UDC: 538.9 Condensed matter Physics, Solid state Physics, Theoretical Condensed matter Physics

FOURTH ORDER PERTURBED HEISENBERG HAMILTONIAN WITH SEVEN MAGNETIC PARAMETERS OF BCC STRUCTURED FERROMAGNETIC ULTRATHIN FILMS-PART 2

M.S.M. Farhan[1] and P. Samarasekara[2]

[1]Department of Physical Sciences, South Eastern University of Sri Lanka, Sammanthurai, Sri Lanka.
[2]Department of Physics, University of Peradeniya, Peradeniya, Sri Lanka

Abstract: *Bcc structured ferromagnetic ultrathin films with two spin layers was described using fourth order perturbed Heisenberg Hamiltonian with all seven magnetic energy parameters. Magnetic energy per unit spin was expressed in terms of spin exchange interaction, second order and fourth order anisotropy, in plane and out plane applied magnetic fields, demagnetization energy and stress induced anisotropy in two spin layers. 3D plot of total magnetic energy versus angle and stress induced anisotropy were plotted for the different values of second order and fourth order magnetic anisotropy constants. All other magnetic energy parameters were fixed at constant values. The order of magnetic energy was changed when the values of fourth order anisotropy constants of two spin layers are interchanged. In addition, the graphs of energy versus angle were plotted to find the magnetic easy and hard directions. The angle between magnetic easy and hard directions is not 90 degrees in this case.*
Keywords: *Fourth order perturbed Heisenberg Hamiltonian, magnetic anisotropy, stress induced anisotropy*

1. Introduction:

Although magnetic properties of ferromagnetic thin films have been previously described using second and third order perturbed Heisenberg Hamiltonian with all seven magnetic parameters and also fourth order perturbed Heisenberg Hamiltonian with only two magnetic parameters, Heisenberg Hamiltonian has never been investigated up to seven magnetic parameters using fourth order perturbation. Ferromagnetic ultrathin films are used in biomedical applications, sensors, potential applications in magnetic memory devices and microwave devices. Ferromagnetic films have been described using many different models. The quasistatic magnetic hysteresis of ferromagnetic thin films grown on a vicinal substrate has been theoretically investigated using Monte carlo simulations [1]. Magnetic properties of ferromagnetic thin films with alternating super layer were studied by Ising model[2]. Structural and magnetic properties of two dimensional FeCo ordered alloys have been determined by first principles band structure theory [3]. Magnetic layers of Ni on Cu have been theoretically studied by using the Korringa-Kohn-Rostoker Green's function metho[4].

The interfacial coupling dependence of the magnetic ordering in ferro-antiferromagnetic bilayers has been investigated using the Heisenberg Hamiltonian [5]. Second order perturbed Heisenberg Hamiltonian incorporated with spin exchange interaction, magnetic dipole interaction, applied magnetic field, second and fourth order magnetic anisotropy terms has been solved for ferromagnetic thin films [6-8]. Magnetic thin films are heated and subsequently cooled during the annealing process. During the heating and annealing process, the stress induced anisotropy (Ks) occurs due to the difference between the thermal expansion coefficients in magnetic film and substrate. For soft magnetic materials, the stress induced anisotropy is comparable to magneto crystalline anisotropy. Therefore, the coercivity depends on the stress induced anisotropy [9-11]. The magnetic properties of ferromagnetic and ferrite thin and thick films have been investigated by us using second, third and fourth order perturbed Heisenberg Hamiltonian by us [12-18].

In this manuscript, the total energy of ferromagnetic thin films will be described by solving the classical Heisenberg Hamiltonian. In this approach, all energy terms such as magnetic energy, spin dipole interaction energy, spin exchange interaction energy, second and fourth order anisotropy and stress induced anisotropy demagnetization factor terms will be considered. Fourth order perturbed Heisenberg Hamiltonian with all seven magnetic energy parameters are explained for bcc structure. MATLAB computer software was used to plot 3D and 2D graphs of energy versus stress induced anisotropy and azimuthal angle of spin.

