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1BOUND $q\bar{q}$ SYSTEMS IN THE FRAMEWORK OF TWO-BODY DIRAC EQUATIONS OBTAINED FROM DIFFERENT VERSIONS OF 3D-REDUCTIONS OF THE BETHE-SALPETER EQUATION

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ABSTRACT. The two-body Dirac equations for bound $q\bar{q}$ systems are obtained from the different (five) versions of the 3D-equations derived from Bethe-Salpeter equation with the instantaneous kernel in the momentum space using the additional approximations. There are formulated the normalization conditions for the wave functions satisfying the obtained two-body Dirac equations. The spin structure of the confining $q\bar{q}$ interaction is taken in the form , with $0 \leq x \leq 1$. It is shown that the two-body Dirac equations obtained from the Salpeter equation does not depend on *x*. As to other four versions such dependence is left. For the systems $(u\bar{s})$, $(c\bar{u})$, $(c\bar{s})$ the dependence of the stable solutions of the Dirac equations obtained in the different version on the mixture parameter *x* is investigated. Results are compared with such dependence of 3D-equations derived from Bethe-Salpeter equations without the additional approximation and some new conclusions are obtained.

1. INTRODUCTION

The Bethe-Salpeter (BS) equation provides natural basis for the relativistic treatment of bound $q\bar{q}$ systems in the framework of the constituent quark model. But due to fact that the BS wave function (amplitude) has not probability interpretation, three-dimensional (3D) reduction is necessary. Review of investigations of bound $q\bar{q}$ systems (mesons) on basis of equations for the wave function obtained in different versions of 3D-reduction of BS equation in the instantaneous (static) approximation for kernel of BS equation is given in Ref. 7. In literature there are known five such versions formulated in Refs. \cite{b1}-\cite{b7}, below noted as SAL 7, GR 7, MW 7, CJ 7 and MNK 7, 7, versions. The last four 3D-equations have correct one-body limit (the Dirac equation) when the mass of one of the particles tends to infinity. As it is well-known the Salpeter has not such a limit. Note that Gross equation is obtained only for $m_1 \neq m_2$ case, while other versions work for the equal masses ($m_1 = m_2$) too.

In our previous papers [7-9] the dependence of the existence of the stable solutions of above mentioned 3D-equations on the Lorentz (spin) structure of $q\bar{q}$-confining interaction potential was investigated. In the literature (see e.g [10,11]) this problem was considered in the framework of two-body Dirac equation (TBDE). There arises the problem, what kind of Lorentz (spin) structure must be used in the TBDE. It seems theoretically natural to begin from the above mentioned 3D-relativistic equations obtained from BS equation and use some additional kinematical approximations. Below such approach is used for derivation of the TBDE for wave function of bound $q\bar{q}$ systems and the corresponding normalization conditions for wave function are formulated.

Then these equations are used for investigation of some aspects of the problem connected to the mass spectra of $q\bar{q}$ bound systems (mesons), namely, dependence of the existence of the stable solutions of these equations and mass spectra on the Lorentz (spin) structure of $q\bar{q}$ interaction potential.

2

32.THE TWO BODY DIRAC EQUATION FOR BOUND $q\bar{q}$ SYSTEMS AND NORMALIZATION CONDITIONS FOR THE CORRESPONDING WAVE FUNCTION

To derive such an equation note that all 3D-equations given in Ref.1 can be written in the common form (c.m.f.)

