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## FOURTH ORDER PERTURBED HEISENBERG HAMILTONIAN WITH SEVEN MAGNETIC PARAMETERS OF BCC STRUCTURED FERROMAGNETIC ULTRATHIN FILMS-PART 1

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**Abstract:** *Bcc structured ferromagnetic ultrathin films with two spin layers was described using fourth order perturbed Heisenberg Hamiltonian with all seven magnetic energy parameters. Magnetic energy per unit spin was expressed in terms of spin exchange interaction, second order and fourth order anisotropy, in plane and out plane applied magnetic fields, demagnetization energy and stress induced anisotropy in two spin layers. 3D plot of total magnetic energy versus angle and stress induced anisotropy were plotted for the different values of second order and fourth order magnetic anisotropy constants. All other magnetic energy parameters were fixed at constant values. The order of magnetic energy was changed when the values of fourth order anisotropy constants of two spin layers are interchanged. In addition, the graphs of energy versus angle were plotted to find the magnetic easy and hard directions. The angle between magnetic easy and hard directions is 90 degrees in this case.*

**Keywords:** *Fourth order perturbed Heisenberg Hamiltonian, magnetic anisotropy, stress induced anisotropy*

### 1. Introduction:

Although magnetic properties of ferromagnetic thin films have been previously described using second and third order perturbed Heisenberg Hamiltonian with all seven magnetic parameters and also fourth order perturbed Heisenberg Hamiltonian with only two magnetic parameters, Heisenberg Hamiltonian has never been investigated up to seven magnetic parameters using fourth order perturbation. Ferromagnetic ultrathin films are used in biomedical applications, sensors, potential applications in magnetic memory devices and microwave devices. Ferromagnetic films have been described using many different models. The quasistatic magnetic hysteresis of ferromagnetic thin films grown on a vicinal substrate has been theoretically investigated using Monte carlo simulations [1]. Magnetic properties of ferromagnetic thin films with alternating super layer were studied by Ising model[2].

Structural and magnetic properties of two dimensional FeCo ordered alloys have been determined by first principles band structure theory [3]. Magnetic layers of Ni on Cu have been theoretically studied by using the Korringa-Kohn-Rostoker Green's function method [4].

The interfacial coupling dependence of the magnetic ordering in ferro-antiferromagnetic bilayers has been investigated using the Heisenberg Hamiltonian [5].

Second order perturbed Heisenberg Hamiltonian incorporated with spin exchange interaction, magnetic dipole interaction, applied magnetic field, second and fourth order magnetic anisotropy terms has been solved for ferromagnetic thin films [6-8]. Magnetic thin films are heated and subsequently cooled during the annealing process. During the heating and annealing process, the stress induced anisotropy (Ks) occurs due to the difference between the thermal expansion coefficients in magnetic film and substrate. For soft magnetic materials, the stress induced anisotropy is comparable to magneto crystalline anisotropy. Therefore, the coercivity depends on the stress induced anisotropy [9-11]. The magnetic properties of ferromagnetic and ferrite thin and

thick films have been investigated by us using second, third and fourth order perturbed Heisenberg Hamiltonian by us [12-18].

In this manuscript, the total energy of ferromagnetic thin films will be described by solving the classical Heisenberg Hamiltonian. In this approach, all energy terms such as magnetic energy, spin dipole interaction energy, spin exchange interaction energy, second and fourth order anisotropy and stress induced anisotropy demagnetization factor terms will be considered. Fourth order perturbed Heisenberg Hamiltonian with all seven magnetic energy parameters are explained for bcc structure. MATLAB computer software was used to plot 3D and 2D graph of energy versus stress induced anisotropy and azimuthal angle of spin.