2. **Model:**

The Heisenberg Hamiltonian of ferromagnetic films can be formulated as following [12-14].

$$H = -\frac{J}{2}\sum_{m,n} \vec{S}_m \cdot \vec{S}_n + \frac{\omega}{2}\sum_{m \neq n} \left(\frac{\vec{S}_m \cdot \vec{S}_n}{r_{mn}^3} - \frac{3(\vec{S}_m \cdot \vec{r}_{mn})(\vec{r}_{mn} \cdot \vec{S}_n)}{r_{mn}^5} \right) - \sum_m D_{\lambda_m}^{(2)}(S_m^z)^2 - \sum_m D_{\lambda_m}^{(4)}(S_m^z)^4$$
$$- \sum_{m,n}[\vec{H} - (N_d\vec{S}_n/\mu_0)] \cdot \vec{S}_m - \sum_m K_s \sin 2\theta_m$$

Here $\vec{S}_m$ and $\vec{S}_n$ are two spins. Above equation can be simplified into following form

$$E(\theta) = -\frac{1}{2}\sum_{m,n=1}^{N} \left[\left(JZ_{|m-n|} - \frac{\omega}{4}\Phi_{|m-n|} \right)\cos(\theta_m - \theta_n) - \frac{3\omega}{4}\Phi_{|m-n|}\cos(\theta_m + \theta_n) \right]$$

$$- \sum_{m=1}^{N}(D_m^{(2)}\cos^2\theta_m + D_m^{(4)}\cos^4\theta_m + H_{in}\sin\theta_m + H_{out}\cos\theta_m)$$

$$+ \sum_{m,n=1}^{N} \frac{N_d}{\mu_0}\cos(\theta_m - \theta_n) - K_s\sum_{m=1}^{N}\sin 2\theta_m \qquad (1)$$

Here N, m (or n), J, $Z_{|m-n|}$, ω, $\Phi_{|m-n|}$, $\theta_m(\theta_n)$, $D_m^{(2)}$, $D_m^{(4)}$, H_{in}, H_{out}, N_d and K_s are total number of layers, layer index, spin exchange interaction, number of nearest spin neighbors, strength of long range dipole interaction, partial summations of dipole interaction, azimuthal angles of spins, second and fourth order anisotropy constants, in plane and out of plane applied magnetic fields, demagnetization factor and stress induced anisotropy constants respectively.

The spin structure is considered to be slightly disoriented. Therefore, the spins could be considered to have angles distributed about an average angle θ. By choosing azimuthal angles as

$$\theta_m = \theta + \varepsilon_m \text{ and } \theta_n = \theta + \varepsilon_n$$

Where the ε's are small positive or negative angular deviations.

Then, $\theta_m - \theta_n = \varepsilon_m - \varepsilon_n$ and $\theta_m + \theta_n = 2\theta + \varepsilon_m + \varepsilon_n$.

After substituting these new angles in above equation number (1), the cosine and sine terms can be expanded up to the fourth order of ε_m and ε_n as following.

$$E(\theta) = E_0 + E(\varepsilon) + E(\varepsilon^2) + E(\varepsilon^3) + E(\varepsilon^4) + \ldots\ldots\ldots$$

If the fifth and higher order perturbations are neglected, then

$$E(\theta) = E_0 + E(\varepsilon) + E(\varepsilon^2) + E(\varepsilon^3) + E(\varepsilon^4) \qquad (2)$$

Here

$$E_0 = -\frac{1}{2} \sum_{m,n=1}^{N} \left(JZ_{|m-n|} - \frac{\omega}{4} \Phi_{|m-n|} \right) + \frac{3\omega}{8} cos2\theta \sum_{m,n=1}^{N} \Phi_{|m-n|} - cos^2\theta \sum_{m=1}^{N} D_m^{(2)}$$

$$-cos^4\theta \sum_{m=1}^{N} D_m^{(4)} - N(H_{in}sin\theta + H_{out}cos\theta + K_s sin2\theta) + \frac{N_d N^2}{\mu_0} \quad (3)$$

$$E(\varepsilon) = -\frac{3\omega}{8} sin2\theta \sum_{m,n=1}^{N} \Phi_{|m-n|}(\varepsilon_m + \varepsilon_n) + sin2\theta \sum_{m=1}^{N} D_m^{(2)} \varepsilon_m + 2cos^2\theta sin2\theta \sum_{m=1}^{N} D_m^{(4)} \varepsilon_m$$

$$-H_{in}cos\theta \sum_{m=1}^{N} \varepsilon_m + H_{out}sin\theta \sum_{m=1}^{N} \varepsilon_m - 2K_s cos2\theta \sum_{m=1}^{N} \varepsilon_m \quad (4)$$

