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IMITATING MODELING – IMPORTANT METHOD OF THE MATHEMATICAL TRAINING THEORY

Robert Valerievich Mayer
Doctor of Pedagogical Sciences, Professor of Physics and Didactic of Physics Chair
The Glazov Korolenko State Pedagogical Institute, Glazov, Russia

Abstract:
The development of theoretical pedagogics and theory of training is closely connected to an use of the mathematical and computer (imitating) modeling methods. In the article the difficulties and features of an application of the formal methods for research of the didactic systems are discussed, the basic stages of the imitating models creation are revealed, the computer model of training, based on a metaphor "student – communication channel" is offered. It is taken into account that: 1) the teacher and the student form an information semantic system, and the training is reduced to the perception (listening or reading) of the increasing complexity texts sequence; 2) as the student learns more complex ideas (phrases), the brain decoder transfer capacity increases due to the "near development zone"; 3) the greater the complexity of the acquired ideas (phrases), the higher the rate of forgetting.

Key words: *didactics, modeling, knowledge, learning, student, teacher.*

Introduction

The problem of a reasonable combination of content-humanitarian and formal-logical approaches for the educational process analysis has been repeatedly raised by various scientists-didactics. The result of the formal-logical approach was the creation of training mathematical theory. It contains a system of axioms based on the psychological research results, on the basis of which various mathematical models of the learning process are created and the consequences are derived [1, 2]. Development of ICT predetermined emergence and use of the simulation (or computer imitation) method consisting in writing of the computer program imitating behavior of the "teacher – pupil"– system and carrying out a series of computing experiments with it. It allows to study different mathematical models of the didactic system (DS) using the computer to investigate their behavior under different pupil's parameters or the educational material distributions and to establish regularities of the different training strategies functioning and estimate their effectiveness.

The situation is complicated by the fact that the presence of human (student and/or teacher) in the didactic system turns its analysis into a poorly formalized problem [3]. Its solution requires the use of modeling methods for weakly structured and poorly formalized systems functioning under uncertainty; this is caused by a lack of information about the object state and ignorance of the laws of the studied processes. A convenient method for the study of DS is fuzzy cognitive modeling [3, 4], based on the soft systems methodology [5] and combining analytical, statistical, linguistic descriptions of the analyzed processes. Its advantage consists in the possibility of mathematical formalization of numerically immeasurable objects qualities and the use of fuzzy systemology [6].

This article is devoted to: 1) the determination of major principles of the didactic system (DS) simulation and discussion the pupil's characteristics; 2) the DS model of training creation using the metaphor "student – communication channel with increasing bandwidth". It is based on works on psychology and didactics [7, 8], general modeling theory [9, 10, 11], mathematical theory of training [2], modeling of poorly-structured systems [12], cognitive control [3], modeling of didactic objects [13], fuzzy logic [6], soft systems methodology [5].

1. The difficulties of the didactic systems research

Application the soft systems methodology and fuzzy logic to the analysis of training process allows to allocate the following DS features, which essentially have complicated their research:

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1. Incompleteness of information about DS: it is fundamentally impossible to fully describe the state of the teacher and student, taking into account all the features of their life experience, knowledge and skills, business qualities, psychological characteristics.
2. The subjectivity of the student's condition assessments, the training material complexity, given by experts: due to the experts' individual characteristics, their stereotypes and preferences, almost always the expert's estimations in addition to the objective component contains a subjective component.
3. Poor-defined and multicriterial goals and tasks of DS: the goal and objectives of the "teacher–student"– system are often formulated indistinctly, which leads to the "tolerance range" appearance. In addition, they break up to several sub-goals and sub-tasks.
4. The uncertainty of the DS characteristics: the number and composition of input and output variables describing the training condition and student's characteristics, as well as the pedagogical impact on him, is uncertain and is set by the researcher.
5. Uncertainty of information about the DS state: accurate information about the student's level of knowledge and skills is unknown, the state of DS is determined with an error. This leads to the use of vague formulations of the qualitative nature, which are expressed in verbal form: "solid knowledge", "formed abilities" , "mastered skills", etc.
6. The absence of formal description of the DS functioning laws: it is not possible to characterize mathematically strictly the laws of assimilation and forgetting, the relationship between the teacher's pedagogical influence and the increase in the student's knowledge, as well as other interdependencies of the system elements; this is caused by their complexity and lack of knowledge.
7. Low predictability of DS behavior: due to the influence of a huge number of different factors, the result of the teacher's control and didactic influence on students cannot be accurately predicted. The DS response to external influence can be foreseen with some probability.
8. The presence of a large number of the DS state characteristics: the state of the system "teacher–student" is determined by a set of quantitative and qualitative characteristics of the educational material and the DS, which are linked with each other.
9. The complexity and low structurability of the studied material: the student's and teacher's knowledge form complex system that has indistinct structure and contains large number of elements linked to each other by numerous fuzzy connections.

