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Definition Problem of Algorithmic Music Composition. Re-evaluation of the Concepts and Technological Approach

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***Summary**:*

In the 20th–21st centuries, the practice of the constructive composing of music is a diverse result in which mathematical manipulations of earlier epochs and modern innovations coexist. The establishment of the conception of algorithmic composing of music and application of algorithmic processes in the sphere of composing contemporary music makes an ever wider use of computer device and has opened immense possibilities for generating musical ideas and their implementation.

Phenomenon of algorithm in music itself covers a wide range of creative procedures. The application of algorithm in music creative process may be distinguished in one more point of view: 1) algorithm as a tool for composing and 2) systematic effect on the whole composition, its musical language and structure, etc. Conceptually algorithmic music composition may be considered as: 1) imitation of certain style; 2) original music result. Also, music composition may be differentiated into computer generated music and computer assisted composition. Besides, this practice provokes the re-evaluation of composer's role. The definition of the function of a computer (or software) is becoming complicated too – whether it is merely a tool of generating the music according to certain instructions. Therefore, the practice of algorithmic music has caused the need for new directions and methodologies in music analysis, peculiarly initiating value-based transformations of the creator and categories of musical compositions.

Keywords:*Algorithmic Music, Computer Assisted Composition, Role of Composer, Advanced Mathematics Theories in Music*

Introduction

Most likely, one of the answers to the question where the vitality of the idea of the interaction between music and mathematics – which can be traced for more than two thousand years and which has been developed in the modern world – lies in the perception that mathematics is the principal cause and source of an all-embracing beauty. The attitude towards mathematics as the art of the beauty of numbers has had an effect on the environment as well: operations with numbers, regularities of symmetry and proportions, have become beauty formulas in different spheres of art. The idea of a mathematically substantiated world was developed as far back as Antiquity, and through its expression has attracted thinkers of this epoch. The name of Aristotle is related to a scholastic definition of beauty: the criteria of beauty, "orderly arrangement, proportion, and definiteness", are "especially manifested by the mathematical sciences." [1] From the perspective of a European perception, the definition of the science of music was a result of the Pythagorean concept, which is based on the universal harmony of numerical proportions. The popular concept that "there is geometry in the humming of strings, there is music in the spacing of the spheres" is attributed to Pythagoras [2].

The idea that a group of mathematical sciences united music, astronomy, geometry and arithmetic belongs to the Pythagoreans as well. These four sciences were called quadrivium by Boethius in the 6th century and were included in the system of seven disciplines – septem artes liberales – taught at universities in the Middle Ages. Attributing music to the sphere of mathematics

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was characteristic of the Baroque epoch as well. The type of music found at that time, *musica scientia* (also *musica contemplativa/speculativa/theorica/theoretica*), was called by Gottfried Leibniz, Johannes Lippius, Jakob Adlung or Andreas Werckmeister “sounding mathematics”, “mathematical knowledge”, “the daughter of mathematics” or “the science of mathematics creating harmonious singing” [3].

In the first dictionary of music written in the German language, *Musicalisches Lexicon Oder Musicalische Bibliothec* (1732) by Johann Gottfried Walther, among different types of music, the type *musica arithmetica* is indicated, that is, arranging the sounds in proportions and numbers [4]; Johannes Kepler called the theory of music perfect science (German *vollkommene Wissenschaft*), whose eternal truth is mathematics [5]; while Lorenz Mizler stated that mathematics is the heart and soul of music, and “without question the bar, the rhythm, the proportion of the parts of a musical work and so on must all be measured. […] Notes and other signs are only tools in music, the heart and soul is the good proportion of melody and harmony. It is ridiculous to say that mathematics is not the heart and soul of music.” [6]

In the 20th–21st centuries, the practice of the constructive composing of music is a diverse result in which mathematical manipulations of earlier epochs and modern innovations coexist. One might say that the compendium of *musica mathematica* of earlier epochs engenders a peculiar eclectic combination, one that employs a combination of a variety of different constructive manipulations. In contemporary musical composition the traditions of applying antique proportions, Kabbalistic numbers, Christian numerology, and numericalized semantics have been revived. The variety of tools used in composing music helps determine contemporary composers’ aspirations for individuality and exclusiveness, which dominates our modern worldview and which manifests itself in the especially personified intentions of composers. However, in the diverse practice of creating music in the 20th–21st centuries, the innovations of mathematical processes related to the application of new mathematical theories (e.g., the principles taken over from more advanced mathematics, fast developing spheres of information technologies), which considerably broadened the space of creative possibilities and the problem field, became established on equal terms alongside traditionally determined phenomena of a mathematical nature. This is a trend of writing music in complicated algorithms that has become possible in the age of the computer, allowing the creation of geometrical, graphic algorithms as a prototype of a composition and ways of a schematized expression of a musical composition, as well as the explication of complicated mathematical formulas or models, mathematical theories (fractals, chaos, groups, probabilities and others), practice of scholastic music, etc.

Concept of algorithm in music

The establishment of the conception of algorithmic composing of music and application of algorithmic processes in the sphere of composing contemporary music makes an ever wider use of computer device and has opened immense possibilities for generating musical ideas and their implementation. According to Kristine Burns, the concept of the musical algorithm is close to the use of a medical algorithm to pinpoint a clinical diagnosis, because of its gradual process and the chain of interconnected questions and answers [7].

The concept of the algorithm [8] has been tied with the field of musical composition since the beginning of the 20th century. Still, it is important to precisely differentiate the use of algorithms in music, as this allows one to study the influence of algorithm on various styles and genres of professional music. For example, the creative principles of Mozart’s dice game (*Würfelspiel*) can be applied to algorithmic processes, as well as the Renaissance mensural system, the various types of isorhythmic motet and the canon, in addition to other examples of music where the results were achieved through the use of certain rule-based operations. Gerhard Nierhaus notes examples of algorithms used in music as far back as Bach’s *Die Kunst der Fuge* or Schönberg’s dodecaphonic system [9].

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Adam Alpern [10] describes the algorithmic music composing as follows: “The area of automated composition refers to the process of using some formal process to make music with *minimal human intervention*.” [11] This statement suggests the narrowing of this problem to the contemporary active use of the computer as a creative tool within the field of electronic music. The growing interest in the composing of computer-generated algorithmic music may be motivated by the fact that contemporary artists, with the aids of computers, have been able to reach a level that is equal to that of a highly mathematized tonal space. There are now a variety of opportunities to adapt, for example, nature’s mathematical principles of evolution or algorithms of genetics to the field of musical composition. Complex computer programs are written to accommodate this interest. The first program to generate musical compositions was written in the 1950s by Lejaren Hiller and Leonard Isaacson. The first algorithmic computer-generated composition, *Illiac Suite* for string quartet (1955–6), was composed by these same authors (see Fig. 1). The interval sequence of this quartet used Markov chain algorithms. The rule-based principle of statistical processes was applied to program computer-generated music [12]. Later examples are *Algorithms* (1968) by Hiller and Isaacson, Xenakis’ experimental pieces from 1971, and his anthological instance of computer composition, *Gendy 3* (1991), where the tonal synthesis was generated by GENDYN [13].