$$E(\varepsilon^2) = \frac{1}{4} \sum_{m,n=1}^{N} \left(JZ_{|m-n|} - \frac{\omega}{4} \Phi_{|m-n|} \right)(\varepsilon_m - \varepsilon_n)^2 - \frac{3\omega}{16} cos2\theta \sum_{m,n=1}^{N} \Phi_{|m-n|}(\varepsilon_m + \varepsilon_n)^2$$

$$+cos2\theta \sum_{m=1}^{N} D_m^{(2)} \varepsilon_m^2 + 2cos^2\theta(cos^2\theta - 3sin^2\theta) \sum_{m=1}^{N} D_m^{(4)} \varepsilon_m^2 + \frac{H_{in}}{2} sin\theta \sum_{m=1}^{N} \varepsilon_m^2$$

$$+\frac{H_{out}}{2}cos\theta \sum_{m=1}^{N} \varepsilon_m^2 - \frac{N_d}{2\mu_0} \sum_{m,n=1}^{N} (\varepsilon_m - \varepsilon_n)^2 + 2K_s sin2\theta \sum_{m=1}^{N} \varepsilon_m^2 \quad (5)$$

$$E(\varepsilon^3) = \frac{\omega}{16} sin2\theta \sum_{m,n=1}^{N} \Phi_{|m-n|}(\varepsilon_m + \varepsilon_n)^3 - \frac{4}{3} sin\theta cos\theta \sum_{m=1}^{N} D_m^{(2)} \varepsilon_m^3$$

$$-4sin\theta cos\theta \left(\frac{5}{3} cos^2\theta - sin^2\theta\right) \sum_{m=1}^{N} D_m^{(4)} \varepsilon_m^3 + \frac{H_{in}}{6} cos\theta \sum_{m=1}^{N} \varepsilon_m^3 - \frac{H_{out}}{6} sin\theta \sum_{m=1}^{N} \varepsilon_m^3$$

$$+\frac{4}{3} K_s cos2\theta \sum_{m=1}^{N} \varepsilon_m^3 \quad (6)$$

$$E(\varepsilon^4) = -\frac{1}{48} \sum_{m,n=1}^{N} \left(JZ_{|m-n|} - \frac{\omega}{4} \Phi_{|m-n|} \right)(\varepsilon_m - \varepsilon_n)^4 + \frac{\omega}{64} cos2\theta \sum_{m,n=1}^{N} \Phi_{|m-n|}(\varepsilon_m + \varepsilon_n)^4$$

$$-\frac{1}{3} cos2\theta \sum_{m=1}^{N} D_m^{(2)} \varepsilon_m^4 - \left(\frac{5}{3} cos^4\theta - 8cos^2\theta sin^2\theta + sin^4\theta\right) \sum_{m=1}^{N} D_m^{(4)} \varepsilon_m^4 - \frac{H_{in}}{24} sin\theta \sum_{m=1}^{N} \varepsilon_m^4$$

$$-\frac{H_{out}}{24}cos\theta \sum_{m=1}^{N} \varepsilon_m^4 + \frac{N_d}{24\mu_0} \sum_{m,n=1}^{N} (\varepsilon_m - \varepsilon_n)^4 - \frac{2}{3} K_s sin2\theta \sum_{m=1}^{N} \varepsilon_m^4 \quad (7)$$

For films with two spin layers, $N = 2$. Therefore, m and n change from 1 to 2.

$$E_0 = -JZ_0 + \frac{\omega}{4}\Phi_0 - JZ_1 + \frac{\omega}{4}\Phi_1 + \frac{3\omega}{4}cos2\theta(\Phi_0 + \Phi_1) - cos^2\theta\left(D_1^{(2)} + D_2^{(2)}\right)$$

$$-cos^4\theta\left(D_1^{(4)} + D_2^{(4)}\right) - 2(H_{in}sin\theta + H_{out}cos\theta + K_s sin2\theta) + \frac{N_d N^2}{\mu_0} \qquad (8)$$

$$E(\varepsilon) = -\frac{3\omega}{4}sin2\theta[(\Phi_0 + \Phi_1)(\varepsilon_1 + \varepsilon_2)] + sin2\theta\left(D_1^{(2)}\varepsilon_1 + D_2^{(2)}\varepsilon_2\right)$$