2. The DS simulation modeling methodology

Mathematical modeling of DS requires the coarsening of the problem, abstraction from non-essential factors and properties of the system. The resulting model should reflect the essential aspects of the didactic process and be useful from a practical point of view. It is easy to disprove opinion on uselessness of the DS mathematical description. For example, the marks received by a student during a semester can be considered as an information model of his educational activity. It is possible to tell, that teacher, estimating the student's answer in a concrete subject, projects student's knowledge on the appropriate plane of the requirements. Although the error of using the five-point scale is large, the results of such a "measurement" of the knowledge amount is certainly useful: they stimulate the student's learning activity and increase the effectiveness of learning.

As noted by R. Shannon, an expert in simulation modeling, should possess the art of selecting parameters and functional dependencies [14]. A computer program that simulates the functioning of the "teacher – student"– system can be considered as a device that calculates the variables $Z(t) = (Z_1, Z_2, ..., Z_k)$, characterizing the student's knowledge of a certain set of the learning material elements (LMEs). This takes into account the psychological regularities of assimilation and forgetting of the reported information by the student. The simulation result depends on the student's parameters, the distribution of educational material, the teacher's requirements, etc. The model must meet **the principle of compliance:** its parameters must be selected so that at given reasonable teacher's impact on the student, his level of knowledge predicted by the model, correspond to the real amount of the successful student's knowledge, which can be measured by the test method.

It should be remembered that the simulation method has no strict justification and is heuristic. We highlight the main stages of creating the DS simulation model:

1. The modeling purpose is determined, which can consist in studying the dynamics of skill formation, building a graph of changes in the student's knowledge, substantiating the hypothesis about the advantage of a particular technique, etc.

2. The specific didactic system is selected and its qualitative model is constructed, which is based on the teacher, students, connections; the main factors influencing its behavior are revealed, the training material distribution and its characteristics are set.

3. The quantitative model of a student, teacher or didactic process is created. It is a system of mathematical equations that reflect the psychological regularities of assimilation and forgetting, especially the method of training.

4. Certain assumptions about the nature of the functional dependencies linking the system response with a particular impact are made, the time scale is selected and the training material distribution the most appropriate to the analyzed situation is set.

5. The algorithm that simulates the DS behavior is developed; on its basis, a computer program is created that calculates the DS state in the subsequent moments of time, builds graphs, diagrams, etc.

6. The student parameters (coefficients of assimilation and forgetting, etc.) are selected so that they correspond to the real course of the learning process and the pedagogical observation or experiment results.

7. The series of DS computer simulations functioning at various parameters of the student's, teacher's influences, educational information distributions, dependences of its subsystems response on external and internal factors influence is carried out.

8. The analysis of turning out results and their comparison to the known facts is carried out. The modeling purposes, qualitative model (that is DS structure), quantitative model (the equations system), the student's parameters, the training material distribution, dependence of the teacher's requirements level on time are corrected until the new unobvious result of modeling will be received.

The didactic process consists of the interrelated actions set; it is a system too. In accordance with the principle of multi-modeling, any sufficiently complex system can be structured and modeled in several ways. This leads to the existence of a number of different approaches to the construction of the mathematical and computer learning process models. The author used different methods of DS modeling; the proposed models can be divided into the following categories [1, 15, 16]: 1) continuous one-component and multi-component models that require solving differential equations system; 2) stochastic discrete models in which the student is modeled by a probabilistic automaton; 3) models with a changing forgetting coefficient; 4) models that take into account the learning coefficient dependence on the speed of information transfer and student's fatigue; 5) the optimizing models allowing to find an effective way of training; 6) models in which multi-agent approach and a method of statistical tests is used; 7) models that allow to study the assimilation and forgetting of meaningful (logically related) information.