$$\left[M - h_1(\mathbf{p}) - h_2(-\mathbf{p})\right]\widetilde{\Phi}_M(\mathbf{p}) =$$
$$= \Pi(M;\mathbf{p})\gamma_1^0 \otimes \gamma_2^0 \int \frac{d^3\mathbf{p}'}{(2\pi)^3} V(\mathbf{p},\mathbf{p}')\widetilde{\Phi}_M(\mathbf{p}') \qquad (2.1)$$

where

$$\Pi(M;p) = \begin{cases} \frac{1}{2}\left(\frac{h_1}{\omega_1} + \frac{h_2}{\omega_2}\right), & \text{(SAL)} \\ \frac{1}{2}\left(1 + \frac{h_1}{\omega_1}\right), & \text{(GR)} \\ \frac{1}{2}\left(\frac{h_1}{\omega_1} + \frac{h_2}{\omega_2}\right) + \frac{M}{\omega_1 + \omega_2}\left(1 + \frac{h_1}{\omega_1} \otimes \frac{h_2}{\omega_2}\right), & \text{(MW)} \\ \frac{1}{2}\left[\frac{M + h_1 + h_2}{\omega_1 + \omega_2}\right], & \text{(CJ)} \\ \frac{1}{2}\left[\frac{M\left(a - p_0^{(+)2}\right) - p_0^{(+)} M\left(h_1 - h_2\right) + B\left(h_1 + h_2\right)}{BR}\right], & \text{(MNK)} \end{cases} \qquad (2.2)$$

$$a = E_1^2 - E_2^2 = \frac{1}{4}\left(M^2 + b_0^2 - 2(\omega_1^2 - \omega_2^2)\right), \quad b_0 = E_1 - E_2,$$

$$E_{\binom{1}{2}} = \frac{M}{2}\left(1 \pm d_{12}\right)$$

$$d_{12} = \frac{m_1^2 - m_2^2}{M^2}, \qquad p_0^{(+)} = \frac{R - b}{2y}, \qquad y = \frac{m_1 - m_2}{m_1 + m_2}$$

$$, \qquad b = M + b_0 y,$$

$$h_i = \boldsymbol{\alpha}_{\mathbf{i}}\mathbf{p}_{\mathbf{i}} + m_i\gamma_i^0, \quad \omega_i = \sqrt{m_i^2 + p_i^2}, \quad O_1 = O_1 \otimes I_2, \quad O_1 = I_1 \otimes O_2.$$

Note that from the eq.(2.1) with the operators Π (2.2) immediately the system of equations (3.61) in Ref.1 follows with definition (3.62,63), if eq. (2.1) is multiplied from left by projection operator $\Lambda_{12}^{(\alpha_1\alpha_2)}$ and their properties are used:

$$\Lambda_{12}^{(\alpha_1\alpha_2)} = \Lambda_1^{(\alpha_1)} \otimes \Lambda_2^{(\alpha_2)}, \Lambda_i^{(\alpha_i)} = \frac{\omega_i + \alpha_i h_i}{2\omega_i}, \Lambda_i^{(\alpha_i)}\Lambda_i^{(\beta_i)} = \delta_{(\alpha_i\beta_i)}\Lambda_i^{(\alpha_i)}$$

$$\Pi^{SAL} = \Lambda_{12}^{(++)} - \Lambda_{12}^{(--)} = \frac{1}{2}\left(\frac{h_1}{\omega_1} + \frac{h_2}{\omega_2}\right),$$

$$\Pi^{GR} = \Lambda_{12}^{(++)} + \Lambda_{12}^{(+-)} = \frac{1}{2}\left(1 + \frac{h_1}{\omega_1}\right) \quad (2.3)$$

Now if in the operator Π^{SAL} we use the approximation

$$\frac{h_i}{\omega_i} \Rightarrow \left.\frac{h_i}{\omega_i}\right|^{\mathbf{p}_i \to 0} = \gamma_i^0 \quad (2.4)$$

then the TBDE is obtained:

$$[M - h_1(\mathbf{p}) - h_2(-\mathbf{p})]\Psi_M(\mathbf{p}) =$$

$$= \Pi_0^{SAL}\gamma_1^0 \otimes \gamma_2^0 \int \frac{d^3\mathbf{p'}}{(2\pi)^3} V(\mathbf{p},\mathbf{p'})\Psi_M(\mathbf{p'}), \quad (2.5)$$

$$\Pi_0^{SAL} = \frac{1}{2}\left(\gamma_1^0 + \gamma_2^0\right) \quad (2.6)$$

This has already been used in coordinate space for bound $q\bar{q}$ systems in Refs.10, 11 and corresponds to Lorentz (spin) structure of confining potential (see below(2.14)).