**2. Model:**

The Heisenberg Hamiltonian of ferromagnetic films can be formulated as following [12-14].

$$H = -\frac{J}{2} \sum_{m,n} \vec{S}_m \cdot \vec{S}_n + \frac{\omega}{2} \sum_{m \neq n} \left( \frac{\vec{S}_m \cdot \vec{S}_n}{r_{mn}^3} - \frac{3(\vec{S}_m \cdot \vec{r}_{mn})(\vec{r}_{mn} \cdot \vec{S}_n)}{r_{mn}^5} \right) - \sum_m D_{\lambda m}^{(2)} (S_m^z)^2 - \sum_m D_{\lambda m}^{(4)} (S_m^z)^4 - \sum_{m,n} [H - (N_d \vec{S}_n / \mu_0)] \cdot \vec{S}_m - \sum_m K_s \sin 2\theta_m$$

Here

$\vec{S}_m$  and  $\vec{S}_n$  are two spins. Above equation can be simplified into following form

$$E(\theta) = -\frac{1}{2} \sum_{m,n=1}^N \left[ \left( JZ_{|m-n|} - \frac{\omega}{4} \Phi_{|m-n|} \right) \cos(\theta_m - \theta_n) - \frac{3\omega}{4} \Phi_{|m-n|} \cos(\theta_m + \theta_n) \right] - \sum_{m=1}^N (D_m^{(2)} \cos^2 \theta_m + D_m^{(4)} \cos^4 \theta_m + H_{in} \sin \theta_m + H_{out} \cos \theta_m) + \sum_{m,n=1}^N \frac{N_d}{\mu_0} \cos(\theta_m - \theta_n) - K_s \sum_{m=1}^N \sin 2\theta_m \tag{1}$$

Here

$N, m$  (or  $n$ ),  $J, Z_{|m-n|}, \omega, \Phi_{|m-n|}, \theta_m(\theta_n), D_m^{(2)}, D_m^{(4)}, H_{in}, H_{out}, N_d$  and  $K_s$  are total number of layers, layer index, spin exchange interaction, number of nearest spin neighbors, strength of long range dipole interaction, partial summations of dipole interaction, azimuthal angles of spins, second and fourth order anisotropy constants, in plane and out of plane applied magnetic fields, demagnetization factor and stress induced anisotropy constants respectively.

The spin structure is considered to be slightly disoriented. Therefore, the spins could be considered to have angles distributed about an average angle  $\theta$ . By choosing azimuthal angles as

$$\theta_m = \theta + \varepsilon_m \text{ and } \theta_n = \theta + \varepsilon_n$$

Where the  $\varepsilon$ 's are small positive or negative angular deviations.

Then,  $\theta_m - \theta_n = \varepsilon_m - \varepsilon_n$  and  $\theta_m + \theta_n = 2\theta + \varepsilon_m + \varepsilon_n$ .

After substituting these new angles in above equation number (1), the cosine and sine terms can be expanded up to the fourth order of  $\varepsilon_m$  and  $\varepsilon_n$  as following.

$$E(\theta) = E_0 + E(\varepsilon) + E(\varepsilon^2) + E(\varepsilon^3) + E(\varepsilon^4) + \dots$$

If the fifth and higher order perturbations are neglected, then

$$E(\theta) = E_0 + E(\varepsilon) + E(\varepsilon^2) + E(\varepsilon^3) + E(\varepsilon^4) \tag{2}$$

Here

$$E_0 = -\frac{1}{2} \sum_{m,n=1}^N \left( JZ_{|m-n|} - \frac{\omega}{4} \Phi_{|m-n|} \right) + \frac{3\omega}{8} \cos 2\theta \sum_{m,n=1}^N \Phi_{|m-n|} - \cos^2 \theta \sum_{m=1}^N D_m^{(2)} - \cos^4 \theta \sum_{m=1}^N D_m^{(4)} - N(H_{in} \sin \theta + H_{out} \cos \theta + K_s \sin 2\theta) + \frac{N_d N^2}{\mu_0} \quad (3)$$