3. Creation of the DS simulation model

As an example of using the simulation method for the DS study, we consider the computer model, which is based on the metaphor "student – communication channel" [17, 18]. This metaphor consists in the mental replacement of the student with the information system including sense organs, "the brain decoder" and memory (Fig. 1). The teacher states the teaching material, the student perceives it by means of sense organs and, having carried out semantic decoding, understands and assimilates it. Let's assume that the teacher correctly pronounces sounds, speaks with normal speed so that the student unmistakably perceives all his words. At the same time, the result of understanding and memorizing the reported information depends strongly on the student's ability to decode the received message, embed it into the system of his concepts. The student can decode and understand not all sentences. The capacity of the "brain decoder" depends on the phrases complexity and the degree of student's training. The student quickly understands the phrases that contain familiar words and simple thoughts. Phrases containing scientific terms, rarely used words or non-obvious ideas are decoded slowly and not correctly. The understood part of the message goes to short-term memory and then to long-term memory [19]. Something similar occurs, when the student reads the educational text (ET).

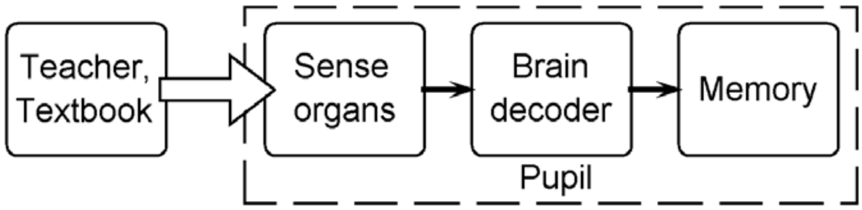

Fig 1. The essence of the metaphor "student – communication channel".

Consider three texts of $V = N = 200$ phrases, the complex profiles $p_1(s)$, $p_2(s)$ and $p_3(s)$, of which are shown in Fig 2.1. For example, in the texts for young schoolchildren there are mainly simple LMEs with $s < 0{,}3$ (curve 1), in ETs for older pupils – LME having complexity $s \in [0{,}3; 0{,}6]$ (curve 2), and in ETs for students – complex LME with $s > 0{,}7$ (curve 3). General informative this texts is equal to 27, 80, 131 CUI (conditional information units) respectively.

The student's "brain decoder", which is engaged in semantic processing of arriving information, is the communication channel with a limited bandwidth. Its transmission coefficient depends on the complexity of the incoming phrases and the student's training degree. A person with high probability p_n understands simple LMEs ($K = 1$) and with low probability – complex LMEs (K tends to 0). It is logical to assume that:

$$K(s) = \frac{p_n(s)}{p(s)} = \frac{1}{1 + \exp(a(s-b))}.$$

If $s = b$, then $K = 0.5$. The learning process develops the student's ability to decode (i.e. understand) the information given to him. On Fig 2.2 the graphs $K = K(s)$ at $a = 10$ and $b = 0$, 0,2, 0,4, 0,6, 0,8 (curves 1, 2, 3, 4 and 5) are shown. At training volume b increases, the graph $K = K(s)$ is shifted to the right, the bandwidth of the "brain decoder" is growing. It is possible to tell, that curve 1 corresponds to the first-grader level, and curve 5 – to the student's level.