In approximation (2.4) from (2.2) it follows

$$\Pi_0^{GR} = \frac{1}{2}\left(1 + \gamma_1^0\right) . \qquad (2.7)$$

As to MW, CJ and MNK versions for derivation of corresponding TBDE the additional to (2.4) approximation is need, namely,

$$\Pi(M;0) \Rightarrow \Pi(m_1 + m_1;0) \equiv \Pi_0 , \qquad (2.8)$$

which is quite natural because it corresponds to zero approximation in iteration procedure for solving nonlinear over M eq. (2.1) for the MW, CJ and MNK versions. As a result from (2.2) it follows

$$\Pi_0^{MW} = \frac{1}{2}\left[\gamma_1^0 + \gamma_2^0 + (1 - \gamma_1^0 \otimes \gamma_2^0)\right] \qquad (2.9)$$

$$\Pi_0 = \frac{1}{2}\left[1 + \mu_1\gamma_1^0 + \mu_2\gamma_2^0\right], \quad \begin{array}{ll} \mu_i = \dfrac{m_i}{m_1 + m_2}, & \text{(CJ)} \\ \mu_i = \dfrac{m_i^2}{m_1^2 + m_2^2}. & \text{(MNK)} \end{array} \qquad (2.10)$$

Thus, we have the following TBDE obtained from, (2.1), (2.2)

$$\left[M - h_1(p) - h_2(-p)\right]\Psi_M(p) = = \Pi_0\gamma_1^0 \otimes \gamma_2^0 \int \frac{d^3p'}{(2\pi)^3} V(p,p')\Psi_M(p') , \qquad (2.11)$$

where the operator Π_0 is given by the formulae (2.6), (2.7), (2.9), (2.10).

Note that there is the another approach for formulation the TBDE, namely, generation of the one-body Dirac equation to two-body one, using constrain dynamics and relation to quantum field theory. Review of such an approach is given in Ref.12.

Representing the wave function $\Psi_M(\mathbf{p})$ as sum of "frequency" components

$$\Psi_M(\mathbf{p}) = \sum_{\alpha_1\alpha_2} \Lambda_{12}^{(\alpha_1\alpha_2)}(\mathbf{p})\Psi_M(\mathbf{p}) = \sum_{\alpha_1\alpha_2} \Psi_M^{(\alpha_1\alpha_2)}(\mathbf{p}) \qquad (2.12)$$

from the eq. (2.11) the system of the equation for the functions $\Psi_M^{(\alpha_1\alpha_2)}(\mathbf{p})$ follows

.

$$\left[M - (\alpha_1\omega_1 + \alpha_2\omega_2)\right]\Psi_M^{(\alpha_1\alpha_2)}(\mathbf{p}) = \Lambda_{12}^{(\alpha_1\alpha_2)}\Pi_0\gamma_1^0 \otimes \gamma_2^0$$

$$\int \frac{d^3\mathbf{p'}}{(2\pi)^3} V(\mathbf{p},\mathbf{p'}) \sum_{\alpha_1'\alpha_2'} \Psi_M^{(\alpha_1'\alpha_2')}(\mathbf{p'}). \qquad (2.13)$$