$$E(\varepsilon) = -\frac{3\omega}{8} \sin 2\theta \sum_{m,n=1}^N \Phi_{|m-n|} (\varepsilon_m + \varepsilon_n) + \sin 2\theta \sum_{m=1}^N D_m^{(2)} \varepsilon_m + 2\cos^2 \theta \sin 2\theta \sum_{m=1}^N D_m^{(4)} \varepsilon_m - H_{in} \cos \theta \sum_{m=1}^N \varepsilon_m + H_{out} \sin \theta \sum_{m=1}^N \varepsilon_m - 2K_s \cos 2\theta \sum_{m=1}^N \varepsilon_m \quad (4)$$

$$E(\varepsilon^2) = \frac{1}{4} \sum_{m,n=1}^N \left( JZ_{|m-n|} - \frac{\omega}{4} \Phi_{|m-n|} \right) (\varepsilon_m - \varepsilon_n)^2 - \frac{3\omega}{16} \cos 2\theta \sum_{m,n=1}^N \Phi_{|m-n|} (\varepsilon_m + \varepsilon_n)^2 + \cos 2\theta \sum_{m=1}^N D_m^{(2)} \varepsilon_m^2 + 2\cos^2 \theta (\cos^2 \theta - 3\sin^2 \theta) \sum_{m=1}^N D_m^{(4)} \varepsilon_m^2 + \frac{H_{in}}{2} \sin \theta \sum_{m=1}^N \varepsilon_m^2 + \frac{H_{out}}{2} \cos \theta \sum_{m=1}^N \varepsilon_m^2 - \frac{N_d}{2\mu_0} \sum_{m,n=1}^N (\varepsilon_m - \varepsilon_n)^2 + 2K_s \sin 2\theta \sum_{m=1}^N \varepsilon_m^2 \quad (5)$$

$$E(\varepsilon^3) = \frac{\omega}{16} \sin 2\theta \sum_{m,n=1}^N \Phi_{|m-n|} (\varepsilon_m + \varepsilon_n)^3 - \frac{4}{3} \sin \theta \cos \theta \sum_{m=1}^N D_m^{(2)} \varepsilon_m^3 - 4\sin \theta \cos \theta \left( \frac{5}{3} \cos^2 \theta - \sin^2 \theta \right) \sum_{m=1}^N D_m^{(4)} \varepsilon_m^3 + \frac{H_{in}}{6} \cos \theta \sum_{m=1}^N \varepsilon_m^3 - \frac{H_{out}}{6} \sin \theta \sum_{m=1}^N \varepsilon_m^3 + \frac{4}{3} K_s \cos 2\theta \sum_{m=1}^N \varepsilon_m^3 \quad (6)$$

$$E(\varepsilon^4) = -\frac{1}{48} \sum_{m,n=1}^N \left( JZ_{|m-n|} - \frac{\omega}{4} \Phi_{|m-n|} \right) (\varepsilon_m - \varepsilon_n)^4 + \frac{\omega}{64} \cos 2\theta \sum_{m,n=1}^N \Phi_{|m-n|} (\varepsilon_m + \varepsilon_n)^4 - \frac{1}{3} \cos 2\theta \sum_{m=1}^N D_m^{(2)} \varepsilon_m^4 - \left( \frac{5}{3} \cos^4 \theta - 8\cos^2 \theta \sin^2 \theta + \sin^4 \theta \right) \sum_{m=1}^N D_m^{(4)} \varepsilon_m^4 - \frac{H_{in}}{24} \sin \theta \sum_{m=1}^N \varepsilon_m^4 - \frac{H_{out}}{24} \cos \theta \sum_{m=1}^N \varepsilon_m^4 + \frac{N_d}{24\mu_0} \sum_{m,n=1}^N (\varepsilon_m - \varepsilon_n)^4 - \frac{2}{3} K_s \sin 2\theta \sum_{m=1}^N \varepsilon_m^4 \quad (7)$$

For films with two spin layers,  $N = 2$ . Therefore,  $m$  and  $n$  change from 1 to 2.