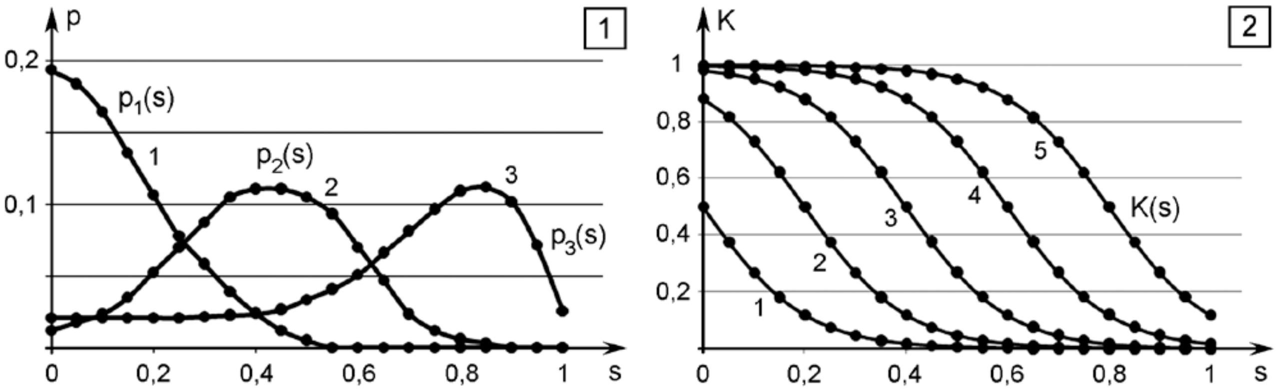

Fig 2. Text complexity profile (1). Change $K(s)$ at training (2).

For example, student reads the text in which the probability of the different complexity phrases are distributed according to the law: $p(s) = 0{,}146\exp(-3s)$. In ET there are large number of easy phrases with $s < 0.4$, but complex phrases with $s > 0.6$ are much less (Fig 3.1). If the transfer coefficient of the "cognition decoder" is equal to $K(s) = 1/(1+\exp(10(s-0{,}4)))$, then the light LMEs pass almost all, but the difficult LMEs – no. On Fig. 3.1 it shows distribution $p_n(s)$ of the LMEs, which were understood by the student (scales $p(s)$ and $p_n(s)$ increased 10 times). On Fig 3.2 the graphs corresponding to the text with another complexity profile $p(s)$, which is read by student with $b = 0.5$, are presented, and the resulting graph $p_n(s)$ is shown. It follows from them that at working with student having proficiency b, it is necessary to use ET, mainly consisting of phrases with complexity $s < b$.

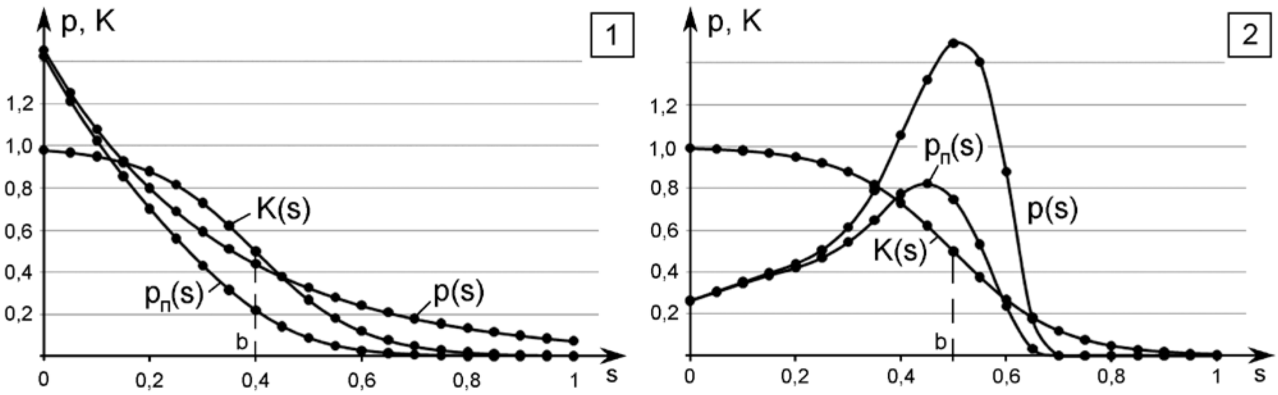

Fig 3. Partial understanding of the educational text by an untrained student.