$$+2cos^2\theta sin2\theta\left(D_1^{(4)}\varepsilon_1 + D_2^{(4)}\varepsilon_2\right) - H_{in}cos\theta(\varepsilon_1 + \varepsilon_2) + H_{out}sin\theta(\varepsilon_1 + \varepsilon_2)$$

$$-2K_s cos2\theta(\varepsilon_1 + \varepsilon_2) \qquad \textbf{(9)}$$

$$E(\varepsilon^2) = \frac{1}{2}\left(JZ_1 - \frac{\omega}{4}\Phi_1\right)(\varepsilon_1 - \varepsilon_2)^2 - \frac{3\omega}{8}cos2\theta[2\Phi_0(\varepsilon_1{}^2 + \varepsilon_2{}^2) + \Phi_1(\varepsilon_1 + \varepsilon_2)^2]$$

$$+cos2\theta\left(D_1^{(2)}\varepsilon_1{}^2 + D_2^{(2)}\varepsilon_2{}^2\right) + 2cos^2\theta(cos^2\theta - 3sin^2\theta)\left(D_1^{(4)}\varepsilon_1{}^2 + D_2^{(4)}\varepsilon_2{}^2\right)$$

$$+\frac{H_{in}}{2}sin\theta(\varepsilon_1{}^2 + \varepsilon_2{}^2) + \frac{H_{out}}{2}cos\theta(\varepsilon_1{}^2 + \varepsilon_2{}^2) - \frac{N_d}{\mu_0}(\varepsilon_1 - \varepsilon_2)^2$$

$$+2K_s sin2\theta(\varepsilon_1{}^2 + \varepsilon_2{}^2) \qquad (10)$$

$$E(\varepsilon^3) = \frac{\omega}{8}sin2\theta[4\Phi_0(\varepsilon_1{}^3 + \varepsilon_2{}^3) + \Phi_1(\varepsilon_1 + \varepsilon_2)^3] - \frac{4}{3}sin\theta cos\theta\left(D_1^{(2)}\varepsilon_1{}^3 + D_2^{(2)}\varepsilon_2{}^3\right)$$

$$-4sin\theta cos\theta\left(\frac{5}{3}cos^2\theta - sin^2\theta\right)\left(D_1^{(4)}\varepsilon_1{}^3 + D_2^{(4)}\varepsilon_2{}^3\right) + \frac{H_{in}}{6}cos\theta(\varepsilon_1{}^3 + \varepsilon_2{}^3)$$

$$-\frac{H_{out}}{6}sin\theta(\varepsilon_1{}^3 + \varepsilon_2{}^3) + \frac{4}{3}K_s cos2\theta(\varepsilon_1{}^3 + \varepsilon_2{}^3) \qquad (11)$$

$$E(\varepsilon^4) = -\frac{1}{24}\left(JZ_1 - \frac{\omega}{4}\Phi_1\right)(\varepsilon_1 - \varepsilon_2)^4 + \frac{\omega}{32}cos2\theta[8\Phi_0(\varepsilon_1{}^4 + \varepsilon_2{}^4) + \Phi_1(\varepsilon_1 + \varepsilon_2)^4]$$

$$-\frac{1}{3}cos2\theta\left(D_1^{(2)}\varepsilon_1{}^4 + D_2^{(2)}\varepsilon_2{}^4\right) - \left(\frac{5}{3}cos^4\theta - 8cos^2\theta sin^2\theta + sin^4\theta\right)\left(D_1^{(4)}\varepsilon_1{}^4 + D_2^{(4)}\varepsilon_2{}^4\right)$$

$$-\frac{H_{in}}{24}sin\theta(\varepsilon_1{}^4 + \varepsilon_2{}^4) - \frac{H_{out}}{24}cos\theta(\varepsilon_1{}^4 + \varepsilon_2{}^4) + \frac{N_d}{12\mu_0}(\varepsilon_1 - \varepsilon_2)^4$$

$$-\frac{2}{3}K_s sin2\theta(\varepsilon_1{}^4 + \varepsilon_2{}^4) \qquad (12)$$

First order perturbation term can be expressed in terms of a row and column matrix with all seven terms in each as following.

$$E(\varepsilon) = \vec{\alpha}.\vec{\varepsilon}$$

Here terms of α are given by α_1 and α_2.