Figure 1. The first algorithmic computer-generated composition: the first part from *Illiac Suite* for string quartet (1955–6) by Lejaren Hiller and Leonard Isaacson.

Towards the definitions for algorithmic music

Most of the time, computer-generated musical compositions are referred to as algorithmic. This is evidenced by terminology, such as “computer aided” or “computer assisted” composition [14]. Electronic compositions are differentiated into computer composed and computer realized music as well [15]. Often the musical concepts “algorithmic composition” (AC) and “computer assisted composition” (CAC) are used synonymously.

The composition of computer-generated music based on algorithms became more intense from the second half of the 20th century onwards, encouraging one not only to come to certain analytical generalizations, but also influencing more complex directions in musical analysis. This is because the concept of the algorithm takes up an especially wide spectrum of creative procedures. On the one hand, when analyzing the concept of algorithmic composition, it is important to

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differentiate between imitative work (for example, imitations of a specific style) and an original musical composition generated by an algorithm. On the other hand, according to Miller Puckette [16], a purely technical differentiation is applied. That is “computer generated music” (CGM, Denis Baggi’s concept) and the aforementioned “computer assisted composition” (CAC):

- computer generated music, CGM, is equated with a synthesized, processed and/or designed sound. The computer takes on the role of the music instrument;
- computer assisted composition, CAC, is entrusted with the task of performing complex mathematical calculations, using complex algorithms, that is, functions that formerly were attributed to human thinking or creative acts.

We may expand Puckette’s description by David Cope’s triad [17]:

- first, “computer generated sound” (CGS), that is close to the sound generated by a synthesizer;
- second, “computer generated assistance” (CGA). The computer is an aid that is used to organize the various elements of music and the parameters of musical language;
- third, “computer generated composition” (CGC) that is close to the aforementioned CGM.

The variety of algorithmic processes in music can be grouped according to characteristic technological principles as well. The result of their application is a unique reproduction of the original musical material. The most popular are: sonification (sound processing, the use of non-sound data), meaning a computerized translation into sound; and a variety of mathematical operations, the use of calculations, the choice of the mathematical operation for a musical prototype, or the composition of new musical material generated by applied musical data analysis (*Data-Driven*); for example the analysis of already existing musical compositions.

While trying to generalize how algorithmic actions are targeted to create music, in the practice of music, I would divide their ideas into two directions, as follows:

1. The universal application of the concept of “algorithm.” This encompasses a variety of determined (set, defined) processes, a variety of constructive actions, which are performed by the composer himself, while holding to respective rules. Part of these algorithms function mechanically. They are easy to distinguish in tonal material. All the examples of aspects of musical composition based on certain mathematical elements, from all of the epochs we have discussed thus far – numbers, proportions, progressions, and so on – could also be attributed to this direction of algorithmic procedures. We can also see the algorithmic nature in Pythagoras’ theory of tone-number equivalents. In the opinion of Järvelainen [18], algorithmic aspects have a unique influence on formal compositions from the Middle Ages that were created according to the graphs of Guido of Arezzo, on rhythmic models of 15th century isorhythmic motets, or in Dufay’s compositions, which show the Golden Ratio in their inner structure. The performance marks for Renaissance mensural canons are also interpreted by applying algorithms. In this case, the composer is merely the author of the initial motif, or the core of the composition, from which the composition is constructed, and of a complex of rules. Schonberg’s twelve-tone series, which Webern expanded into total serialization, according to John A. Maurer, is noted for controlling absolutely all musical parameters, maximally abstracting the composition process [19]. For this reason the matrixes (series) that are made up of the dodecaphonic principle encourage to name them as algorithms.

2. Application of complex mathematical algorithms, as well as algorithms from other fields of science, are applied to the composition of contemporary music. However, their development and application is difficult to notice, and they are often only brought to life in the computerized realm. The composer concentrates on the complicated process of composing music, which is based on complex mathematical procedures. An example would be the experimental music composer Charles Dodge’s composition *The Earth’s Magnetic Field*

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(1970). The number sequences of the movement of the Earth's gravitational field was used as the prototype for the tonal fabric of this piece. The computer converted the numbers into tones [20]. In Dodge's composition *Profile* (1984) the pitch, rhythm, and amplitude were influenced by an adaptation of the 1/f noise algorithm. According to Järvelainen [21], the principles of Markov chains are especially appropriate for generating musical melodies.

The adaptation of algorithms to the process of musical composition can be differentiated according to whether the algorithm is only one of the tools of composition, or whether an algorithm is systematically applied to the musical language, structure of the composition [22]. In the first instance, the given algorithm is controlled by the composer and "taken care of." The algorithm influences the different musical parameters. An example would be pitch, rhythm, dynamics, and their combinations. The second instance would be where the algorithm, or a combination of algorithms, is given a more independent function, one that would encompass the entirety of compositional codes. David Cope has been thinking along these lines [23]: he sees two aspects of the composer's intention in the creative production of computer-generated music. One group, for example, Bryan Ferneyhough, composes music in a traditional manner, but uses the computer as an additional tool for composition. This group of composers experiments with certain models, but modulates the final result according to their own "taste" (meaning, giving the work a human touch with the space of the computer-generated composition). The second group, like Gottfried Michael Koenig, in most of his compositions uses the computer program as a complete method for problem solving, for organizing all of a work's parameters, and to control the musical material. However, I would like to raise a debate within the realm of creative dimension – does the algorithm take over the role of the composer completely, or at least in part.

Practical implementation of algorithms in music

The practice of computer programmed algorithms in contemporary music generalized from a few different sources [24] can be examined according to a choice of respective mathematical phenomena. Therefore, we may distinguish eight algorithmic principles for musical composition and/or analysis as following:

- the principles of generative grammars (based on structural linguistics analysis) [25];
- genetic algorithms;
- Markov chains/model [26];
- the adaptation of the chaos, self-similarity principles (Verhulst equation, various theories of noise [27], chaos systems, Cantor set, Mandelbrot set, the Koch snowflake and other fractals);
- cellular automata [28];
- analogies of transition networks;
- artificial neural networks [29];
- artificial intelligence (AI) processes that are based on the right choice for the next step [30].

