THE DEGREE OF LIGHT POLARIZATION IN NONSTATIONARY PROCESSES

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ABSTRACT. The model consideration of the description of polarization state of light is received when fully polarized light passed through a nonstationary anisotropic device is presented. In this case fully polarized light becomes partially polarized. It is shown that the degree of the polarization of light is immediately connected with the time profile of nonstationary process.

As is known partially polarized radiation is obtained from nonpolarized during reflection or scattering [1]. The reception of partially polarized radiation as a result of passing of originally fully polarized light through the nonstationary device was first investigated in [2,3]. In this work the further development of this approach is made for the nonstationary device, the coefficient of birefringence of which sinusoidaly changes in time.

The model polarization device is presented in the form of a medium single thickness (for the simplicity of further discussion) with a birefringent coefficient $\Delta n = n_x - n_y$ (n_x, n_y are the value of n coefficient according to the respective axes) and the orientation of the axis of anisotropy ρ. If $\boldsymbol{\Delta n}$ and ρ are sufficiently quick functions of time and with this $n_x + n_y$ with sufficient accuracy can be accepted as independent of time, then the model device turns out to be nonstationary. Such a device causes depolarization of the fully polarized wave front and the field of the transmitted wave immediately after the device becomes partially polarized. The degree of light polarization connected with the time profile of the nonstationary process can be presented as [2]:

$$V \approx \left| \frac{\int\limits_{-T'}^{T'} \cos^2\left[\frac{æ_o}{2}\Delta n(t)\right]dt + \int\limits_{-T'}^{T'} \sin^2\left[\frac{æ_o}{2}\Delta n(t)\right]\cos 4\rho(t)dt}{\int\limits_{-T}^{T'} \cos^2\left[\frac{æ_o}{2}\Delta n(t)\right]dt + \int\limits_{-T'}^{T'} \sin^2\left[\frac{æ_o}{2}\Delta n(t)\right]dt} \right| \qquad (1)$$

where $æ_o = \dfrac{2\pi}{\lambda_0}$ (λ_o - is the length in a general case of an elliptically polarized wave coming on the device), T' is the time of observation.

As an example of a nonstationary model polarization device let us choose the law of time change in the following form:

$$n_x + n_y \approx 2n_o$$

$$, \qquad \rho(t) = kt\ , \qquad (2)$$

where $\overline{\omega}$ is a cyclic frequency of the device, n_o and k are characteristics of the model. Thus we have:

$$V \approx \left| \frac{\int\limits_{-T'}^{T'} \cos^2\left(\frac{æ_o}{2} n_0 \sin\overline{\omega}t\right)dt + \int\limits_{-T'}^{T'} \sin^2\left(\frac{æ_o}{2} n_0 \sin\overline{\omega}t\right)\cos 4ktdt}{\int\limits_{-T'}^{T'} \cos^2\left(\frac{æ_o}{2} n_0 \sin\overline{\omega}t\right)dt + \int\limits_{-T'}^{T'} \sin^2\left(\frac{æ_o}{2} n_0 \sin\overline{\omega}t\right)dt} \right| \qquad (3)$$

The transformation of the last expression gives:

$$V \approx \left| \frac{1}{2} + \frac{1}{2T'}\int\limits_{0}^{T'} \cos\left(æ_o n_0 \sin\overline{\omega}t\right)dt + \frac{1}{2T'}\int\limits_{o}^{T'} \cos 4ktdt - \right.$$

$$\left. - \frac{1}{2T'}\int\limits_{0}^{T'} \cos\left(æ_o n_o \sin\overline{\omega}t\right)\cos 4ktdt \right|, \qquad (4)$$

where:

$$\int_0^{T'} (\text{æ}_o n_o \sin \overline{\omega} t)dt = \frac{1}{\overline{\omega}} \int_0^{\overline{\omega} T'} \cos(\text{æ}_o n_0 \sin x)dx =$$

$$= \frac{1}{\overline{\omega}} \frac{\overline{\omega} T'}{\pi} \int_0^{\pi} \cos\left[\text{æ}_0 n_o \sin\left(\frac{\overline{\omega} T'}{\pi} x \right) \right] d\left(\frac{\overline{\omega} T'}{\pi} x \right) = T' J_o(\text{æ}_0 n_o), \quad (5)$$

J_o is the Bessel function of the first kind of zero order [4].