$$\alpha_1 = -\frac{3\omega}{4}sin2\theta(\Phi_0 + \Phi_1) + sin2\theta D_1^{(2)} + 2cos^2\theta sin2\theta D_1^{(4)} - H_{in}cos\theta + H_{out}sin\theta$$

$$-2K_s cos2\theta \tag{13}$$

$$\alpha_2 = -\frac{3\omega}{4} sin2\theta(\Phi_0 + \Phi_1) + sin2\theta D_2^{(2)} + 2cos^2\theta sin2\theta D_2^{(4)} - H_{in}cos\theta + H_{out}sin\theta$$

$$-2K_s cos2\theta \tag{14}$$

Second order perturbation term can be expressed in terms of a two by two matrix, a row matrix and a column matrix as following.

$$E(\varepsilon^2) = \frac{1}{2}\vec{\varepsilon}.C.\vec{\varepsilon}$$

Elements of 2×2 matrix (C) are delineated by

$$C_{11} = JZ_1 - \frac{\omega}{4}\Phi_1 - \frac{3\omega}{4}cos2\theta(2\Phi_0 + \Phi_1) + 2cos2\theta D_1^{(2)} + 4cos^2\theta(cos^2\theta - 3sin^2\theta)D_1^{(4)}$$

$$+H_{in}sin\theta + H_{out}cos\theta - \frac{2N_d}{\mu_0} + 4K_s sin2\theta \tag{15}$$

$$C_{12} = C_{21} = -JZ_1 + \frac{\omega}{4}\Phi_1 - \frac{3\omega}{4}cos2\theta\Phi_1 + \frac{2N_d}{\mu_0} \tag{16}$$

$$C_{22} = JZ_1 - \frac{\omega}{4}\Phi_1 - \frac{3\omega}{4}cos2\theta(2\Phi_0 + \Phi_1) + 2cos2\theta D_2^{(2)} + 4cos^2\theta(cos^2\theta - 3sin^2\theta)D_2^{(4)}$$

$$+H_{in}sin\theta + H_{out}cos\theta - \frac{2N_d}{\mu_0} + 4K_s sin2\theta \tag{17}$$

Third order perturbation term can be expressed in terms of a two by two matrix. A row matrix and a column matrix as following.

$$E(\varepsilon^3) = \varepsilon^2.\beta.\vec{\varepsilon}$$

Elements of two by two matrix (β) are specified by

$$\beta_{11} = \frac{\omega}{8}sin2\theta(4\Phi_0 + \Phi_1) - \frac{4}{3}sin\theta cos\theta D_1^{(2)} - 4sin\theta cos\theta\left(\frac{5}{3}cos^2\theta - sin^2\theta\right)D_1^{(4)}$$

$$+\frac{H_{in}}{6}cos\theta - \frac{H_{out}}{6}sin\theta + \frac{4}{3}K_s cos2\theta \tag{18}$$

$$\beta_{12} = \beta_{21} = \frac{3\omega}{8}sin2\theta\Phi_1 \tag{19}$$

$$\beta_{22} = \frac{\omega}{8}sin2\theta(4\Phi_0 + \Phi_1) - \frac{4}{3}sin\theta cos\theta D_2^{(2)}$$
$$- 4sin\theta cos\theta\left(\frac{5}{3}cos^2\theta - sin^2\theta\right)D_2^{(4)}$$

$$+\frac{H_{in}}{6}cos\theta - \frac{H_{out}}{6}sin\theta + \frac{4}{3}K_s cos2\theta \tag{20}$$

Fourth order perturbation term can be expressed in terms of two by two matrices, row matrices and column matrices as following.

$$E(\varepsilon^4) = \varepsilon^3.F.\vec{\varepsilon} + \varepsilon^2.G.\varepsilon^2$$

Elements of 2×2 matrix (F and G) are delineated by

$$
\begin{aligned}
F_{11} = &-\frac{1}{24}\left(JZ_1 - \frac{\omega}{4}\Phi_1\right) + \frac{\omega}{32}\cos2\theta(8\Phi_0 + \Phi_1) - \frac{1}{3}\cos2\theta D_1^{(2)} \\
&-(\frac{5}{3}\cos^4\theta - 8\cos^2\theta\sin^2\theta + \sin^4\theta)D_1^{(4)} - \frac{H_{in}}{24}\sin\theta - \frac{H_{out}}{24}\cos\theta + \frac{N_d}{12\mu_0} \\
&-\frac{2}{3}K_s\sin2\theta
\end{aligned}
\tag{21}
$$