Taking the $q\bar{q}$ interaction operator V in the form 7 (combination of one-gluon exchange and confining part of potential)

$$V = \gamma_1^0 \otimes \gamma_2^0 V_{OG} + \left[x\gamma_1^0 \otimes \gamma_2^0 + (1-x)I_1 \otimes I_2\right]V_C \qquad (2.14)$$

and representing the function $\Psi_M^{(\alpha_1\alpha_2)}(\mathbf{p})$ as

$$\Psi_M^{(\alpha_1\alpha_2)}(\mathbf{p}) = \qquad (2.15)$$

$$= N_{12}^{(\alpha_1\alpha_2)}(p)\left[\begin{pmatrix} 1 \\ \dfrac{\alpha_1\boldsymbol{\sigma_1 p}}{\omega_1+\alpha_1 m_1}\end{pmatrix}\otimes\begin{pmatrix} 1 \\ \dfrac{-\alpha_2\boldsymbol{\sigma_2 p}}{\omega_2+\alpha_2 m_2}\end{pmatrix}\equiv f_{12}^{(\alpha_1\alpha_2)}(\mathbf{p})\right]\chi_M^{(\alpha_1\alpha_2)}(\mathbf{p}),$$

where

$$N_{12}^{(\alpha_1\alpha_2)}(p)=\sqrt{\frac{\omega_1+\alpha_1 m_1}{2\omega_1}}\sqrt{\frac{\omega_2+\alpha_2 m_2}{2\omega_2}}, \qquad (2.16)$$

then for the wave functions $\chi_M^{(\alpha_1\alpha_2)}(\mathbf{p})$ from (2.13) can be obtained the following system of equations

$$\left[M-(\alpha_1\omega_1+\alpha_2\omega_2)\right]\chi_M^{(\alpha_1\alpha_2)}(\mathbf{p})=$$

$$=\sum_{\alpha_1'\alpha_2'}\int\frac{d^3\mathbf{p'}}{(2\pi)^3}V_{eff}^{(\alpha_1\alpha_2\alpha_1'\alpha_2')}(\mathbf{p},\mathbf{p'})\chi_M^{(\alpha_1'\alpha_2')}(\mathbf{p'}) \qquad (2.17)$$

where

$$V_{eff}^{(\alpha_1\alpha_2\alpha_1'\alpha_2')}(\mathbf{p},\mathbf{p'})=N_{12}^{(\alpha_1\alpha_2)}(\mathbf{p})B^{(\alpha_1\alpha_2\alpha_1'\alpha_2')}(\mathbf{p},\mathbf{p'})N_{12}^{(\alpha_1'\alpha_2')}(\mathbf{p'}), \quad (2.18)$$

$$\left[1-\frac{\alpha_1\alpha_2\alpha_1'\alpha_2'(\boldsymbol{\sigma_1 p})(\boldsymbol{\sigma_2 p})(\boldsymbol{\sigma_1 p'})(\boldsymbol{\sigma_2 p'})}{(\omega_1+\alpha_1 m_1)(\omega_2+\alpha_2 m_2)(\omega_1'+\alpha_1' m_1)(\omega_2'+\alpha_2' m_2)}\right]V_1(\mathbf{p},\mathbf{p'}), \text{ (SAL)}$$

(2.19)

$$=\left[V_1(\mathbf{p},\mathbf{p}')+\frac{\alpha_2\alpha_2'(\boldsymbol{\sigma}_2\mathbf{p})(\boldsymbol{\sigma}_2\mathbf{p}')}{(\omega_2+\alpha_2 m_2)(\omega_2'+\alpha_2' m_2)}V_2(x;\mathbf{p},\mathbf{p}')\right],\ (GR) \qquad (2.20)$$

$$=\left[1-\frac{\alpha_1\alpha_2\alpha_1'\alpha_2'(\boldsymbol{\sigma}_1\mathbf{p})(\boldsymbol{\sigma}_2\mathbf{p})(\boldsymbol{\sigma}_1\mathbf{p}')(\boldsymbol{\sigma}_2\mathbf{p}')}{(\omega_1+\alpha_1 m_1)(\omega_2+\alpha_2 m_2)(\omega_1'+\alpha_1' m_1)(\omega_2'+\alpha_2' m_2)}\right]V_1(\mathbf{p},\mathbf{p}')+$$