Generative grammar

The origins of generative grammar in music can be identified already in the early 20th century with Heinrich Schenker's Ursatz theory. Later the generative grammar model was successfully adapted in Fred Lerdahl's and Ray Jackendoff's research. In order to reduce the composition to its most essential tones, they strove to purify the hierarchy of layers of musical structure by analyzing the relationship of groups (when a piece is divided into motifs, phrases, and sections), metric structure, and the reduction of a duration, which is connected with pitch, group and metric indicators. Lelio Camilleri's research was connected with the principles of generative grammar, which he applied to the analysis of few songs from Schubert's vocal cycles Op. 23, 25, and 89. Using the derived data of initial phrases, Camilleri generated this style's melody copies [31]. Together with a group of scientists, Mario Baroni created a few computer projects that analyzed examples of classical music or composed music in the chosen style according to the principles of

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generative grammar. The program Melos 2 was adapted to analyze the structure of Lutheran chorales; the program Harmony generated a bass and a harmonic vertical for a given melody [32].

Genetic algorithm

Genetic algorithm principles are essentially similar to the traditional process of music composing. According to the algorithm, the initial model experiences its structural modifications, mutations, conflict, and other actions in order for biological genes to form; the adequacy of the result is compared with other generative models, and so on. In the sound space, it resembles the development and modification of the musical material, or the principles of inversion, retrograde, and other transformations. The principles of genetic algorithms can be applied to the analysis of classical music as well. For example, Michael Towsey et al. used the computer to analyze Renaissance compositions and popular music, as well as children's songs. They analyzed a total of 36 pieces and put together a chart to compare all of them according to 21 features – pitch, tonality, melodic relief, rhythm, repeating models, motifs, as well as a complex of other qualities [33].

Andrew Horner and David E. Goldberg were among the first to write about thematic bridging in the generating of music. They give an example how, according to the principles of genetic algorithms, out of the chosen five-tone motif it is possible to generate a certain melody [34] (see Fig. 2):

- firstly, the last tone is discarded, and the result is the four-tone motif;
- second, the new motif 's tones change places;
- third, from the last motif the final tone is discarded;
- fourth, the first tone of the resulting motif is changed;
- fifthly, the tones once again change places;
- sixthly, the result is achieved – a sequence of all the tones.

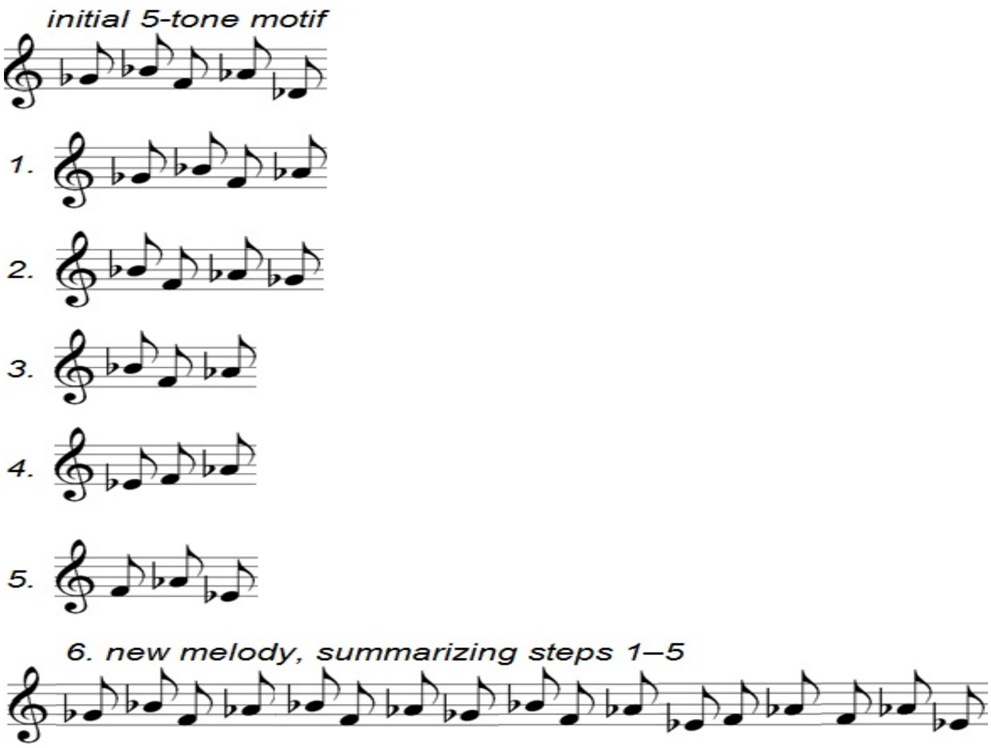

Figure 2. Generation of a certain melody according to the principles of genetic algorithms (the arrangement of music fragments based on the description by Andrew Horner and David E. Goldberg).

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Markov chain/model

The Markov chain theory has become one of innovative means of composing music and analyzing it as well. The first time this method was used in 1950, when Harry F. Olson applied it to analyze the songs of Stephen Foster. Wei Chai and Barry Vercoe used the principle of the hidden Markov model to analyze and compare pitch, interval structures, and duration data of the melodies of folk songs from Ireland, Germany, and Austria. According to Nierhaus, Markov chains could be applied to a one-dimensional symbol sequence analysis. Therefore, it is never completely adequate to thoroughly analyze a complex musical composition to evaluate its horizontal and vertical structure [35].

The use of the model and the composing practice can be explained as the aims of composers in the creative process to smooth any cognitive possibility. The application of the probability process can be seen in Cage's declaration seeking to free himself from an individual approach that influences his creative process [36]. Already at the beginning of the 1950s, when Cage read the *I Ching* (*Book of Changes*), those texts inspired him to take an interest in probability theory. The composer used the process of tossing coins in his creative process. An intermediary, an oracle, chose the pitch from the charts that he created. Xenakis, protesting against the strict serialized control of tone, chose the more common concept of "stochastic music" as a scholarly synonym of chance and applied the distribution of probabilities to musical composition. For example, in the composition *Pithoprakta* (1955) probability processes determined the duration of tones that were formed based on the kinetic theory of gas, while the tonal changes revealed the analogies of the stochastic process [37]. Other compositions by Xenakis that are based on the logic of the Markov chain are *Analogique A* for string orchestra (1959), *Analogique B* for sinusoidal sounds (1958–9) and *Syrmos* for eighteen strings (1959); in 1962 he completed *Morsima-Amorsima* for four instruments. Based on Xenakis' experiments, it is possible to differentiate several compositional methods of stochastic music [38] as follows:

- free-form stochastic music (based on the probability theory);
- Markov's stochastic music (musical composition according to the Markov model);
- a musical strategy (the group theory is used in the composition).

