there is an uncountable set of decisions $\gamma_1, \gamma_2, \gamma_3$.

We will consider several classes of decisions and according to the first integrals (2.16)

1. $$\begin{cases} \gamma_1 = \beta_6 / \beta_1 \\ \gamma_2 = -\beta_5 / \beta_3 \\ \gamma_3 = 1 \end{cases}$$

$$\frac{u^{\beta_6/\beta_1} w}{v^{\beta_5/\beta_3}} = \frac{u_0^{\beta_6/\beta_1} w_0}{v_0^{\beta_5/\beta_3}} \tag{2.17}$$

2. $$\begin{cases} \gamma_1 = -\beta_4 / \beta_2 \\ \gamma_2 = 1 \\ \gamma_3 = -\dfrac{\beta_3}{\beta_5} \end{cases}$$

$$\frac{u^{\beta_4/\beta_2} w^{\beta_3/\beta_5}}{v} = \frac{u_0^{\beta_4/\beta_2} w_0^{\beta_3/\beta_5}}{v_0} \tag{2.18}$$

3. $$\begin{cases} \gamma_1 = -1 \\ \gamma_2 = -\dfrac{\beta_2}{\beta_4} \\ \gamma_3 = \dfrac{\beta_1}{\beta_6} \end{cases}$$

$$\frac{u w^{\beta_1/\beta_6}}{v^{\beta 2/\beta_4}} = \frac{u_0 w_0^{\beta_1/\beta_6}}{v_0^{\beta 2/\beta_4}} \tag{2.19}$$

We will consider the first case

$$\begin{cases} \beta_1 = \beta_6 \\ \beta_5 = \beta_3 \end{cases} \tag{2.20}$$

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Then from (2.11), (2.12) we will receive

$$\begin{cases} \beta_4 = \beta_2 \\ \alpha_3 = \alpha_2 - \alpha_1 \end{cases} \tag{2.21}$$

And from (2.17) we will receive

$$\frac{uw}{v} = \frac{u_0 w_0}{v_0} \;. \tag{2.22}$$

In phase space (O,u,v,w) (2.22) represents a hyperbolic paraboloid.

From (2.22) we will receive

$$v = puw, \quad p = \frac{v_0}{u_0 w_0} \tag{2.23}$$

Taking into account (2.21), (2.23) task (2.1), (1.2) will take a form

$$\begin{cases} \dfrac{du(t)}{dt} = \alpha_1 u(t) + \beta_1 p u^2(t) w(t) + \beta_2 u(t) w(t) \\ \dfrac{dw(t)}{dt} = (\alpha_2 - \alpha_1) w(t) - \beta_3 u(t) w(t) - \beta_1 p u(t) w^2(t) \end{cases} \tag{2.24}$$

$$u(0) = u_0, w(0) = w_0.$$

(2.24) it is easy to rewrite in the following look

$$\begin{cases} \dfrac{\dot{u}}{u} = \alpha_1 + \beta_1 puw + \beta_2 w \\ \dfrac{\dot{w}}{w} = \alpha_3 - \beta_1 puw - \beta_3 u \end{cases} \tag{2.25}$$

We will assume that

$$\begin{cases} \alpha_2 = 0 \\ \beta_2 = \beta_3 \end{cases} \tag{2.26}$$

Then from (2.25) we will receive

$$(uw)^{\bullet} = -\beta_2 uw(u - w). \tag{2.27}$$

Having multiplyed the first equation of system (2.24) and the second equation on the u and w after of some transformations we will receive

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$$\left(u^2+w^2\right)^{\bullet}=2(u-w)\left[\alpha_1(u+w)+\beta_2 uw+\beta_1 puw(u+w)\right]. \tag{2.28}$$

Having divided (2.28) on (2.27) we will receive

$$\frac{d(u^2+w^2)}{d(uw)}=\frac{2\left[\alpha_1(u+w)+\beta_2 uw+\beta_1 puw(u+w)\right]}{-\beta_2 uw}. \tag{2.29}$$

We will enter designations

$$\begin{cases} u+w\equiv z \\ \\ uw\equiv y \end{cases} \tag{2.30}$$

Then the differential equation (2.29), taking into account (2.30) will correspond in the following look

$$\frac{d(z^2-2y)}{dy}=\frac{2\left[\alpha_1 z+\beta_2 y+\beta_1 pyz)\right]}{-\beta_2 y}. \tag{2.31}$$

