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ON THE QUESTION OF DEPOLARIZATION OF LIGHT IN NONSTATIONARY ABSORBING OBJECTS

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ABSTRACT. The change of the polarization state of light in passing through nonstationary anisotropic medium with complex birefringence which is described by the sinusoidally changing square function of time has been considered. It has been shown that with definite meanings of characteristics the medium causes a full depolarization of originally fully polarized light.

The passing of light through the nonstationary anisotropic medium results in the factor that the originally fully polarized light becomes partially polarized [1,2]. In those works the nonstationary anisotropic medium was characterized by nonstationary birefringence $\Delta n(t)$ (in a general case complex) and nonstationary angle of orientation of the axis of anisotropy $\rho(t)$, in this case the axis of anisotropy was considered with respect to the laboratory coordinate system if we admit that the axes of birefringence and anisotropic absorption coincide. [1,2] contains information about the degree of polarization of the field which has passed through the nonstationary medium in the most general way without concretizing the kind of temporal dependence the of $\Delta\hat{n}(t)$ and $\rho(t)$. As an example in the work [3] there has been considered the nonstationary medium with a sinusoidal change of coefficient of birefringence with linear change of $\rho(t)$.

In this work we consider the nonstationary anisotropic medium which is characterized by complex birefringence (taking absorption into consideration) in the form of sinusoidally changing square function of time. In this case it is admitted that $\rho(t)$ is the linear function of time. This admission is due to little change of $\rho(t)$ with change of $\Delta\hat{n}(t)$ [4]:

$$\begin{cases} \hat{n}_x + \hat{n}_y \approx 2\hat{n}_o, \\ \Delta\hat{n}(t) = \hat{n}_o A\cos^2 \omega t, \qquad \rho(t) = kt, \qquad t < T', \\ \Delta\hat{n}(t) = 0, \quad \rho(t) = 0, \qquad t > T', \end{cases} \tag{1}$$

where $\hat{n}_o = \hat{n}_o - in\tau_o$ is the initially complex refraction coefficient; , are complex refraction coefficients according to the respective axes (n_o, n_x, n_y are real refraction coefficients, τ_o, τ_x, τ_y are extinction coefficients); A, ω, k are characteristics of the nonstationary medium depending on the kind of the medium; T' in the time of observation. Here $\Delta\hat{n}(t)$ and $\rho(t)$ are sufficiently quick functions of time which are comparable with optical frequency.

Let the fully elliptically polarized wave E with frequency ω_0 with the orientation of the major axis of the ellipse along the axis X of the laboratory coordinate system which is spreading along the axis Z be incident on the nonstationary anisotropic medium. When passing through this medium the wave is partially depolarized and the degree of the polarization of the past field is defined by the ratio [5]

$$V = \frac{I_{pol}}{I_{pol} + I_{nonpol}}, \qquad 0 \le V \le 1, \tag{2}$$

when I_{pol} is the intensity of the polarized part, I_{nonpol} is the intensity of the nonpolarized part.

The expression of the degree of polarization in the most general way without taking into consideration the apparent kind of temporal dependence for the characteristics of the nonstationary medium has the form [2]:

$$V = \frac{\int\limits_{-\infty}^{+\infty} \sin^2 2\rho(t)\left[\mathrm{sh}^2\left(\frac{æ_o d}{2}\Delta n\tau(t)\right) + \sin^2\left(\frac{æ_o d}{2}\Delta n(t)\right)\right]dt}{\int\limits_{-\infty}^{+\infty}\left(1 + 2\,\mathrm{sh}^2\left(\frac{æ_o d}{2}\Delta n\tau(t)\right)\right)dt + \frac{1-\varepsilon^2}{1+\varepsilon^2}\int\limits_{-\infty}^{+\infty}\mathrm{sh}\left(æ_o d\Delta n\tau(t)\right)\cos 2\rho(t)dt}, \tag{3}$$

where $E = E_x \exp i(\omega_o t - æ_o z)\begin{pmatrix} 1 \\ \pm i\varepsilon \end{pmatrix} \left(0 \le \varepsilon = \frac{E_y}{E_x} \le 1\right)$ is a wave which is incident on the medium, E_x, E_y are components of electrical vector according to the respective axis; $æ_o = \frac{2\pi}{\lambda_0}$, λ_o is a wavelength; *d* is the thickness of the medium.