$$E_0 = -JZ_0 + \frac{\omega}{4} \Phi_0 - JZ_1 + \frac{\omega}{4} \Phi_1 + \frac{3\omega}{4} \cos 2\theta (\Phi_0 + \Phi_1) - \cos^2 \theta (D_1^{(2)} + D_2^{(2)}) - \cos^4 \theta (D_1^{(4)} + D_2^{(4)}) - 2(H_{in} \sin \theta + H_{out} \cos \theta + K_s \sin 2\theta) + \frac{N_d N^2}{\mu_0} \quad (8)$$

$$E(\varepsilon) = -\frac{3\omega}{4} \sin 2\theta [(\Phi_0 + \Phi_1)(\varepsilon_1 + \varepsilon_2)] + \sin 2\theta (D_1^{(2)} \varepsilon_1 + D_2^{(2)} \varepsilon_2) + 2\cos^2 \theta \sin 2\theta (D_1^{(4)} \varepsilon_1 + D_2^{(4)} \varepsilon_2) - H_{in} \cos \theta (\varepsilon_1 + \varepsilon_2) + H_{out} \sin \theta (\varepsilon_1 + \varepsilon_2) - 2K_s \cos 2\theta (\varepsilon_1 + \varepsilon_2) \quad (9)$$

$$E(\varepsilon^2) = \frac{1}{2} \left( JZ_1 - \frac{\omega}{4} \Phi_1 \right) (\varepsilon_1 - \varepsilon_2)^2 - \frac{3\omega}{8} \cos 2\theta [2\Phi_0 (\varepsilon_1^2 + \varepsilon_2^2) + \Phi_1 (\varepsilon_1 + \varepsilon_2)^2] + \cos 2\theta (D_1^{(2)} \varepsilon_1^2 + D_2^{(2)} \varepsilon_2^2) + 2\cos^2 \theta (\cos^2 \theta - 3\sin^2 \theta) (D_1^{(4)} \varepsilon_1^2 + D_2^{(4)} \varepsilon_2^2)$$

$$\begin{aligned}
 & + \frac{H_{in}}{2} \sin\theta(\varepsilon_1^2 + \varepsilon_2^2) + \frac{H_{out}}{2} \cos\theta(\varepsilon_1^2 + \varepsilon_2^2) - \frac{N_d}{\mu_0}(\varepsilon_1 - \varepsilon_2)^2 \\
 & + 2K_s \sin 2\theta(\varepsilon_1^2 + \varepsilon_2^2)
 \end{aligned} \tag{10}$$

$$\begin{aligned}
 E(\varepsilon^3) = & \frac{\omega}{8} \sin 2\theta [4\Phi_0(\varepsilon_1^3 + \varepsilon_2^3) + \Phi_1(\varepsilon_1 + \varepsilon_2)^3] - \frac{4}{3} \sin\theta \cos\theta (D_1^{(2)} \varepsilon_1^3 + D_2^{(2)} \varepsilon_2^3) \\
 & - 4 \sin\theta \cos\theta \left( \frac{5}{3} \cos^2\theta - \sin^2\theta \right) (D_1^{(4)} \varepsilon_1^3 + D_2^{(4)} \varepsilon_2^3) + \frac{H_{in}}{6} \cos\theta(\varepsilon_1^3 + \varepsilon_2^3) \\
 & - \frac{H_{out}}{6} \sin\theta(\varepsilon_1^3 + \varepsilon_2^3) + \frac{4}{3} K_s \cos 2\theta(\varepsilon_1^3 + \varepsilon_2^3)
 \end{aligned} \tag{11}$$

$$\begin{aligned}
 E(\varepsilon^4) = & -\frac{1}{24} \left( JZ_1 - \frac{\omega}{4} \Phi_1 \right) (\varepsilon_1 - \varepsilon_2)^4 + \frac{\omega}{32} \cos 2\theta [8\Phi_0(\varepsilon_1^4 + \varepsilon_2^4) + \Phi_1(\varepsilon_1 + \varepsilon_2)^4] \\
 & - \frac{1}{3} \cos 2\theta (D_1^{(2)} \varepsilon_1^4 + D_2^{(2)} \varepsilon_2^4) - \left( \frac{5}{3} \cos^4\theta - 8 \cos^2\theta \sin^2\theta + \sin^4\theta \right) (D_1^{(4)} \varepsilon_1^4 + D_2^{(4)} \varepsilon_2^4) \\
 & - \frac{H_{in}}{24} \sin\theta(\varepsilon_1^4 + \varepsilon_2^4) - \frac{H_{out}}{24} \cos\theta(\varepsilon_1^4 + \varepsilon_2^4) + \frac{N_d}{12\mu_0} (\varepsilon_1 - \varepsilon_2)^4 \\
 & - \frac{2}{3} K_s \sin 2\theta(\varepsilon_1^4 + \varepsilon_2^4)
 \end{aligned} \tag{12}$$