Group theory

The mathematical direction of contemporary music is reflected in the use of the mathematical group theories. The author of transformational music theory, David Lewin, based his work on group theory when he researched the relationships of musical intervals as an expression of the transformational net. The group theory model is applied to the analysis of musical scales as well, because the transcription of the 12-tone system into a number sequence matches the module 12 principle; group C12 (a cyclic group of order 12) is made up of 12 of its elements, the tones (*c* – 0, *c-sharp* – 1 and so on).

Chaos theory

The concept of "chaos" is in opposition to the "cosmos" concept that represents harmony. However, chaos is an organized system based on inside rules and brings an order to the creative music process. That is because "from the variety of endless possibilities, that is chaos, the orderly respective musical elements are selected" [39]. The creation of chaos in contemporary musical composition can be mathematically based using its mathematical formulas as compositional algorithms. Examples of the application of the chaos principle in contemporary music is quite varied. It could be an expression of Granular Synthesis [40] which was first used by the Canadian composer Barry Truax in his piece *Riverun* (1986). Xenakis applied this phenomenon to his work *Gendy 3* (1991). The model for counting population expansion $f(X) = P \times X \times (1 - X)$, the so-called Verhulst equation [41], has been applied to the practice of contemporary music as well.

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Gary Lee Nelson's composition *The Voyage of the Golah Iota* (1993) made use of this mathematical formula. The composer chose the numbers from 1 to 4 for the measurement *P*, for *X* – from 0 to 1, and using the computer, graphically recreated the equation data. The graphic visualization was transformed into a musical space. Additionally, to program this piece Nelson applied granular synthesis as well as the principles of genetic algorithms, chaos actions that determined the repetition of elementary motifs and complex sequences [42].

The phenomena of chaos may be expressed in music on abstract or semantic level as well. This approach is suitable to rationally characterize György Ligeti's creative work. Ligeti was intrigued by the effect of illusionary rhythm. According to the composer himself [43], the best example would be the third part of his Concerto for piano (1985–8); here the bar line is rendered unnecessary. The polyrhythmic principles he applied to his piece for two pianos, *Monument* (1976). In this composition, two pianists perform the same phrase in a different meter – duple time and triple time. This is also illustrated in an earlier idea by Ligeti, the effect of which the composer compares with the house of an old widow that is filled with old ticking clocks [44]. This is a composition for a hundred metronomes called *Poeme symphonique* (Symphonic Poem, 1962). In this piece he uses mechanical metronomes, whose ticking grows slower in varying tempos and which creates a complicated rhythmic micropolyphony.

Ligeti's etude *Désordre* (1985) for piano is one more example of logical construction of polymeter that finally evokes the chaotic sound. The sense of disorder was created, holding on to constructively defined rhythmic sequences and a strict order. Typical of the composition is a rational blending of metro-rhythm and accents, as well as the drama of their exchange (tendencies of polymeter). The analysis of the metro-rhythmic structure shows that the piece is compiled with accentuated and shifted structures that are based on sequences of certain rhythmic values. For example, in the first section of the etude (mm. 1–33) the left hand is continually playing the line of 8 eighth notes. Its perpetual motion is combined with the periodically shortening of this rhythmic group by one eighth note in the part for the right hand (shortening happens every fourth measure; see Fig. 3). This results in displacement of metric accents (downbeats). After a certain time, at the end of the first section, the part for the right hand becomes a measure longer (in the right – m. 33 = in the left – m. 32). The derived graphic scheme of the metro-rhythmic shifting of the first section I compared with the scheme of the third section. In this manner we come up with a polymeter palindrome that is formed from a distance, because in the third section an analogical shift of accents is made up in the part for right hand, where one eighth note is added every fourth measure (see Fig. 4 & 5).

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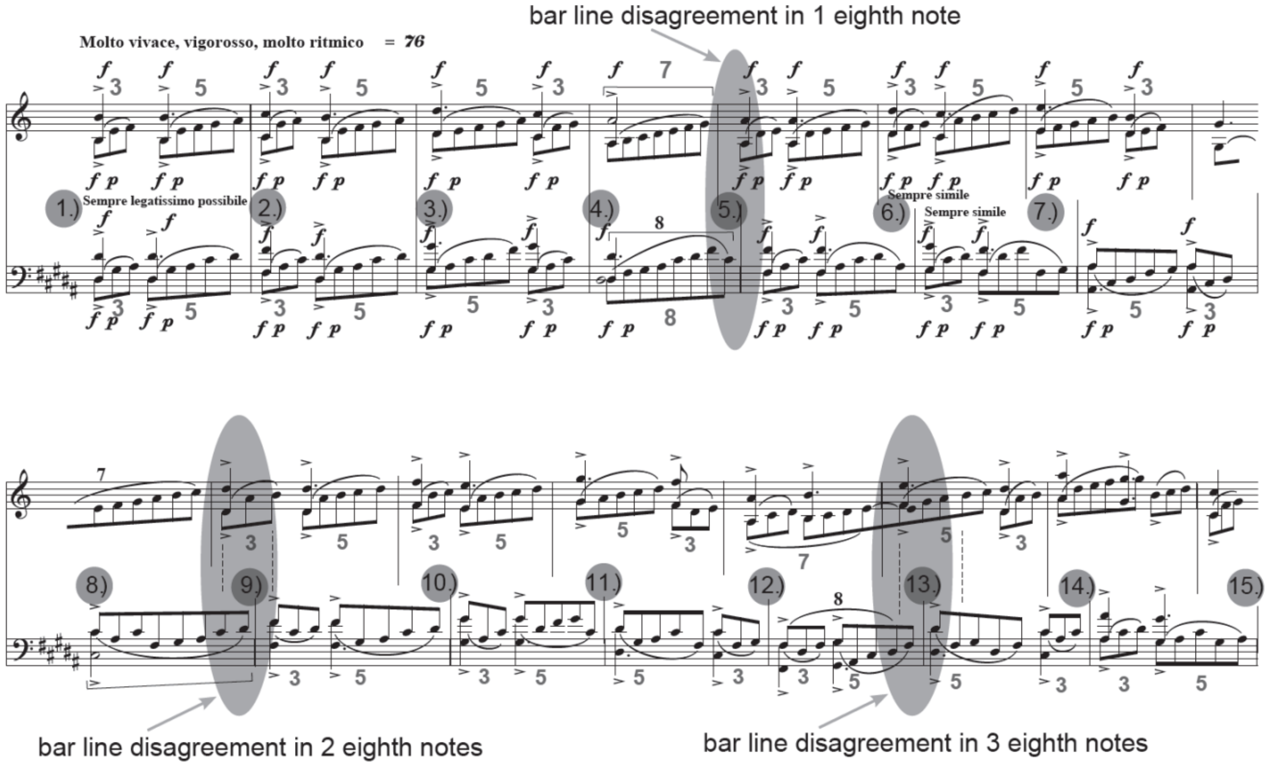

Figure 3. Liget, *Désordre*. Shift of metric accents in mm. 1–29.

