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## NEUTRINO OSCILLATIONS IN EXTERNAL FIELDS IN CURVED SPACETIME

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**Abstract.** We study spin and flavor oscillations of neutrinos under the influence of gravitational waves (GWs). First, we consider neutrino spin oscillations in nonmoving and unpolarized matter, a transverse magnetic field, and a plane GW. We show that a parametric resonance can take place in this system. We also study neutrino flavor oscillations in GWs. The equation for the density matrix of flavor neutrinos is solved when we discuss the neutrino interaction with stochastic GWs emitted by coalescing supermassive black holes. We find the fluxes of cosmic neutrinos, undergoing flavor oscillations in such a gravitational background, which can be potentially measured by a terrestrial detector. Some astrophysical applications of our results are considered.

**Keywords:** spin and flavor oscillations of neutrinos, influence of gravitational waves

### 1. GENERAL

Neutrinos are known to be massive and mixed particles. These neutrino properties lead to neutrino oscillations [1]. There are various types of neutrino oscillations. We can mention neutrino flavor oscillations, when transitions between different flavor eigenstates happen,  $\nu_\alpha \leftrightarrow \nu_\beta$ , where  $\alpha, \beta = e, \mu, \tau$ . Transitions between different helicity states within one neutrino generation  $\nu_L \leftrightarrow \nu_R$  are called neutrino spin oscillations.

We consider one neutrino eigenstate, which is supposed to be a Dirac particle, and neglect the mixing between different neutrino types. The wave equation for a massive Dirac neutrino with the anomalous magnetic moment  $\mu$ , interacting with background matter and the electromagnetic field  $F_{\mu\nu}$  in curved spacetime, reads

$$\left[ i\gamma^\mu \nabla_\mu - \frac{\mu}{2} F_{\mu\nu} \sigma^{\mu\nu} - \frac{V^\mu}{2} \gamma_\sigma (1 - \gamma^5) - m \right] \psi = 0, \quad (1)$$

where  $m$  is the neutrino mass,  $\nabla_\mu$  is the covariant derivative, and  $V^\mu$  is the effective potential of the neutrino interaction with background matter.

Using Eq. (1) we get the effective Schrodinger equation for the neutrino wave function  $\nu^T = (\nu_L, \nu_R)$  if a neutrino interacts with a plane GW, transverse magnetic field, and background matter,

$$\tilde{H}_{eff} = \begin{pmatrix} -V^0/2 & \mu B[1 - he^{-\dot{\phi}t}/2] \\ \mu B[1 - he^{\dot{\phi}t}/2] & V^0/2 \end{pmatrix}. \quad (2)$$

where  $\dot{\phi}$  and  $h$  are the phase and the amplitude of GW. The solution of Eq. (2) for the parameters of a neutrino and external fields corresponding to a particle propagating in the vicinity of merging black holes, surrounded by a dense magnetized accretion disk, was found in [2]. Using these results, one find that there is a parametric resonance in neutrino spin oscillations, i.e. the transition probability for the process  $\nu_L \leftrightarrow \nu_R$  can reach great values.

## 2. NEUTRINO FLAVOR OSCILLATIONS

Now we study neutrino flavor oscillations under the influence of GW. We suppose that we deal with three flavor neutrinos  $(\nu_e, \nu_\mu, \nu_\tau)$  which are related to the neutrino mass eigenstates  $\psi_a$ ,  $a = 1, 2, 3$ , with masses  $m_a$ , by means of the matrix transformation  $\nu = U\psi$ . These neutrinos are taken to interact with GWs. Let us consider the interaction of a neutrino with a stochastic GW background. In this situation, the density matrix  $\rho_I$  obeys the equation [3],

$$\frac{d}{dt}\langle \rho_I \rangle = -\frac{3}{64}\langle h^2 \rangle \tau [M, [M, \langle \rho_I \rangle]], \quad (3)$$

where  $\tau$  is the correlation time and

$$M = \frac{1}{2E} U \cdot \text{diag}(0, \Delta m_{21}^2, \Delta m_{31}^2) \cdot U^+. \quad (4)$$

Here  $\Delta m_{ab}^2 = m_a^2 - m_b^2$  is the standard definition for the mass squared differences.

## 3. NUMERICAL REALIZATION

The numerical solution of Eq.(3) was found in [3]. We consider the following parameters:  $\langle h^2 = 1.6 \times 10^{-32} \rangle$ , and  $L = 1Gpc$  is the propagation length, which correspond to the neutrino interaction with stochastic GWs emitted by supermassive black holes. We get that, for the normal mass ordering, the asymptotic fluxes (at the Earth) are  $F_{e\oplus} = 0.3127$ ,  $F_{\mu\oplus} = 0.3504$  and  $F_{\tau\oplus} = 0.3369$ . For the inverted ordering, one has  $F_{e\oplus} = 0.3154$ ,  $F_{\mu\oplus} = 0.3497$ , and  $F_{\tau\oplus} = 0.3349$ . It means that, at the Earth, the predicted fluxes are close to the case  $(F_e : F_\mu : F_\tau)_{\oplus} = (1 : 1 : 1)$ . However, there is a small deviation from this prediction of [4] for both normal and inverted mass orderings. Moreover, one can see that there is a small dependence of our

results on the hierarchy of the neutrino masses. The recent measurement of the flavor content of cosmic neutrinos was made in [5].

In Fig 1, we show the This numerical solution is represented in Fig. 1. In Fig. 1(a), we show the fluxes of  $\nu_e$  and  $\nu_\mu$  versus the propagation distance. This solution is based on the fluxes at a source (the initial condition) in the form,  $F_e(0) \equiv 1/3$  and  $F_\mu(0) \equiv 2/3$ . The characteristics of neutrinos and GW are taken as in Ref. [3]. One can see in Fig. 1(b) that the asymptotic value of the fluxes ratio is  $(F_e/F_\mu)_{Earth} = 0.9$ , which is in the agreement with the results of Ref. [3].