First order perturbation term can be expressed in terms of a row and column matrix with all seven terms in each as following.

$$E(\varepsilon) = \vec{\alpha} \cdot \vec{\varepsilon}$$

Here terms of  $\alpha$  are given by  $\alpha_1$  and  $\alpha_2$ .

$$\alpha_1 = -\frac{3\omega}{4} \sin 2\theta (\Phi_0 + \Phi_1) + \sin 2\theta D_1^{(2)} + 2 \cos^2\theta \sin 2\theta D_1^{(4)} - H_{in} \cos\theta + H_{out} \sin\theta - 2K_s \cos 2\theta \tag{13}$$

$$\alpha_2 = -\frac{3\omega}{4} \sin 2\theta (\Phi_0 + \Phi_1) + \sin 2\theta D_2^{(2)} + 2 \cos^2\theta \sin 2\theta D_2^{(4)} - H_{in} \cos\theta + H_{out} \sin\theta - 2K_s \cos 2\theta \tag{14}$$

Second order perturbation term can be expressed in terms of a two by two matrix, a row matrix and a column matrix as following.

$$E(\varepsilon^2) = \frac{1}{2} \vec{\varepsilon} \cdot C \cdot \vec{\varepsilon}$$

Elements of  $2 \times 2$  matrix (C) are delineated by

$$\begin{aligned}
 C_{11} = & JZ_1 - \frac{\omega}{4} \Phi_1 - \frac{3\omega}{4} \cos 2\theta (2\Phi_0 + \Phi_1) + 2 \cos 2\theta D_1^{(2)} + 4 \cos^2\theta (\cos^2\theta - 3 \sin^2\theta) D_1^{(4)} \\
 & + H_{in} \sin\theta + H_{out} \cos\theta - \frac{2N_d}{\mu_0} + 4K_s \sin 2\theta
 \end{aligned} \tag{15}$$

$$C_{12} = C_{21} = -JZ_1 + \frac{\omega}{4} \Phi_1 - \frac{3\omega}{4} \cos 2\theta \Phi_1 + \frac{2N_d}{\mu_0} \tag{16}$$

$$\begin{aligned}
 C_{22} = & JZ_1 - \frac{\omega}{4} \Phi_1 - \frac{3\omega}{4} \cos 2\theta (2\Phi_0 + \Phi_1) + 2 \cos 2\theta D_2^{(2)} + 4 \cos^2\theta (\cos^2\theta - 3 \sin^2\theta) D_2^{(4)} \\
 & + H_{in} \sin\theta + H_{out} \cos\theta - \frac{2N_d}{\mu_0} + 4K_s \sin 2\theta
 \end{aligned} \tag{17}$$

Third order perturbation term can be expressed in terms of a two by two matrix. A row matrix and a column matrix as following.