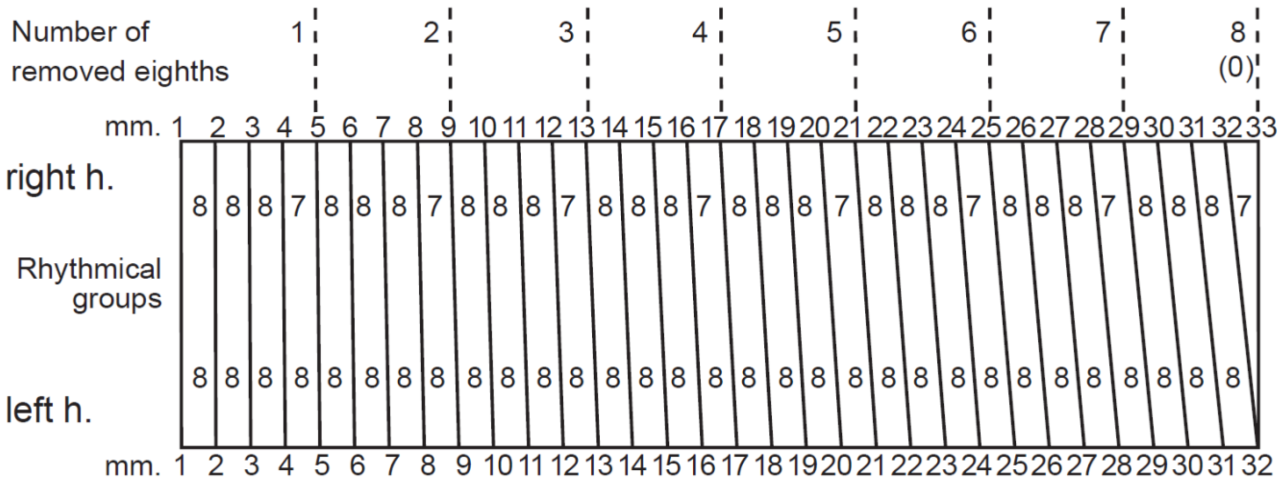

Figure 4. Liget, *Désordre*. Diagram of the first section, mm. 1–33.

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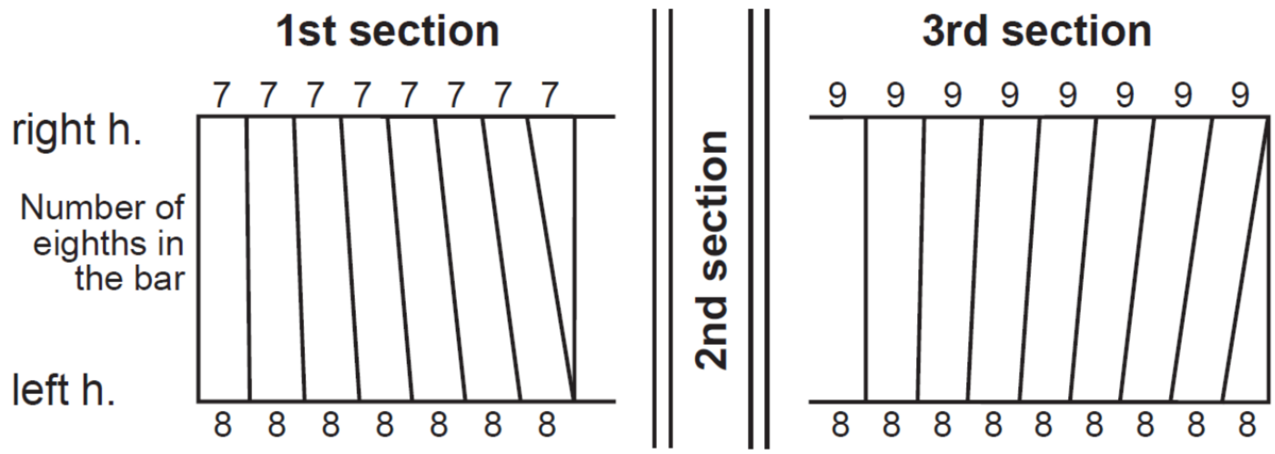

Figure 5. Liget, *Désordre*. Palindromic diagram of the first and third sections.

Fractal theory analogies

After Benoit Mandelbrot's [45] discoveries in the field of fractal geometry, this theory found its place in the imaginations of contemporary composers as a tonal realization of fractal geometry [46]. The study of the expression of geometric fractals in music inspired new analytical works. According to two Swiss brothers, geologist Kenneth Hsü and musicologist Andrew Hsü, fractals are typical even in Bach's and Mozart's music. That is because in these composers' work the acoustic model, or the fundamental structure, remains when compressed according to fractal principle. For example, the fundamental tonal scale of Bach's clavier inventions remains the same even after getting rid of 1/2, then 1/3, 1/4 and so on, numbers of tones [47].

The self-repetition of the fractal macromodel in smaller levels can be seen in Schenker's theory as well, because the reductive experiments in the musical composition, the derivation of *Vordergrund*, *Hintergrund* and *Urlinie*, recall the principles of self-similarity.

Often composers themselves discuss the implications of fractals in musical composition. According to Bruno Degazio the piece *Roads to Chaos* (1986) was composed using fractal processes [48]. In his computer music piece *Profile* (1984) Charles Dodge used the computer to match pitch, rhythm, elements of amplitude to the 1/f noise algorithm. Dodge describes this work as a recursive structure that fills in time, analogical to the principle of filling space with fractals. This is because the melody for three parts (voices) was constructed as follows: each upper voice tone in the middle (second) voice is expanded to a phrase of a few tones. And then each tonal unit of this voice becomes the "seed" of the bottom (third) voice phrase [49] (see Fig. 6).