The decision (2.31), taking into account initial conditions (2.24) and designations (2.30), has an appearance

$$z-z_0=-\frac{\alpha_1}{\beta_2}\ln\frac{y}{y_0}-\frac{\beta_1 p}{\beta_2}(y-y_0). \tag{2.32}$$

Thus taking into account designations (2.30) from (2.32) we will receive an integrated curve in the phase plane (O,w,u)

$$u+w-(u_0+w_0)=-\frac{\alpha_1}{\beta_2}\ln\frac{uw}{u_0w_0}-\frac{\beta_1 p}{\beta_2}(uw-u_0w_0)\ . \tag{2.33}$$

We will enter designations

$$\begin{cases} F_1(u,w)\equiv\alpha_1 u(t)+\beta_1 pu^2(t)w(t)+\beta_2 u(t)w(t) \\ F_2(u,w)\equiv(\alpha_2-\alpha_1)w(t)-\beta_3 u(t)w(t)-\beta_1 pu(t)w^2(t) \end{cases}. \tag{2.34}$$

It is easy to receive

$$\frac{\partial F_1}{\partial u}=\alpha_1+2\beta_1 pu(t)w(t)+\beta_2 w(t)$$

(2.35)

$$\frac{\partial F_2}{\partial w}=\alpha_2-\alpha_1-\beta_3 u(t)-2\beta_1 pu(t)w(t)$$

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$$\operatorname{div}\vec{F} = \frac{\partial F_1}{\partial u} + \frac{\partial F_2}{\partial w} = \alpha_1 + 2\beta_1 pu(t)w(t) + \beta_2 w(t) +$$

$$\alpha_2 - \alpha_1 - \beta_3 u(t) - 2\beta_1 pu(t)w(t) =$$

$$= \alpha_2 + \beta_2 w - \beta_3 u \equiv G(u,w) \tag{2.36}$$

$$G(u,w) = 0$$

$$u = \frac{\beta_2}{\beta_3} w + \frac{\alpha_2}{\beta_3}, \qquad \alpha_2 = 0 \tag{2.37}$$

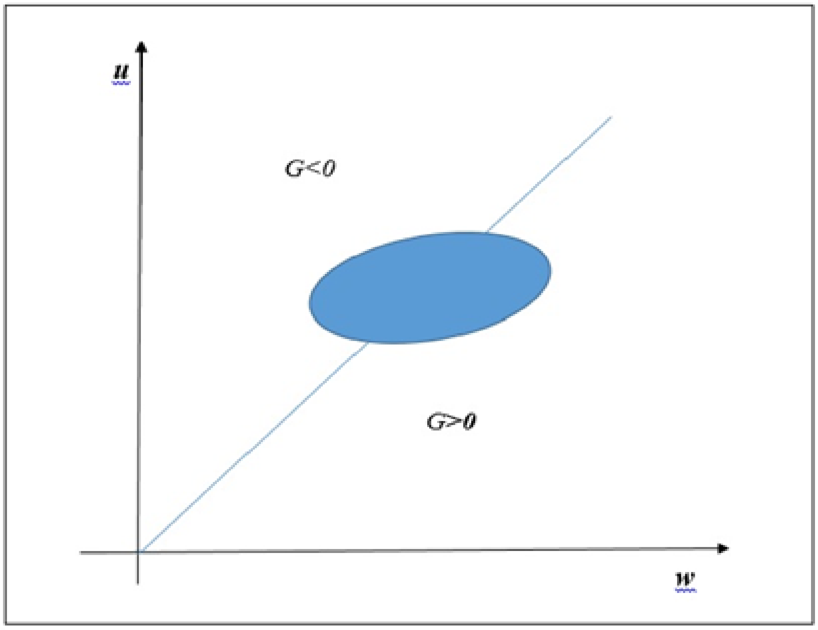

Fig. 2

Theorem 1. A task (2.24) in some one-coherent area $D \subset (O, u(t), w(t))$ the first quadrant has the decision in the form of the closed trajectory which is completely lying in this area.

Thus according to (2.37) in the first quadrant of the phase plane (O, w, u) there is such area in which $G(u; w)$ function of a sign change and according to Bendikson's kriterium in this area existence of the closed integrated curve is possible, i.e. in this case $w$function doesn't become equal to zero and there is no full assimilation of the third side.