$$E(\varepsilon^3) = \varepsilon^2 \cdot \beta \cdot \vec{\varepsilon}$$

Elements of two by two matrix ( $\beta$ ) are specified by

$$\beta_{11} = \frac{\omega}{8} \sin 2\theta (4\Phi_0 + \Phi_1) - \frac{4}{3} \sin \theta \cos \theta D_1^{(2)} - 4 \sin \theta \cos \theta \left( \frac{5}{3} \cos^2 \theta - \sin^2 \theta \right) D_1^{(4)} + \frac{H_{in}}{6} \cos \theta - \frac{H_{out}}{6} \sin \theta + \frac{4}{3} K_s \cos 2\theta \tag{18}$$

$$\beta_{12} = \beta_{21} = \frac{3\omega}{8} \sin 2\theta \Phi_1 \tag{19}$$

$$\beta_{22} = \frac{\omega}{8} \sin 2\theta (4\Phi_0 + \Phi_1) - \frac{4}{3} \sin \theta \cos \theta D_2^{(2)} - 4 \sin \theta \cos \theta \left( \frac{5}{3} \cos^2 \theta - \sin^2 \theta \right) D_2^{(4)} + \frac{H_{in}}{6} \cos \theta - \frac{H_{out}}{6} \sin \theta + \frac{4}{3} K_s \cos 2\theta \tag{20}$$

Fourth order perturbation term can be expressed in terms of two by two matrices, row matrices and column matrices as following.

$$E(\varepsilon^4) = \varepsilon^3 \cdot F \cdot \vec{\varepsilon} + \varepsilon^2 \cdot G \cdot \varepsilon^2$$

Elements of  $2 \times 2$  matrix ( $F$  and  $G$ ) are delineated by

$$F_{11} = -\frac{1}{24} \left( JZ_1 - \frac{\omega}{4} \Phi_1 \right) + \frac{\omega}{32} \cos 2\theta (8\Phi_0 + \Phi_1) - \frac{1}{3} \cos 2\theta D_1^{(2)} - \left( \frac{5}{3} \cos^4 \theta - 8 \cos^2 \theta \sin^2 \theta + \sin^4 \theta \right) D_1^{(4)} - \frac{H_{in}}{24} \sin \theta - \frac{H_{out}}{24} \cos \theta + \frac{N_d}{12\mu_0} - \frac{2}{3} K_s \sin 2\theta \tag{21}$$

$$F_{12} = F_{21} = \frac{1}{6} \left( JZ_1 - \frac{\omega}{4} \Phi_1 \right) + \frac{\omega}{8} \cos 2\theta \Phi_1 + \frac{N_d}{3\mu_0} \tag{22}$$

$$F_{22} = -\frac{1}{24} \left( JZ_1 - \frac{\omega}{4} \Phi_1 \right) + \frac{\omega}{32} \cos 2\theta (8\Phi_0 + \Phi_1) - \frac{1}{3} \cos 2\theta D_2^{(2)} - \left( \frac{5}{3} \cos^4 \theta - 8 \cos^2 \theta \sin^2 \theta + \sin^4 \theta \right) D_2^{(4)} - \frac{H_{in}}{24} \sin \theta - \frac{H_{out}}{24} \cos \theta + \frac{N_d}{12\mu_0} - \frac{2}{3} K_s \sin 2\theta \tag{23}$$

$$G_{11} = G_{22} = 0 \tag{24}$$

$$G_{12} = G_{21} = -\frac{1}{8} \left( JZ_1 - \frac{\omega}{4} \Phi_1 \right) + \frac{3\omega}{32} \cos 2\theta \Phi_1 + \frac{N_d}{4\mu_0} \tag{25}$$

Therefore, the total magnetic energy given in equation (2) can be deduced to

$$E(\theta) = E_0 + \vec{\alpha} \cdot \vec{\varepsilon} + \frac{1}{2} \vec{\varepsilon} \cdot C \cdot \vec{\varepsilon} + \varepsilon^2 \cdot \beta \cdot \vec{\varepsilon} + \varepsilon^3 \cdot F \cdot \vec{\varepsilon} + \varepsilon^2 \cdot G \cdot \varepsilon^2 \tag{26}$$

For the minimum energy of the second order perturbed term

$$\vec{\varepsilon} = -C^+ \cdot \alpha \tag{27}$$

Here  $C^+$  is the pseudo inverse of matrix  $C$ .  $C^+$  can be found using

$$C \cdot C^+ = I - \frac{E}{N}$$

Here  $E$  is the matrix with all elements given by  $E_{mm} = 1$ .  $I$  is the identity matrix.