$$\left[+\frac{\alpha_1\alpha_1'(\boldsymbol{\sigma}_1\mathbf{p})(\boldsymbol{\sigma}_1\mathbf{p}')}{(\omega_1+\alpha_1 m_1)(\omega_1'+\alpha_1' m_1)}+\right.$$

$$\left.+\frac{\alpha_2\alpha_2'(\boldsymbol{\sigma}_2\mathbf{p})(\boldsymbol{\sigma}_2\mathbf{p}')}{(\omega_2+\alpha_2 m_2)(\omega_2'+\alpha_2' m_2)}\right]V_2(x;\mathbf{p},\mathbf{p}'),\ (MW) \qquad (2.21)$$

$$B^{(\alpha_1\alpha_2\alpha_1'\alpha_2')}(\mathbf{p},\mathbf{p}')=V_1(\mathbf{p},\mathbf{p}')+\left[\frac{\alpha_1\alpha_1'(\boldsymbol{\sigma}_1\mathbf{p})(\boldsymbol{\sigma}_1\mathbf{p}')}{(\omega_1+\alpha_1 m_1)(\omega_1'+\alpha_1' m_1)}\mu_2+\right.$$

$$\left.+\frac{\alpha_2\alpha_2'(\boldsymbol{\sigma}_2\mathbf{p})(\boldsymbol{\sigma}_2\mathbf{p}')}{(\omega_2+\alpha_2 m_2)(\omega_2'+\alpha_2' m_2)}\mu_1\right]V_2(x;\mathbf{p},\mathbf{p}'), \qquad (2.22)$$

$$(CJ, MNK)$$

$$\omega_i'=\sqrt{m_i^2+\mathbf{p}'^2},\ V_1(\mathbf{p},\mathbf{p}')=V_{OG}(\mathbf{p},\mathbf{p}')+V_C(\mathbf{p},\mathbf{p}'),$$

$$V_2(x;\mathbf{p},\mathbf{p}')=V_{OG}(\mathbf{p},\mathbf{p}')+(2x-1)V_C(\mathbf{p},\mathbf{p}'). \qquad (2.23)$$

It is very important that the TBDE (2.17) with effective potential (2.18) with (2.19) obtained from the equation (2.1), corresponding to SAL version (2.2) does not depend on parameter x interned in the interaction operator (2.14), which means that from this equation no information can be obtained on the Lorentz (spin) structure of the confining $q\bar{q}$ interaction potential (2.14). Second interesting result is that the wave functions satisfying the TBDE (2.17) with effective potentials defined by formulae (2.18) with expression(2.19), (2.20), obtained for SAL and GR versions (2.2) of the 3D-relativistic equations have all nonzero "frequency components" whereas two components of the wave functions satisfying the equation (2.1) with projection operators (2.2), are zero, namely:

(2.24)

which directly follows (and is well known) from the eq.(2.1) if it is multiplied (from left) by the operators $\Lambda_{12}^{(\pm\mu)}$, $\Lambda_{12}^{(-\pm)}$ and used the formulae (2.3).

For formulation of normalization condition for the wave function (2.12) which satisfies the equation (2.11), we note that normalization condition for Salpeter wave function obtained in Ref.1 (see (3.14)) can be written in the form

(2.25)

The analogous condition can be derived for wave function satisfying Gross equation (2.1) (2.2) if we use equation for full Green operator corresponding to the equation (2.1).

$$\widetilde{G} = g_0 \Pi \Gamma_0 + g_0 \widetilde{U} \widetilde{G}, \qquad g_0 = \left[M - h_1 - h_2\right]^{-1}, \tag{2.26}$$