All these relations have the statistical meaning. Let the teacher say phrases (LMEs) which complexity changes accidentally according to the distribution law $p = p(s)$. The "brain decoder" works as a stochastic filter, skipping the LMEs with probability $K(s)$ (fig. 4). Just as the technical communication channel capacity decreases with increasing signal frequency, the transfer coefficient of the student’s "brain decoder" decreases with increasing complexity of ideas and terms used. Therefore the LMEs with low complexity pass in the majority. This explains the partial understanding of the teacher’s report by student. Within the framework of the considered approach, training can be represented as reading texts of increasing complexity, leading to an increase in the bandwidth of the student’s "brain decoder". At the same time it is necessary to consider that the student has a "nearest development zone": if he managed to acquire the ideas with complexity s, then he with some expenses of time will be able to acquire LMEs which complexity lies in an interval $[s; s+\Delta s]$. When the b grows the “brain decoder bandwidth" increases, the student understands the LMEs, the complexity s of which is in the interval $[0; b]$.

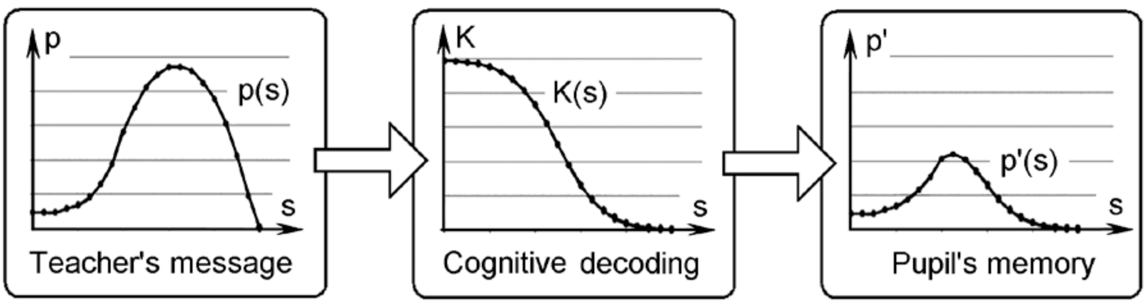

Fig 4. Explanation of the student's partial understanding of the teacher.

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We have created a program in the Free Pascal IDE, which simulates the text reading and its partial understanding depending on the student's knowledge level. The program simulates a situation when a student reads a sequence of texts T_1, T_2, …, T_{20}, the complexity profile of which has a maximum, gradually shifting towards increase s. It contains a cycle on time with step $\Delta\tau$; the interval of complexity change from 0 to 1 is divided into 20 parts with width $\Delta s = 0.05$. The brain decoder bandwidth depends on the knowledge acquired by the student; it is calculated at each time step according to the formula: $K_s^{t+1} = (Z_{s-\Delta s}^t + Z_{s+\Delta s}^t)/2$, where $s = \Delta s \cdot i$, $i = 1, 2, \ldots, 20$. This allows to take into account the student "nearest development zone". In the process of learning the knowledge of LMEs with complexity $s \in [s; s+\Delta s]$ increase according to the law $Z_s^{t+1} = Z_s^t + \alpha K(s) p(s) N \Delta\tau$, where α is the assimilation coefficient. After reading the text the student begin to forget the studied LMEs, the amount of relevant student's knowledge decreases: $Z_s^{t+1} = Z_s^t - \gamma(s) Z_s^t \Delta\tau$, where $\gamma(s)$ is the forgetting coefficient, which is proportional to the complexity of LMEs: $\gamma(s) = gs$, where $g = 0.001 – 0.002$.