$$
F_{12} = F_{21} = \frac{1}{6}\left(JZ_1 - \frac{\omega}{4}\Phi_1\right) + \frac{\omega}{8}\cos2\theta\Phi_1 + \frac{N_d}{3\mu_0}
\tag{22}
$$

$$
\begin{aligned}
F_{22} = &-\frac{1}{24}\left(JZ_1 - \frac{\omega}{4}\Phi_1\right) + \frac{\omega}{32}\cos2\theta(8\Phi_0 + \Phi_1) - \frac{1}{3}\cos2\theta D_2^{(2)} \\
&-(\frac{5}{3}\cos^4\theta - 8\cos^2\theta\sin^2\theta + \sin^4\theta)D_2^{(4)} - \frac{H_{in}}{24}\sin\theta - \frac{H_{out}}{24}\cos\theta + \frac{N_d}{12\mu_0} \\
&-\frac{2}{3}K_s\sin2\theta
\end{aligned}
\tag{23}
$$

$$
G_{11} = G_{22} = 0
\tag{24}
$$

$$
G_{12} = G_{21} = -\frac{1}{8}\left(JZ_1 - \frac{\omega}{4}\Phi_1\right) + \frac{3\omega}{32}\cos2\theta\Phi_1 + \frac{N_d}{4\mu_0}
\tag{25}
$$

Therefore, the total magnetic energy given in equation (2) can be deduced to

$$
E(\theta) = E_0 + \vec{\alpha}.\vec{\varepsilon} + \frac{1}{2}\vec{\varepsilon}.C.\vec{\varepsilon} + \varepsilon^2.\beta.\vec{\varepsilon} + \varepsilon^3.F.\vec{\varepsilon} + \varepsilon^2.G.\varepsilon^2
\tag{26}
$$

For the minimum energy of the second order perturbed term

$$
\vec{\varepsilon} = -C^+.\alpha
\tag{27}
$$

Here C^+ is the pseudo inverse of matrix C. C^+ can be found using

$$
C.C^+ = 1 - \frac{E}{N}
$$

Here E is the matrix with all elements given by $E_{mn} = 1$. I is the identity matrix.

Therefore,

$$
C_{11}^+ = C_{22}^+ = \frac{C_{21} + C_{22}}{2(C_{11}C_{22} - C_{21}^2)}
\tag{28}
$$

$$
C_{12}^+ = C_{21}^+ = \frac{C_{21} + C_{22}}{2(C_{21}^2 - C_{11}C_{22})}
\tag{29}
$$

Therefore, from the matrix equation (27)

$$\varepsilon_1 = (\alpha_2 - \alpha_1)\, C_{11}^+ \tag{30}$$

$$\varepsilon_2 = (\alpha_2 - \alpha_1)\, C_{21}^+ \tag{31}$$

After substituting ε in equation (26), the total magnetic energy can be determined.