Therefore,

$$C_{11}^+ = C_{22}^+ = \frac{C_{21} + C_{22}}{2(C_{11}C_{22} - C_{21}^2)} \tag{28}$$

$$C_{12}^+ = C_{21}^+ = \frac{C_{21} + C_{22}}{2(C_{21}^2 - C_{11}C_{22})} \tag{29}$$

Therefore, from the matrix equation (27)

$$\varepsilon_1 = (\alpha_2 - \alpha_1) C_{11}^+ \tag{30}$$

$$\varepsilon_2 = (\alpha_2 - \alpha_1) C_{21}^+ \tag{31}$$

After substituting  $\varepsilon$  in equation (26), the total magnetic energy can be determined.

### 3. Results and Discussion:

All the graphs in this manuscript were plotted for ferromagnetic films with body center cubic lattice and two spin layers. For ferromagnetic films with bcc(001) structure,  $Z_0=0$ ,  $Z_1=4$ ,  $Z_2=0$ ,  $\Phi_0 = 5.8675$  and  $\Phi_1 = 2.7126$  [6-8]. 3D plot of  $\frac{E(\theta)}{\omega}$  versus angle and

$\frac{K_S}{\omega}$  is given in figure1 for  $\frac{D_1^{(4)}}{\omega} = 5$  and  $\frac{D_2^{(4)}}{\omega} = 10$ . Here other parameters are fixed at  $\frac{J}{\omega} = \frac{H_{in}}{\omega} = \frac{H_{out}}{\omega} = \frac{N_{R1}}{\mu_0\omega} = \frac{D_1^{(2)}}{\omega} = \frac{D_2^{(2)}}{\omega} = 10$  for this simulation. The energy in this graph is in the order of  $10^{51}$ . The energy maximum can be observed at  $\frac{K_S}{\omega} = 3, 7, 26, 33, 42, 50$  and 90. The major maximum was observed at about  $\frac{K_S}{\omega} = 26$ . Minimum value of energy is zero. Compared to the 3D plots obtained using third order perturbed Heisenberg Hamiltonian, peaks are closely packed in 3D plots obtained using fourth order perturbed Heisenberg Hamiltonian [15].