Substituting into (3) the evident kind of the function $\Delta\hat{n}(t)$ and $\rho(t)$ from (1) we shall receive

$$V = \frac{\int_0^{+\infty} \sin^2(2kt)[sh^2\left(\frac{æ_o d}{2} n\tau_o A\cos^2\omega t\right) + \sin^2\left(\frac{æ_o d}{2}\right) n_o A\cos^2\omega t\,]dt}{\int_0^{+\infty}\left(1 + 2sh^2\left(\frac{æ_o d}{2} n\tau_o A\cos^2\omega t\right)\right)dt - \frac{1-\varepsilon^2}{1+\varepsilon^2}\int_0^{+\infty} sh\left(æ_o d n\tau_o A\cos^2\omega t\right)\cos(2kt)dt} \tag{4}$$

Having solved the integrals which enter into (4) for the degree of polarization we shall receive

$$V = \frac{I_1 - I_2 - I_3 + I_4}{4\left(I_1 - \frac{1-\varepsilon^2}{1+\varepsilon^2} I_5\right)}, \tag{5}$$

where is the solution of the integrals:

$$I_1 = \int_0^{+\infty} ch\left(æ_o d n\tau_o A\cos^2\omega t\right)dt =$$

$$= \frac{\pi}{8\omega}\left[e^{-\frac{æ_o d n\tau_o A}{2}} H_o^{(1)}\left(\frac{iæ_o d n\tau_o A}{2}\right) - e^{\frac{æ_o d n\tau_o A}{2}} H_o^{(2)}\left(\frac{iæ_o d n\tau_o A}{2}\right)\right], \tag{6}$$

$$I_2 = \int_0^{+\infty} \cos\left(æ_o dn_o A \cos^2 \omega t\right)dt =$$

$$= \frac{\pi}{8\omega}\left[e^{\frac{iæ_o dn_o A}{2}} H_o^{(1)}\left(\frac{æ_o dn_o A}{2}\right) - e^{-\frac{iæ_o dn_o A}{2}} H_o^{(2)}\left(\frac{æ_o dn_o A}{2}\right)\right], \quad (7)$$

$$I_3 = \int_0^{+\infty} \cos(4kt)\mathrm{ch}\left(æ_o dn\tau_o A \cos^2 \omega t\right)dt =$$

$$= (-1)^{k/\omega} \frac{\pi}{8\omega}\left[e^{-\frac{æ_o dn\tau_o A}{2}} H_{2k/\omega}^{(1)}\left(\frac{iæ_o dn\tau_o A}{2}\right) - \right.$$

$$\left. - e^{\frac{æ_o dn\tau_o A}{2}} H_{2k/\omega}^{(2)}\left(\frac{iæ_o dn\tau_o A}{2}\right)\right], \quad (8)$$

$$I_4 = \int_0^{+\infty} \cos(4kt)\cos\left(æ_o dn_o A \cos^2 \omega t\right)dt =$$

$$= (-1)^{k/\omega} \frac{\pi}{8\omega}\left[e^{\frac{iæ_o dn_o A}{2}} H_{2k/\omega}^{(1)}\left(\frac{æ_o dn_o A}{2}\right) - \right.$$

$$\left. - e^{-\frac{iæ_o dn_o A}{2}} H_{2k/\omega}^{(2)}\left(\frac{æ_o dn_o A}{2}\right)\right], \quad (9)$$

$$I_5 = \int_0^{+\infty} \cos(2kt)\mathrm{sh}\left(æ_o dn\tau_o A \cos^2 \omega t\right)dt =$$

$$= -(-i)^{k/\omega} \frac{\pi}{8\omega}\left[e^{-\frac{æ_o dn\tau_o A}{2}} H_{k/\omega}^{(1)}\left(\frac{iæ_o dn\tau_o A}{2}\right) + \right.$$

$$\left. + e^{\frac{æ_o dn\tau_o A}{2}} H_{k/\omega}^{(2)}\left(\frac{iæ_o dn\tau_o A}{2}\right)\right], \quad (10)$$

where $H_p^{(1)}, H_p^{(2)}$ (Hankel function) are cylindrical functions of the 3rd kind of order p [6].

Substituting into (5) the solution of the integrals in the apparent kind (6)-(10) we shall receive:

$$V = \frac{E}{4F}, \tag{11}$$

where

$$\begin{aligned} E = {} & e^{-\frac{æ_o dn\tau_o A}{2}} \left[H_0^{(1)}\left(\frac{iæ_o dn\tau_o A}{2}\right) - \right. \\ & - e^{\frac{æ_o dn\tau_o A}{2}} \left[H_0^{(2)}\left(\frac{iæ_o dn\tau_o A}{2}\right) - (-1)^{k/\omega} H_{2k/\omega}^{(2)}\left(\frac{iæ_o dn\tau_o A}{2}\right)\right] - \\ & - e^{\frac{iæ_o dn_o A}{2}} \left[H_0^{(1)}\left(\frac{æ_o dn_o A}{2}\right) - (-1)^{k/\omega} H_{2k/\omega}^{(1)}\left(\frac{æ_o dn_o A}{2}\right)\right] - \\ & - e^{-\frac{iæ_o dn_o A}{2}} \left[H_0^{(2)}\left(\frac{æ_o dn_o A}{2}\right) - (-1)^{k/\omega} H_{2k/\omega}^{(2)}\left(\frac{æ_o dn_o A}{2}\right)\right] \end{aligned} \tag{12}$$

$$\begin{aligned} F = {} & e^{-\frac{æ_o dn\tau_o A}{2}} \left[H_o^{(1)}\left(\frac{iæ_o dn\tau_o A}{2}\right) + (-i)^{k/\omega} \frac{1-\varepsilon^2}{1+\varepsilon^2} H_{2k/\omega}^{(1)}\left(\frac{iæ_o dn\tau_o A}{2}\right)\right] - \\ & - e^{\frac{æ_o dn\tau_o A}{2}} \left[H_o^{(2)}\left(\frac{iæ_o dn\tau_o A}{2}\right) - (-i)^{k/\omega} \frac{1-\varepsilon^2}{1+\varepsilon^2} H_{2k/\omega}^{(2)}\left(\frac{iæ_o dn\tau_o A}{2}\right)\right]. \end{aligned} \tag{13}$$