$$\int_o^T \cos(4kt)dt = \frac{1}{4k} \left[\sin 4kt \right]_0^{T'} = \frac{\sin 4kt}{4k} \quad (6)$$

$$\int_0^{T'} \cos(\text{æ}_o n_o \sin \overline{\omega} t) \cos 4kt dt = \frac{1}{\overline{\omega}} \int_0^{\overline{\omega} T'} \cos(\text{æ}_0 n_o \sin x) \cos\left(\frac{4k}{\overline{\omega}} x \right) dx =$$

$$= \frac{1}{2\overline{\omega}} \int_0^{\overline{\omega} T'} \left[\cos\left(\text{æ}_0 n_o \sin x - \frac{4k}{\overline{\omega}} x \right) + \cos\left(\text{æ}_0 n_o \sin x + \frac{4k}{\overline{\omega}} x \right) \right] dx =$$

$$= \frac{1}{2\overline{\omega}} \left[\pi \frac{\sin 4kT'}{\sin\left(\frac{4k\pi}{\overline{\omega}} \right)} J_{\frac{4k}{\overline{\omega}}}(\text{æ}_0 n_o) + \pi \frac{\sin 4kT'}{\sin\left(\frac{4k\pi}{\overline{\omega}} \right)} J_{-\frac{4k}{\overline{\omega}}}(\text{æ}_0 n_o) \right] =$$

$$= \frac{\pi}{2\overline{\omega}} \frac{\sin 4kT'}{\sin\left(\frac{4k\pi}{\overline{\omega}} \right)} \left[J_{\frac{4k}{\overline{\omega}}}(\text{æ}_0 n_o) + J_{-\frac{4k}{\overline{\omega}}}(\text{æ}_0 n_o) \right], \quad (7)$$

J_ν is the function of Anger ν order [4].

Substituting the received integrals into (4) we shall finally receive:

$$V \approx \left| \frac{1}{2} + \frac{1}{2} J_o(æ_o n_o) + \frac{1}{2} \frac{\sin 4kT'}{4kT'} - \right.$$

$$\left. - \frac{1}{4} \frac{\pi}{\varpi T'} \frac{\sin 4kT'}{\sin\left(\frac{4k\pi}{\varpi}\right)} \left[J_{\frac{4k}{\varpi}}(æ_o n_o) + J_{-\frac{4k}{\varpi}}(æ_o n_o) \right] \right| . \qquad (8)$$

We can show according to [5] that

$$J_{\frac{4k}{\bar\omega}}(æ_o n_o) + J_{-\frac{4k}{\bar\omega}}(æ_o n_o) \le 2 \frac{\sin\left(\frac{4k\pi}{\bar\omega}\right)}{\frac{4k\pi}{\bar\omega}}.$$

In this case taking into consideration

$$\max\left[\frac{\sin 4kT'}{4kT'}\right] = 1$$

$$\min\left[\frac{\sin 4kT'}{4kT'}\right] \approx -0.2 \qquad (9)$$

We shall receive from (8)

$$\max[V] = \left| \frac{1}{2} + \frac{1}{2} + \frac{1}{2} - \frac{1}{2} \right| = 1$$

$$\min[V] \approx \left| \frac{1}{2} + \frac{1}{2}(-0.4) + \frac{1}{2}(-0.2) - \frac{1}{2} 0.2 \right| = 0.3 .$$

Thus for the model:

$$0.3 \le \boldsymbol{V} \le 1. \qquad (10)$$

The similar results can be received in the approach of geometrical optics, when $\lambda_o \to 0$. According to [4,5]

$$J_o(æ_0 n_o) \approx \sqrt{\frac{2}{\pi æ_0 n_o}} \cos\left(æ_0 n_o - \frac{\pi}{4}\right) \to 0,$$

$$J_{\frac{4k}{\overline{\omega}}}(æ_0 n_o) \approx \sqrt{\frac{2}{\pi æ_0 n_o}} \cos\left(æ_0 n_o - \frac{2k\pi}{\overline{\omega}} - \frac{\pi}{4}\right) \to 0,$$

$$J_{-\frac{4k}{\overline{\omega}}}(æ_0 n_o) \approx \sqrt{\frac{2}{\pi æ_0 n_o}} \cos\left(æ_0 n_o + \frac{2k\pi}{\overline{\omega}} - \frac{\pi}{4}\right) \to 0,$$

consequently for the degree of polarization we shall have from (8)

$$V \approx \frac{1}{2}\left(1 + \frac{\sin 4kT'}{4kT'}\right), \tag{11}$$

where using (9) we have:

$$0.4 \le \boldsymbol{V} \le 1,$$

which does not contradict (10).

Both in general case (8) and in the approach of geometrical optics (11) in case of final T' the device is stationary when k = 0, then it is evident V = 1 and the polarization device does not influence the degree of polarization of radiation.

Thus, the described method of the definition of the degree of partial polarization of light can be used for any device if the time profile of its nonstationarity is known. The inverse problem can also be considered – the definition of the kind of device's nonstationarity in case of full depolarization of light, which has passed through the device which is supposed to be carried out in future.

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REFERENCES

1. M.Born, E.Wolf. Principles of Optics. 1973. (Russian).
2. Sh. Kakichashvili. The Journal of Technical Physics, **65**, 1995, 200 (Russian).
3. Sh. Kakichashvili, A. Purtseladze. Letter to Journal of Technical Physics, **25**, 1999, 74 (Russian).
4. E.Yanke, F.Emde, F.Lesh. Special Functions. 1977, 342 (Russian).
5. Mathematical Encyclopedia. **1,** 1977, 1152 (Russian).

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