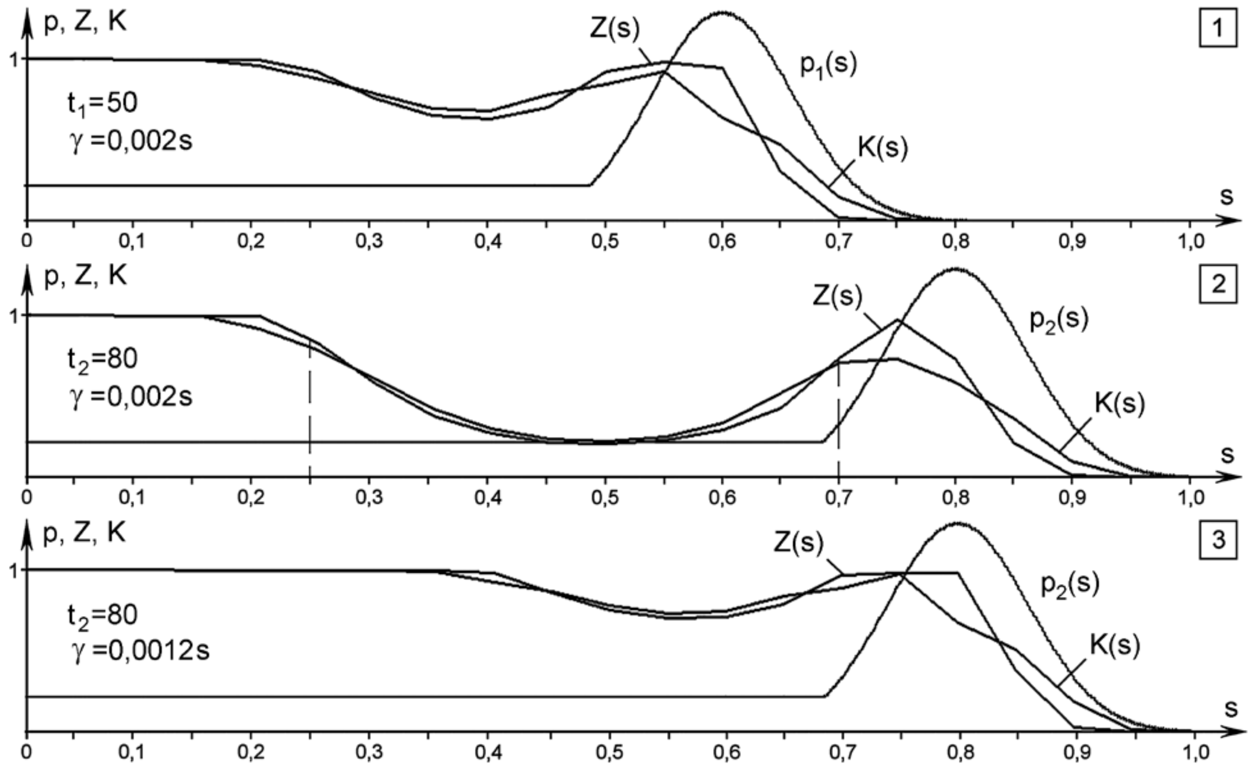

Fig. 5. Training by reading the texts with growing complexity.

The simulation results are shown in Fig 5. At the moment t_1 the student reads the text with a complex profile $p_1(s)$ (Fig. 5.1). There are also the graphs of the dependences of the student's knowledge $Z(s)$ and the transfer coefficient $K(s)$ on the LMEs complexity. At a time $t_2 > t_1$, the student reads a text with a similar distribution $p_2(s)$, the maximum of which is shifted in the direction of increasing complexity (Fig. 5.2).

It can be seen that with these parameters of the model, the student has time to learn the received information (Fig. 5.1). This leads to a gradual increase in the bandwidth of his "brain decoder": he begins to understand more and more complex ideas related to this topic. If training happens seldom or the forgetting coefficient γ is too high ($g = 0{,}002$, Fig. 5.2), the student can begin to forget quite complex LMEs $(s \in [0{,}25;\ 0{,}7])$, which he did not manage to update in the memory, well remembering simple $(s < 0{,}25)$ and recently studied LMEs $(s \in [0{,}7; 0{,}8])$. At small size $g = 0{,}0012$ forgetting occurs slowly (Fig. 5.3).

Summary

The article solves the following problems: 1) the important features of didactic system, that significantly complicate their study, are revealed; 2) the problem of the model parameters correct choice which allows us to talk about the art of modeling are considered, the principle of compliance are formulated; 3) the main stages of creating the DS simulation are identified; 4) the need for the comprehensive application of different approaches to modeling DS are showed. In addition, the computer model of the didactic system is built; it bases on the following ideas: 1) the teacher and the student form an information semantic system; training is reduced to the perception (listening or reading) of the sequence of increasing complexity texts; 2) in process of learning more complex LMEs by the student the capacity of his "brain decoder" increases because of the "nearest development zone"; 3) the greater the LME complexity, the higher the forgetting coefficient. The modeling results allow to explain some features of training.

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