If we know characteristics of the anisotropic nonstationary medium and the wavelength of the incident light λ_0 then the expressions (11)-(13) allow the numerical value of the degree of the partial polarization of the light which has passed through the medium to be found with the help of Tables of cylindrical function (1).

With sufficiently big values of the argument $H_p^{(1)}$ and $H_p^{(2)}\left(\frac{æ_o dn\tau_0 A}{2} >> 1,\ \frac{æ_o dn_0 A}{2} >> 1\right)$, we can make use of an asymptotic approximation [6] and under these conditions (6)-(10) are simplified.

$$I_1 \approx \frac{\pi i}{2\omega\sqrt{\pi iæ_o dn\tau_o A}} \sin\left(iæ_o dn\tau_o A - \frac{\pi}{4}\right), \qquad (6_1)$$

$$I_2 \approx \frac{\pi i}{2\omega\sqrt{\pi æ_o dn_o A}} \sin\left(æ_o dn_o A - \frac{\pi}{4}\right), \qquad (7_1)$$

$$I_3 \approx \frac{(-1)^{k/\omega}\pi i}{2\omega\sqrt{\pi iæ_o dn\tau_o A}} \sin\left(iæ_o dn\tau_o A - \frac{\pi}{4} - \frac{k}{\omega}\pi\right), \qquad (8_1)$$

$$I_4 \approx \frac{(-1)^{k/\omega}\pi i}{2\omega\sqrt{\pi æ_o dn_o A}} \sin\left(æ_o dn_o A - \frac{\pi}{4} - \frac{k}{\omega}\pi\right), \qquad (9_1)$$

$$I_5 \approx \frac{-(-i)^{k/\omega}\pi}{2\omega\sqrt{\pi iæ_o dn\tau_o A}} \cos\left(iæ_o dn\tau_o A - \frac{\pi}{4} - \frac{k}{2\omega}\pi\right). \qquad (10_1)$$

In this case for the degree of polarization from (5) we shall receive:

$$V \approx \frac{\sin\left(i\text{æ}_o dn\tau_o A-\frac{\pi}{4}\right)-(-1)^{k/\omega}\sin\left(i\text{æ}_o dn\tau_o A-\frac{\pi}{4}-\frac{k}{\omega}\pi\right)}{\sin\left(i\text{æ}_o dn\tau_o A-\frac{\pi}{4}\right)+(-i)^{k/\omega+1}\frac{1-\varepsilon^2}{1+\varepsilon^2}\cos\left(i\text{æ}_o dn\tau_o A-\frac{\pi}{4}-\frac{k}{2\omega}\pi\right)}-$$

$$-\sqrt{\frac{in\tau_o}{n_o}}\frac{\sin\left(\text{æ}_o dn_o A-\frac{\pi}{4}\right)-(-1)^{k/\omega}\sin\left(\text{æ}_o dn_o A-\frac{\pi}{4}-\frac{k}{\omega}\pi\right)}{\sin\left(i\text{æ}_o dn\tau_o A-\frac{\pi}{4}\right)+(-i)^{k/\omega+1}\frac{1-\varepsilon^2}{1+\varepsilon^2}\cos\left(i\text{æ}_o dn\tau_o A-\frac{\pi}{4}-\frac{k}{2\omega}\pi\right)} \quad (14)$$

If $\frac{k}{\omega}\to 0$ then from (14) V = 0 which suggests the fully polarized light which has passed through the medium (1) is fully depolarized.

The analog of the medium which is described in (1) can be the device of the Kerr cell type.

In conclusion we shall note that later the experimental test of the results received in this work with the purpose of creating a device, which fully depolarizes polarized radiation, is expected.

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დასკვნა

განხილულია სინათლის პოლარიზაციის მდგომარეობის ცვლილება არასტაციონარულ ანიზოტროპულ არეში გავლის შემდეგ. ორმაგი სხივთტეხის კომპლექსური კოეფიციენტი აღწერილია დროში სინუსოიდალური ცვლილების კვადრატული ფუნქციით. ნაჩვენებია, რომ მახასიათებლების გარკვეული მნიშვნელობებისათვის მოცემული არე იწვევს თავდაპირველად სრულად პოლარიზებული სინათლის სრულ დეპოლარიზაციას.