3. Results and Discussion:

All the graphs in this manuscript were plotted for ferromagnetic films with body center cubic lattice and two spin layers. For ferromagnetic films with bcc(001) structure, $Z_0=0$, $Z_1=4$, $Z_2=0$, $\Phi_0 = 5.8675$ and $\Phi_1 = 2.7126$ [6-8]. Figure1 represents the 3D plot of energy versus angle and $\frac{K_s}{\omega}$ for $\frac{D_1^{(4)}}{\omega} = 10$ and $\frac{D_2^{(4)}}{\omega} = 5$. Here other parameters are fixed at $\frac{J}{\omega} = \frac{H_{in}}{\omega} = \frac{H_{out}}{\omega} = \frac{N_d}{\mu_0 \omega} = \frac{D_2^{(2)}}{\omega} = \frac{D_1^{(2)}}{\omega} = 10$ for this simulation. Several peaks can be observed in all the graphs. However, the gap between two peaks is not a constant. In this graph, the energy maximum can be observed at $\frac{K_s}{\omega} = 12, 21, 30, 47, 62,$ --etc. The major maximum was observed at about $\frac{K_s}{\omega} = 21$. Minimum values of energy are zero in this case. The energy is in the order of 10^{50}. When the fourth order magnetic anisotropy of bottom spin layer is higher than that of the top spin layer, the order of magnetic energy slightly decreases.
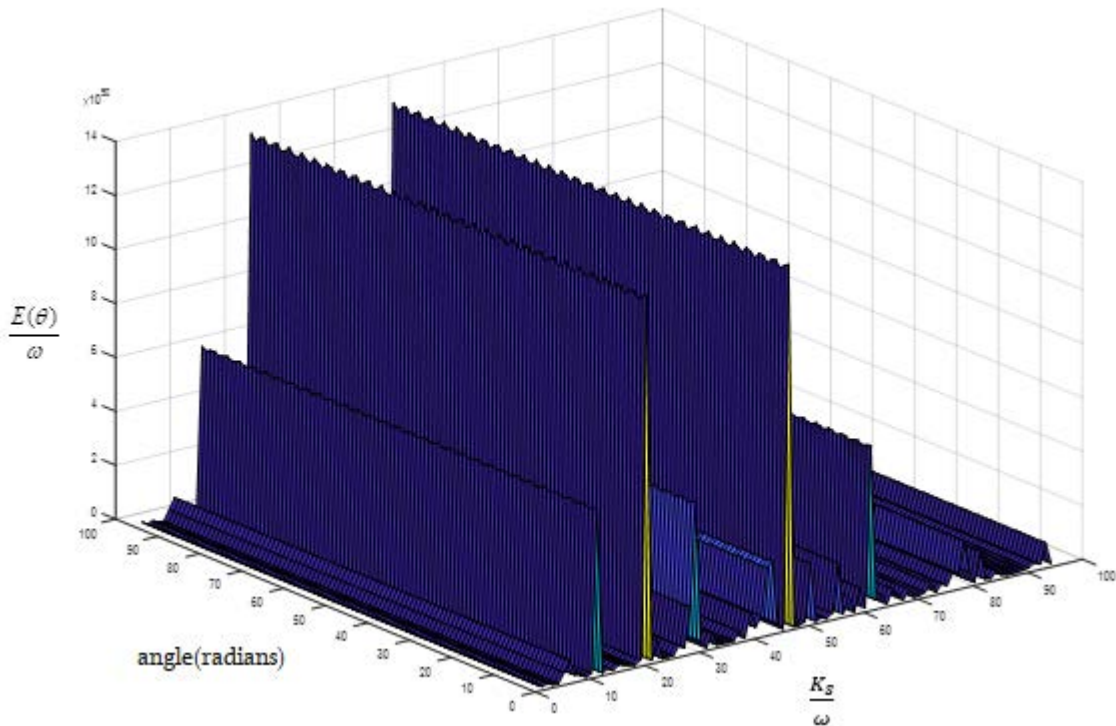

Figure 1: 3-D plot of energy versus angle and stress induced anisotropy constant for $\frac{D_1^{(4)}}{\omega} = 10$, $\frac{D_2^{(4)}}{\omega} = 5$.

Figure 2 shows the graph of energy versus angle for $\frac{K_s}{\omega} = 21$. In this graph, energy minimums can be observed at 0.683 and 3.707 radians. The major minimum was observed at about 0.683 radians. The energy maximums can be found at 2.45 and 5.246 radians. The major maximum was observed

at about 5.246 radians. The angle between consecutive magnetic easy and hard directions is not 90 degrees in this case.
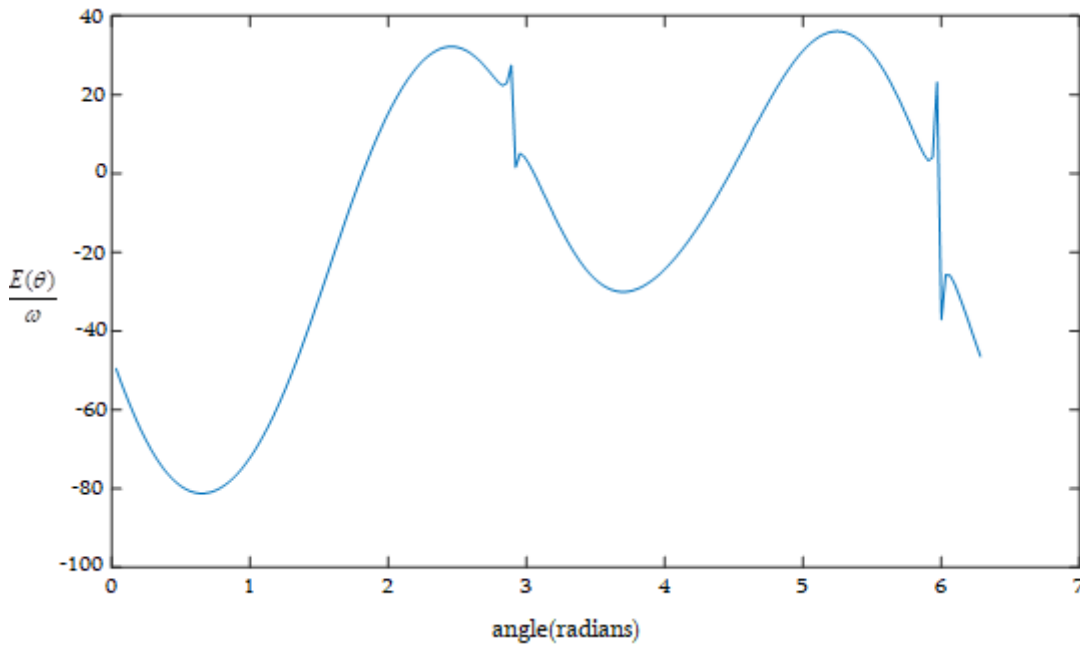