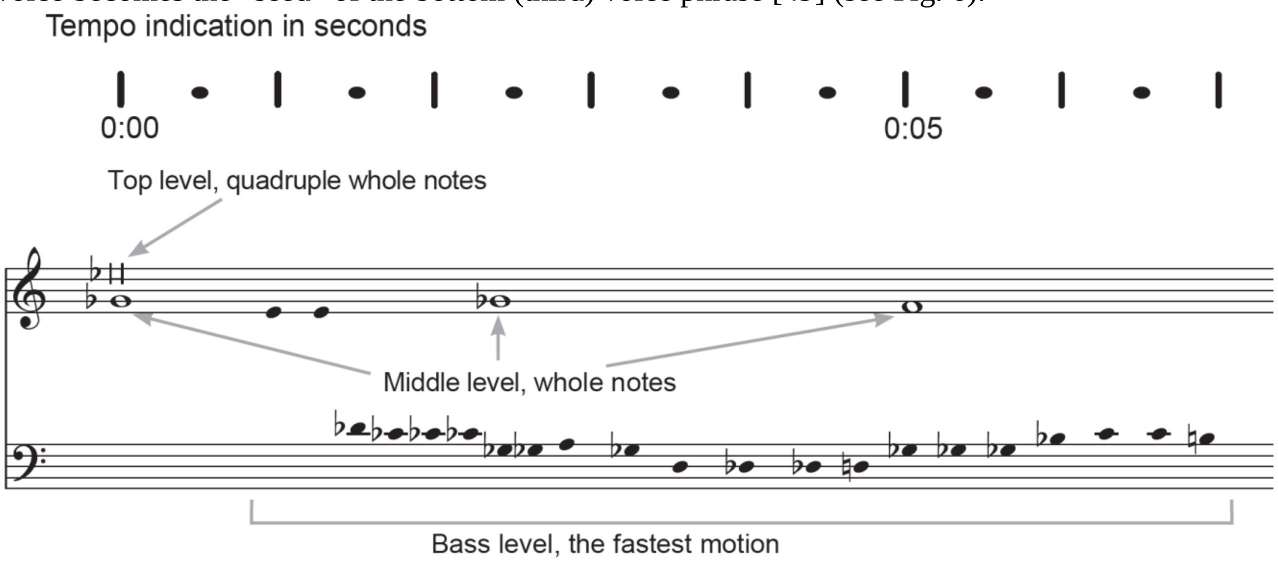

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Figure 6. Dodge, *Profile*. Principle of fractality in creating three-voice composition.

Gary Lee Nelson indicates that he used theories of fractals, chaos, artificial intelligence, quaternions, iterated function systems, and the L-system, as the sources for his work. In his microtone composition *Fractal Mountains* (1988–9), he applied a recursive division of musical time, pitch, and amplitude according to fractal algorithms. The composer merged this piece's form with the contours of fractal mountains, thus creating a microtone system, which divides an octave into 96 even intervals of 12.5 cents [50].

American minimalist Tom Johnson's various music compositions, for example *Kientzy Loops* (2000) for saxophone, illustrate how the principle of self-similarity can be found in tonal material. The perpetually playing phrase is heard as the same melody, selecting every second, fourth, etc. note from the melody, i.e. playing at a tempo that is two, four and more times slower (see Fig. 7).

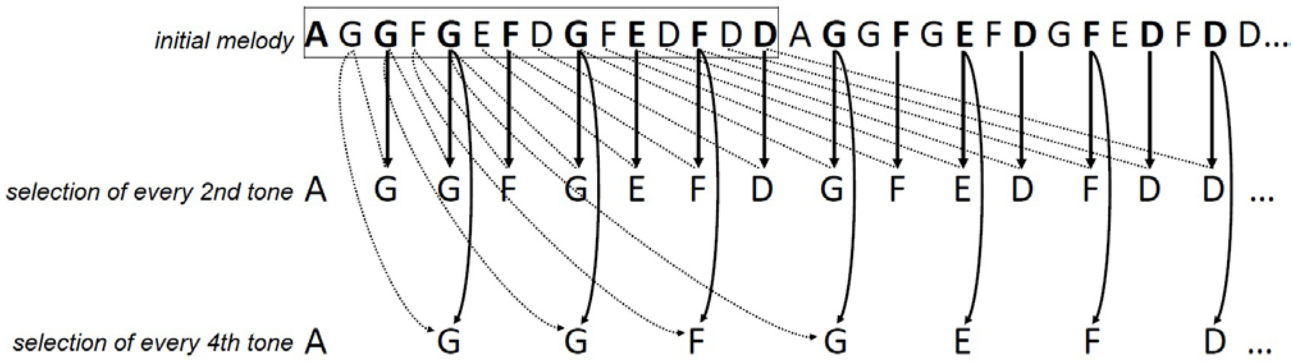

Figure 7. Fractal arrangement of melody according to Johnson.

Implication of mathematical formulas and other phenomena

Research on contemporary music composition reveals the especially complex implication of mathematical formulas. For example, from Study No. 21 on Conlon Nancarrow began to apply complicated mathematical proportions to create the polyphonic tempo. The twelve-voice score of Study No. 37 is based on the elaborate mathematical relationships of the chromatic scale; in Study No. 33 the ratio of the movement of two voices was established by the formula written into the title, *Canon* $\frac{\sqrt{2}}{2}$, bringing together two different forms of the same number – the irrational numerator (square root of 2) and the rational denominator (the natural number 2). Study No. 40, which Kyle Gann called the transcendental cannon [51], has a title that was written down with two irrational numbers: *Canon* $\frac{e}{\pi}$. This could be described as the composer's intention to bring into conflict two opposite mathematical expressions, which were of a different nature, a dynamic and a static [52].

According to Tom Johnson, his five-part piece for orchestra *Dragons in A* (1979) was composed based on the principles of the Dragon formula [53], while the melody of the four-part piece for piano *Cosinus* (1994) is described as a mathematical structure played in a vertical from one to four voices [54]. The formula for a Galileo number (a magnitude known in fluid dynamics) influenced the structure of Johnson's work *Galileo* (2000). In this composition five metal pendulums resemble Nancarrow's idea of a polymetric canon. The pendulums were hung in varying heights. As they moved, they created a sound of an increasingly more complex rhythmic counterpoint. The slowest pendulum was hung at the height of c. 4 meters, while others were hung according to the relationships defined by Galileo, 1/2, 2/3, 3/4 and 4/5. These relationships match the metronome marks of 20 – 25 – 26 2/3 – 30 – 40.

The implication of complicated mathematical calculations are inseparable from Xenakis' musical exploration. An entire chain of mathematical functions are written out in the sketch for his composition *Achorripsis* for 21 instruments (1957). The composer posits that he relied on the probability theory, specifically the Poisson distribution with the formula $Pk = (\lambda k/k!) \times e–\lambda$ (where λ happens to be exactly the mean value of the Poisson distribution). When Xenakis transferred 196

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different cells, which influenced the combinations of tempo and duration. In the composition seven timbre groups were used as well as 28 rhythmic units (196 : 7 = 28), which were laid out in a two-dimensional figure [55].

The polyphonically constructed canon structure in certain instances can also be mathematically proven. For example, Bach’s *Musikalisches Opfer* (BWV 1079) creates a refined musical palindrome example, the so-called crab canon (Latin *canon cancrizans*), after the *Thema Regium*. Bach’s musical composition embodies the phenomenon of infinity, because the melody plays harmoniously, performing both from its beginning and from the end, or while performing a simultaneous canon of its original and retrograde (that is, the beginning of the melody is the same as the end, and vice versa; see Fig. 8). Therefore, the design of this musical canon is analogical to the so-called Möbius strip, a quality of the one-sided (mathematically – non-oriented) surface, which was named in 1858 to honor Austrian mathematician August Ferdinand Möbius.