We will consider the second special case

$$\begin{cases} \beta_1 = \beta_6 \\ 2\beta_3 = \beta_5 \end{cases} \tag{2.38}$$

Then from (2.17) respectively we will receive

$$v^2 = q^2 uw, \qquad q^2 = \frac{v_0^2}{u_0 w_0}. \tag{2.39}$$

In phase space (O, u, v, w) (2.39) represents a **cone.**

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Taking into account (2.39) system of the equations (2.1) will assume an air:

$$\begin{cases} \dfrac{du(t)}{dt} = \alpha_1 u(t) + \beta_1 q u(t)\sqrt{wu} + \beta_2 u(t) w(t) \\ \dfrac{dw(t)}{dt} = (2\alpha_2 - \alpha_1) w(t) - 2\beta_3 u(t) w(t) - \beta_1 q \sqrt{uw} w(t) \end{cases} \tag{2.40}$$

We will enter designations

$$u(t) \equiv \varphi^2(t), \qquad w(t) \equiv \psi^2(t) \ . \tag{2.41}$$

Then from (2.40), (2.41) we will the following task

$$\begin{cases} \dfrac{d\varphi(t)}{dt} = \dfrac{\alpha_1}{2}\varphi(t) + \dfrac{\beta_1 q}{2}\varphi^2(t)\psi(t) + \dfrac{\beta_2}{2}\varphi(t)\psi^2(t) \\ \dfrac{d\psi(t)}{dt} = \dfrac{2\alpha_2 - \alpha_1}{2}\psi(t) - \beta_3 \psi(t)\varphi^2(t) - \dfrac{\beta_1 q}{2}\varphi(t)\psi^2(t) \end{cases} \tag{2.42}$$

$$\varphi(0) \equiv \sqrt{u}_0, \qquad \psi(0) \equiv \sqrt{w}_0$$

We will enter designations

$$\begin{cases} F_1(\varphi,\psi) \equiv \dfrac{\alpha_1}{2}\varphi(t) + \dfrac{\beta_1 q}{2}\varphi^2(t)\psi(t) + \dfrac{\beta_2}{2}\varphi(t)\psi^2(t) \\ F_2(\varphi,\psi) \equiv \dfrac{2\alpha_2 - \alpha_1}{2}\psi(t) - \beta_3 \psi(t)\varphi^2(t) - \dfrac{\beta_1 q}{2}\varphi(t)\psi^2(t) \end{cases} \tag{2.43}$$

It is easy to receive

$$\frac{\partial F_1}{\partial \varphi} = \frac{\alpha_1}{2} + \beta_1 q \varphi(t)\psi(t) + \frac{\beta_2}{2}\psi^2(t)$$

$$\frac{\partial F_2}{\partial \psi} = \frac{2\alpha_2 - \alpha_1}{2} - \beta_3 \varphi^2(t) - \beta_1 q \varphi(t)\psi(t) \tag{2.44}$$

$$div(\vec{F}) = \frac{\partial F_1}{\partial \varphi} + \frac{\partial F_2}{\partial \psi} = \alpha_2 + \frac{\beta_2}{2}\psi^2(t) - \beta_3 \varphi^2(t) \tag{2.45}$$

$$G_1(\varphi(t),\psi(t)) \equiv \alpha_2 + \frac{\beta_2}{2}\psi^2(t) - \beta_3 \varphi^2(t)$$
$$G_1(\varphi(t),\psi(t)) = 0,$$

$$\alpha_2 + \frac{\beta_2}{2}\psi^2(t) - \beta_3 \varphi^2(t) = 0 \, . \tag{2.46}$$

Theorem 2. A task (2.42) in some one- $D \subset (O, \varphi(t), \psi(t))$ coherent area the first

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quadrant has the decision in the form of the closed trajectory which is completely lying in this area.

Thus according to (2.46) in the first quadrant of the phase (O, φ, ψ) plane there is such $G_1(\varphi, \psi)$ area in function of a sign change and according to Bendikson's kriterium in this area existence of the closed integrated curve is possible, i.e. in this case ψ function (according 2.41 $w(t)$ function) doesn't become equal to zero and there is no full assimilation of the third side.

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