Figure 2: 2-D graph of energy versus angle for $\frac{K_s}{\omega} = 21$.

4. Conclusion:

All the graphs between total magnetic energy versus angle and $\frac{K_s}{\omega}$ were plotted using fourth order perturbed Heisenberg Hamiltonian with all seven magnetic parameters for bcc structure. According to 3D plots, the order of the magnetic energy is in the order of 10^{50}. The minimum order of magnetic energy (10^4) was observed, when the second order magnetic anisotropy constant of both the spin layers is higher than other magnetic energy parameters. The values of stress induced anisotropy constant at energy minima and maxima were found using 3D plots. The graph of energy versus angle was plotted for a one value of stress induced anisotropy constant found at an energy maximum. When the fourth order magnetic anisotropy constant of the top spin layer is less than that of the bottom spin layer, the energy maximums were observed at $\frac{K_s}{\omega} = 12, 21, 30, 47,$ and 62. For $\frac{K_s}{\omega} = 21$, energy minimums were observed at 0.683 and 3.707 radians, and the energy maximums were found at 2.45 and 5.246 radians.

References:

1. D. Zhao, Feng Liu, D.L. Huber and M.G. Lagally, Journal of Applied Physics (2002), 91(5), 3150.
2. M. Bentaleb, N. Aouad El and M. Saber, Chinese Journal of Physics (2002), 40(3), 307.
3. D. Spisak and J. Hafner, Journal of Magnetism and Magnetic Materials (2005), 286, 386.
4. A. Ernst, M. Lueders, W.M. Temmerman, Z. Szotek and G. Van der Laan, Journal of Physics: Condensed matter (2000), 12(26), 5599.
5. Tsai Shan-Ho, D. P. Landau and C. Schulthess Thomas, Journal of Applied Physics (2003), 93(10), 8612.
6. A. Hucht and K.D. Usadel, Physical Review B (1997), 55, 12309.
7. A. Hucht and K.D. Usadel, Journal of Magnetism and Magnetic Materials (1999), 203(1), 88.
8. K.D. Usadel and A. Hucht, Physical Review B (2002), 66, 024419.
9. P. Samarasekara and F.J. Cadieu, Japanese Journal of Applied Physics (2001), 40, 3176.
10. P. Samarasekara and F.J. Cadieu, Chinese Journal of Physics (2001), 39(6), 635.
11. P. Samarasekara, Chinese Journal of Physics (2002), 40(6), 631.
12. P. Samarasekara and E.M.P. Ekanayake, STM journals: Journal of Nanoscience, NanoEngineering & Applications (2020), 10(3), 35.
13. P. Samarasekara and W.A. Mendoza, Electronic Journal of Theoretical Physics (2010), 7(24), 197.
14. P. Samarasekara, Electronic Journal of Theoretical Physics (2006), 3(11), 71.
15. N.U.S. Yapa, P. Samarasekara and Sunil Dehipawala, Ceylon Journal of Science (2017), 46(3), 93.
16. P. Samarasekara, M.K. Abeyratne and S. Dehipawalage, Electronic Journal of Theoretical Physics (2009), 6(20), 345.
17. P. Samarasekara and S.N.P. De Silva, Chinese Journal of Physics (2007), 45(2-I), 142.
18. P. Samarasekara, Electronic Journal of Theoretical Physics (2008), 5(17), 227.