Figure 8. Möbius strip and Bach’s canon cancrizans from *Musikalisches Opfer,* BWV 1079, as the example of musical palindrome.

Another intriguing aspect of measurement, and an innovative path to musical composition that was offered to contemporary composers, is the *Vuza tiling rhythmic canons* [56]. According to its rules, the same fragment of the melody, the motif, is laid out in several voices in different versions of duration (augmentation, diminution, etc.), so that the entire line would be filled without pauses (holes) and clashes in the vertical (doublings). The Vuza musical canons present composers with the complicated task of choosing the appropriate melodic part and to correctly set up its variants. Therefore, filling up the complex multi-voiced texture by hand is impossible most of the time and must be accomplished using computer calculations. French composer Fabien Lévy’s piece *Coïncidences* for orchestra (1999) is regarded to be the first musical composition that was composed according to the rules of the Vuza canon (see Fig. 9). The adaptation of this mathematical phenomenon in music reveals the interaction between the polyphony and monody; but from a perceptual position it is not a traditional canon [57].

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Figure 9. Lévy, *Coïncidences*, first page of the score.

The technology of Vuza canons is especially convenient for organizing a rhythmic picture. We see this in Tom Johnson's musical experiments: from 2003 onwards he wrote many compositions with the common title of *Tilework*. For example, in the composition *Tilework for Log Drums* (2005) a problem was "solved" as to how the variants of three-tone motif fill a six-voice score so that the same motif would play in a relationship of five different tempos, 5 : 4 : 3 : 2 : 1. A possible solution of this problem is illustrated in four different tempo graphs, 18 x 6 (see Fig. 10).

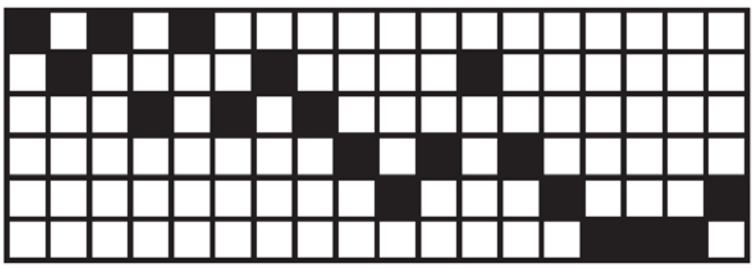

Figure 10. Initial diagram for Johnson's six-voice composition *Tilework for Log Drums* [58].

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Conclusions

The investigations of contemporary musical compositions reveal the phenomenon of the renewal of mathematical traditions of earlier epochs, as well as the original trend representing the process of rendering contemporary music mathematically. Knowledge of the initial idea is often the only key to the correct analysis of a musical composition of the 20th–21st centuries, as well as an inseparable knowledge of other scientific fields that are closely related to the theories of advanced mathematics, fractals, chaos, etc. Therefore, in the frame of the 20th–21st centuries, it becomes more and more difficult to apply the definitions and concepts of traditional music. It is too early to speak about style in the computer music panorama that has been recently accelerated and exploited over the last few decades. From the methodological point of view, there is a lack of tools as, for example, naming the genre of a computer-generated composition. Even by making use of the same compositional models, composers can generate a lot of solutions and absolutely different results.

There is no doubt that the use of computer possibilities, the transformation of algorithmic processes into the space of music, and the manipulations with various mathematical phenomena considerably extend the spectrum of generating musical ideas. Accordingly, the practice of the computerized composing of music has opened the way for innovative analytical approaches. One could argue that it “shook” the fundamentals of traditionally established definitions of music, peculiarly initiating value-based transformations of the categories, such as creator and musical composition.

First of all, in the modern world of information we encounter a *different* position of the composer. Alpern’s comment on human’s minimal participation in the creation of algorithmic music provokes a new definition of the composer’s role within the context of the creative work. This is the gradual establishment of a *composer-programmer* model. The definition of a modern creator of music is focused on the person for whom mastery of computer programming becomes an inseparable part of his agenda. While paraphrasing David Cope, not all contemporary composers manage the complicated processes of computer programming, thus forming a category of *non-scientist composers*: “I presume this concentration of these two categories will change over time, once more accessible information is made available to non-scientist composers.” [59]

Considering aesthetic and functional points of view, I would like to note the increasing separation of a *scientist* and *non-scientist composer* or a *composer* and *art composer* [60] at the turn of the 20th to the 21st centuries. This concept brings to mind the division between professional composers and craftsmen in the musical panorama of the 18th–19th centuries. Additionally, the audience of computer music stimulates the development of a listener of a new kind, a listener who is able to *hear* the algorithm used in a composition.

The definition of the function of a computer, a special software, is becoming complicated as well. On one hand, it is merely a tool that generates sounds according to certain commands. However, on the other hand, it is an equivalent second “author”, because the most complicated calculations, derivation of numerical formulas, and the actions of transcribing them into sounds, are entrusted to the computer. The composer participates in this process by presenting the software with the initial data – the “seed”, then intervening in and modifying the process, and then making an assessment of the proposed result. This leads to the question about the authorship of the final result, a computer-generated musical composition. Should the music piece be attributed to the composer, the person, the initiator of the idea who gives specific instructions to a computer? Or should it be attributed to the developer, the creator of the computer software whose product carried out those procedures?

These considerations lead us to the establishment of *meta-composition* term [61], which focuses on what is more than an elementary or traditional musical composition. The concept of meta-composition has become especially convenient in defining the complexity of a musical composition in a computer space and in unifying the computer software encompassing the ideas of generating music and lots of possibilities as well as the final result.