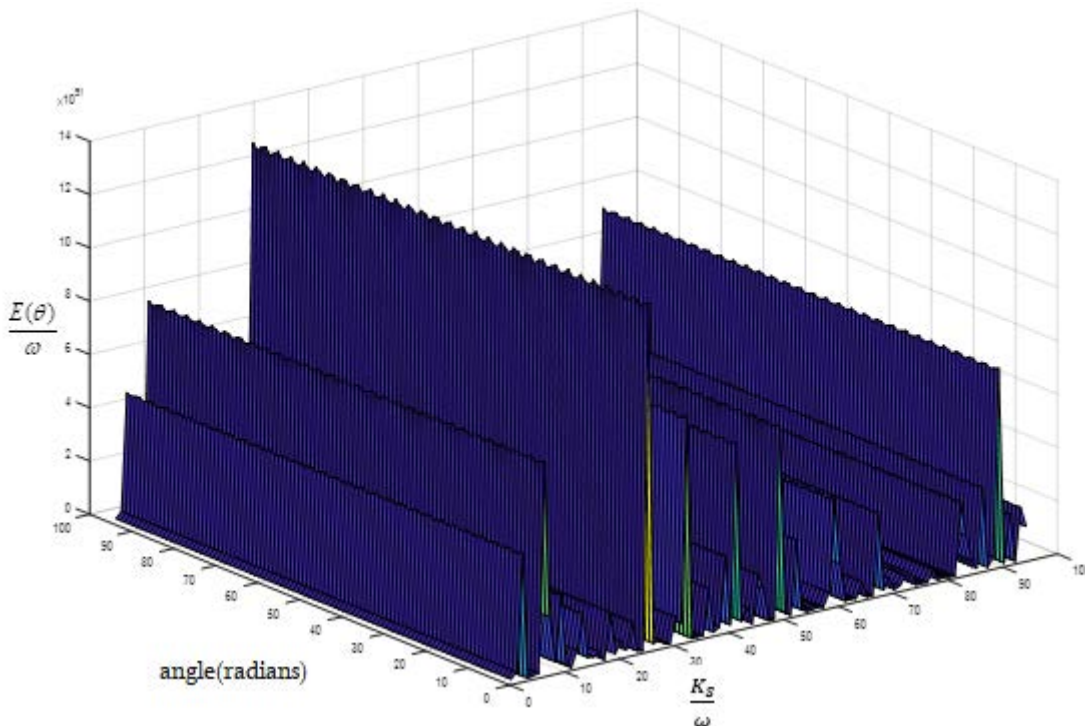


Figure 1: 3-D plot of energy versus angle and stress induced anisotropy constant for  $\frac{D_1^{(4)}}{\omega} = 5, \frac{D_2^{(4)}}{\omega} = 10$  and other parameters = 10.

Figure 2 shows the graph of energy versus angle for  $\frac{K_S}{\omega} = 26$ . Other parameters were kept at the values given above. In this graph, energy minimums can be observed at 0.7226 and 3.86 radians. The major minimum was observed at about 0.7226 radians. The energy maximum can be found at 2.293 and 5.43 radians. The major maximum was observed at about 5.43 radians. Therefore, the angle between consecutive magnetic easy and hard directions is 90 degrees.

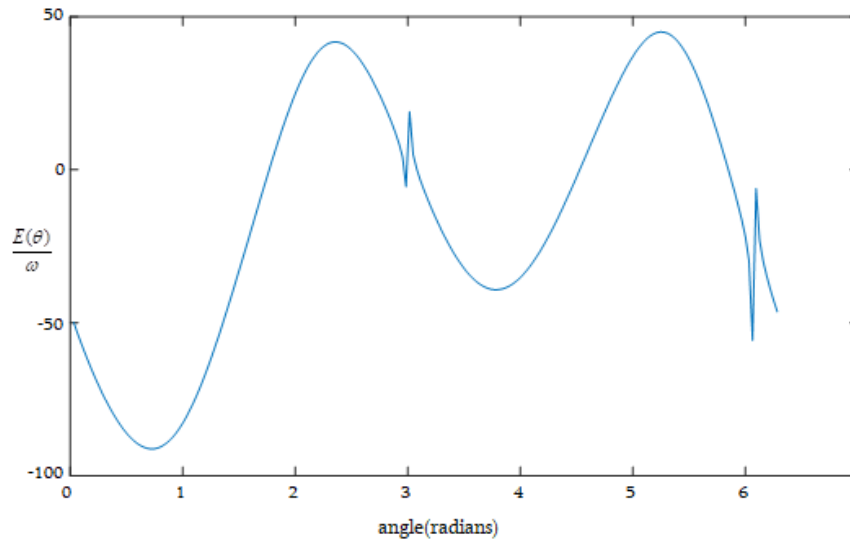


Fig. 2: Graph of energy versus  $\frac{K_S}{\omega} = 26$ .

#### 4. Conclusion:

All the graphs between total magnetic energy versus angle and  $\frac{K_S}{\omega}$  were plotted using fourth order perturbed Heisenberg Hamiltonian with all seven magnetic parameters for bcc structure. According to 3D plots, the order of the magnetic energy is in the order of  $10^{51}$ . The values of stress induced anisotropy constant at energy minima and maxima were found using 3D plots. The graph of energy versus angle was plotted for a one value of stress induced anisotropy constant found at an energy maximum. When the fourth order magnetic anisotropy constant of the top spin layer is higher than that of the bottom spin layer, the energy maximums were observed at  $\frac{K_S}{\omega} = 3, 7, 26, 33, 42, 50$  and 90. For  $\frac{K_S}{\omega} = 26$ , energy minimums were observed at 0.7226 and 3.86 radians, and the energy maximums were found at 2.293 and 5.43 radians.

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