Performing analogous simulations, we show that  $(F_e/F_\tau)_{Earth} = 21.8$  can be also reproduced.

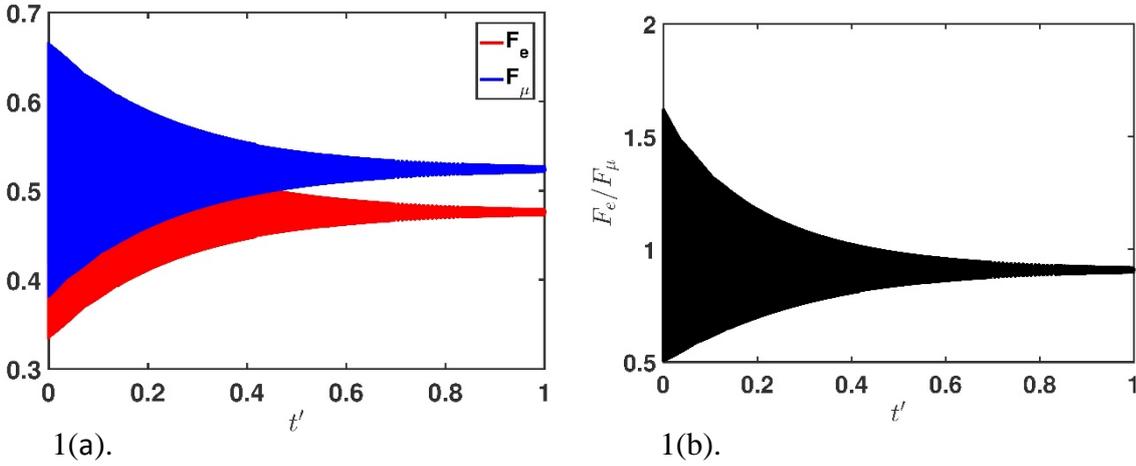


Fig. 1: The numerical solution of Eqs. (3) and (4), rewritten in the two flavors approximation, for neutrino oscillations in stochastic GWs. (a) The fluxes of electron neutrinos  $F_e$  (red line) and muon neutrinos  $F_\mu$  (blue line) versus the propagation distance. (b) The ratio of fluxes  $F_e/F_\mu$  as a function of the propagation distance.

We show the fluxes of different neutrino flavors in Fig. 2 in the general case of three neutrino flavors.

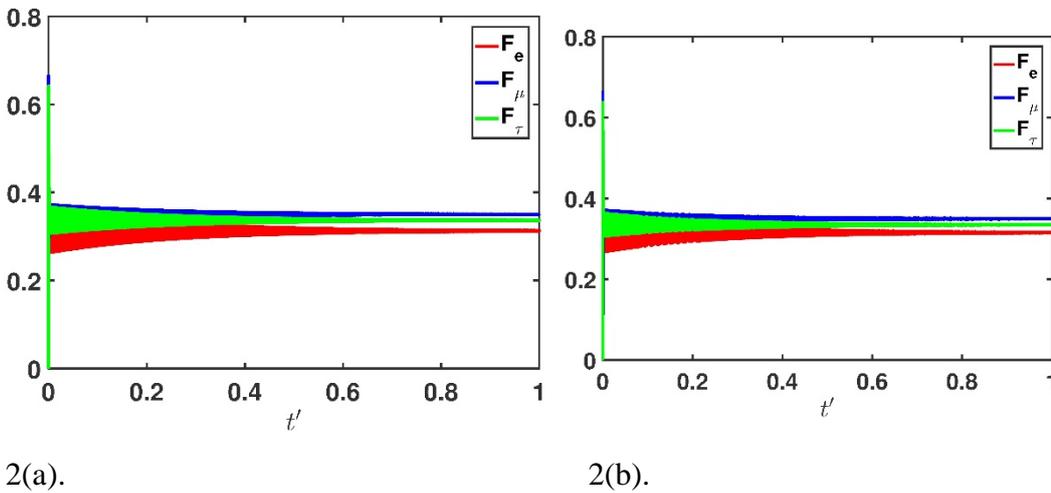


Fig. 2: The numerical solution of Eqs. (3) and (4) in the general case of three neutrino flavors, for neutrino oscillations in stochastic GWs. (a) Normal mass ordering; (b) Inverted mass ordering.

## CONCLUSION

In the present work, we have studied the neutrino spin evolution in background matter and an external electromagnetic field in curved spacetime. This study was motivated by the necessity for the substantiation of the quasiclassical equation for the neutrino spin evolution, which was proposed earlier. We study neutrino spin oscillations in the vicinity of coalescing BHs surrounded by a magnetized accretion disk. We predict the resonant enhancement of spin oscillations if an accretion disk has a sharp outer edge.

In this work, we have also studied neutrino flavor oscillations under the influence of a plain GW with the circular polarization for the first time. We have analyzed the evolution of the mass eigenstates in the quasiclassical approximation. Using the expression for the action for a massive particle, interacting with GW, we have derived the contribution of GW to the effective Hamiltonian for the neutrino mass eigenstates. We have revealed that, in case of the neutrino propagation along GW, GW does not influence neutrino flavor oscillations. However, neutrino flavor oscillations can be influenced by stochastic GWs. Using the numerical solution of the evolution equation, we predicted the neutrino fluxes, measured by a terrestrial neutrino telescope, are in the region not excluded in [5]. Neutrino flavor oscillations in stochastic GWs were further studied in Refs. [6,7].

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