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Endnotes

[1] The definition was presented by Aristotle in his *Metaphysics*, Book 13, Part 3, 1078a–1078b, translated by Hugh Tredennick; also quoted in: Blackwell, 2000, p. 162.
[2] Pythagoras' quotation cited in: Young, 1965, p. 113.
[3] Лобанова, 1994, p. 128.
[4] German: "Musica Arithmetica [lat.ital.] Musique Arithmetique [gall.] betrachtet die Klange nach der Proportion, so sie mit den Zahlen Machen." (Walther, 1732, p. 431)
[5] Quotation in German "in der Mathematik ihre principia aeternae veritatis hat" was cited in: Heher, 1992, p. 30.
[6] Mizler's statement was published in *Neu eroffnete musikalische Bibliothek*, Bd. 2, Leipzig, 1743, p. 54; English translation quoted from Tatlow & Griffiths 2017.
[7] Burns, 1984, p. 2
[8] The etymology of the concept of "algorithm" is connected with the Greek word *arithmos* (number) and the 9th century mathematician and astronomer Muhammad ibn Musa al-Khwārizmī (Khwarizmi), his surname's Latin form, Algoritmi. Al-Khwarizmi's treatise *Kitab al-Jabr val-Mukabala* (c. 820) presented the counting with Indian numerals and greatly contributed to the establishment of the so-called Arabic numeration in European mathematics; the concept 'al-Jabr' was exchanged with an 'algebra' (according to Cope, 2000, p. 1, and Supper, 1997, p. 63). The treatise in 1120 was translated into Latin as *Algoritmi de numero Indorum*. The author's name was written as 'Algorismus'. One more treatise in which the term *algorismus* can be found is the French scholar Alexander of Villedieu's *Carmen de Algorismo* (The Poem about Arithmetic, beginning of the 13th century). Later, the Greek version of *algorithmus* became the more preferred version. It meant the definition of controlled procedures. Today the concept of algorithm is defined as a set of finite rules, or a sequence of operations with the purpose of reaching a concrete goal.
[9] Nierhaus, 2009, p. 1
[10] Alpern, 1995, p. 1.
[11] The words in italics highlighted by the author of this article R.P.
[12] See: Mason et al., 1988, p. 794; Cope, 2008, p. xiv.
[13] GENeration DYNamique, described as Dynamic Stochastic Synthesis.
[14] Cope, 2000, p. 2.
[15] Dodge, 1986, p. 187.
[16] Puckette, 2006, p. ix.
[17] Cope, 2008, pp. ix–x.
[18] Järvelainen, 2000, p. 1.
[19] Maurer, 1999, p. 2.
[20] Alpern, 1995, p. 1.
[21] Järvelainen, 2000, p. 10.
[22] Nierhaus, 2009, p. 261.
[23] Cope, 2008, pp. x–xii.
[24] For example, see: Burns, 1984; Dodge, 1988; Järvelainen, 2000; Nierhaus, 2009.
[25] In the 1980s, after Noam Chomsky created the model theory, which is based on linguistic and hierarchical principles, the study of generative grammar became popular in musicology, and its principles were applied to the analysis of musical structures.
[26] The model is based on the probability principle, when the likelihood of a future act is based on one or a few acts that have already taken place, which influence the later process, which is called the order. In the beginning part of this process there is a high degree of uncertainty (a lack of stability). In the later stages of the chain the certainty becomes stronger (more predictable). The Markov chain theory was developed by the Russian mathematician Andrei Markov. He tried to define the special characteristics of literary language. While analyzing Pushkin's *Eugene Onegin*, he determined when and where in Pushkin's text the same vowel is repeated, where and when one vowel is followed by another, where and when a consonant comes after a vowel.
[27] For example, researchers Richard F. Voss and John Clarke, after studying a variety of noise forms (white, pink, etc.), established that analyzed examples of music from various epochs and styles often have frequencies that match the 1/*f* noise spectrum (more see: Voss, 1978, p. 258–263).
[28] Cellular automata algorithm is often compared to the genetic algorithm theory, although the only thing they have in common is their terminology, and not their principles of functioning. The principle of cellular automaton is closer to the L-system. In order to generate music, most of the time main cellular automaton rules are not applied, but rather a method of graphic representation, a musical cartography, where the cells move according to a certain order in a net (Nierhaus, 2009, p. 201–202).

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[29] Algorithms of artificial neural networks are usually used to compose music, and not to analyze music. According to Nierhaus, it is only possible to match a few artificial network types, because these algorithms typically have a weak processing ability (Nierhaus, 2009, p. 221).
[30] The principles of artificial intelligence, fundamentally, are best suited to generate algorithmic music in the Baroque style (in order to create Baroque melodies, four-voice polyphony, and so on). This algorithm can also be used with other algorithms, like Markov chains, the principles of generative grammar (Nierhaus, 2009, p. 200).
[31] Nierhaus, 2009, pp. 94 & 99.
[32] More see: Baroni et al., 1984, p. 201–218.
[33] See: Towsey et al., 2001, p. 54–65.
[34] Horner, 1991, p. 479–480.
[35] Nierhaus, 2009, p. 81.
[36] Pritchett, 2001, p. 690–696.
[37] Järvelainen, 2000, p. 3.
[38] *Теория современной композиции*, 2005, p. 515.
[39] Ibid., p. 514.
[40] Granular synthesis is a method in computer-generated music sound that operates on microtonal structures; the techniques of analogs and selections are applied.
[41] Mathematician Pierre Francois Verhulst developed a chaos theory formula, the Verhulst equation; it is used to explain population growth through a genetic algorithm. The critical value is $P = 4$.
[42] Nierhaus, 2009, p. 1.
[43] Ligeti, 1988, p. 10.
[44] Clendinning, 1996, p. 2.
[45] Polish born mathematician Benoit Mandelbrot tried to prove that according to the main theory of fractal geometry, fractals can be found in various phenomena in nature and in the structures of objects. Because of the findings of this scholar, it became possible to use mathematical relationships to express non-mathematical or non-geometric forms, such as clouds, mountains, trees, and so on.
[46] Latin *fractus* – broken, smashed, split. Fractal is a geometric object that is similar to itself. Its main properties remain intact if we investigate the part of the fractal, similar (in one way or another) to the whole object.
[47] Hsü & Hsü, 1991, p. 98. Also see: Lewin, 1991.
[48] Degazio, 1986, p. 440.
[49] Dodge, 1988, p. 11–14.
[50] Nelson, 1993, p. 2; Nelson, 1994, p. 3.
[51] Gann, 1995, p. 200.
[52] The expression $e : \pi$ (where e is a natural logarithm basis) is still written using the numerical formula 2.7182818284… : 3.1415926536…
[53] The Dragon curve (Dragon formula) is a special example of a fractal, self-similar curve, which is obtained from the so-called IFS, iterated function system.
[54] Johnson, *Editions 75 Catalogue.*
[55] Xenakis, 1992, p. 29–31.
[56] This method of composition is based on the Romanian mathematician Dan Tudor Vuza's (born 1955) canon theory. This phenomenon is associated with the art of mosaics, which was especially perfected in the Byzantine and Islamic cultures and which master-fully adapts the rules of geometry, so that a certain area would be carefully filled in with the details of varying forms, which links everything into an impressive sight.
[57] Lévy, 2011, p. 27–30.
[58] The figure reproduced from Johnson, 2011, p. 19.
[59] Cope, 2008, p. xiii.
[60] For example, Gerhard Nierhaus in his book *Algorithmic Composition. Paradigms of Automated Music Generation* (2009) uses two conceptions: *composer* and *art composer*.
[61] *Greek μετά* – after, beyond, adjacent, self.

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