Assuming that the operator $\widetilde{G}^{-1}$ exists (being natural at any rate in the bound states, we need) from eq. (2.26) after some transformations the following relation can be obtained

$$\tilde{G}\Gamma_0\Pi\left[g_0^{-1} - \tilde{U}\right]\tilde{G}\Gamma_0 = \tilde{G}\Gamma_0\Pi\Pi\,. \tag{2.27}$$

Noting that from (\ref{eq.20}) we have

$$\tilde{G}\Gamma_0\Pi^{GR}\left[g_0^{-1} - \tilde{U}\right]\tilde{G}\Gamma_0\Pi^{GR} = \tilde{G}\Gamma_0\Pi^{GR}. \tag{2.28}$$

Now using the spectral representation of Green operator $\tilde{G}$

$$\tilde{G}(\mathbf{p}) = \sum_B \frac{\left|\tilde{\Phi}_{P_B}\right\rangle\left\langle\tilde{\bar{\Phi}}_{P_B}\right|}{P^2 - M_B^2} + \tilde{R}(P), \left\langle\tilde{\bar{\Phi}}_{P_B}\right| = \left\langle\tilde{\Phi}_{P_B}\right|\Gamma_0, \tag{2.29}$$

from (2.28) it can be obtained the relation

$$\left\langle\tilde{\Phi}_M\right|\Pi^{GR}\left|\tilde{\Phi}_M\right\rangle\Pi^{GR} = 2M\Pi^{GR}. \tag{2.30}$$

It means that the normalization condition analogous to (2.25)

$$\Big| \tag{2.31}$$

holds only in corresponding subspace of the Gilbert space. Note that the condition (2.31) can be obtained from the formula (3.28) of ref.1, which was not derived, but supposed with an analogy to (3.14).

Now, noting that the TBDE (2.11) for the SAL and GR versions of the 3D-relativistic equation (2.1) were obtained in the approximation (2.4) for the projection operators Π^{SAL} and Π^{GR}, the corresponding condition for wave function can be obtained from (2.25), (2.31) by replacement (2.6) and $\Pi^{GR} \Rightarrow \Pi_0^{GR}$ (2.7). Thus we have

|, (2.32)

where Π_0^{SAL} and Π_0^{GR} are given by formulae (2.6), (2.7).

Further, noting that the projection operator Π_0^{MW} (2.9) satisfies condition , from relation (2.27) can be obtained the normalization condition analogous to (2.32) i.e.

| (2.33)

As to normalization conditions for wave functions satisfying the TBDE (2.11), corresponding to the CJ and MNK versions, they can not be derived analogously because the corresponding projection operators Π_0 (2.10) does not satisfy the conditions $\Pi_0\Pi_0 = \Pi_0$ or $\Pi_0\Pi_0 = 1$. But bellow we assume (suppose) that the condition analogous to (\ref{eq.25}) can be written in common form

|, (2.34)

where operator Π_0 is given by the formulae (2.6), (2.7), (2.9), (2.10) for all versions. As a result with an account of the formulae (2.12), (2.15), (2.16) the normalization condition for the components of the wave functions $\chi_M^{(\alpha_1\alpha_2)}$ takes the form

$$\sum_{\alpha_1\alpha_2\beta_1\beta_2} \left\langle \chi_M^{(\alpha_1\alpha_2)} \right| N_{12}^{(\alpha_1\alpha_2)} f_{12}^{(\alpha_1\alpha_2)+} \Pi_0 f_{12}^{(\beta_1\beta_2)} N_{12}^{(\beta_1\beta_2)} \left| \chi_M^{(\beta_1\beta_2)} \right\rangle = 2M \quad (2.35)$$

from which follows

$$\int \frac{d^3p}{(2\pi)} \sum_{\alpha_1\alpha_2\beta_1\beta_2} \frac{1}{4}[E_{12}^{(\alpha_1\alpha_2\beta_1\beta_2)} \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix} +$$

$$+\alpha_1\beta_1 E_{12}^{(-\alpha_1\alpha_2-\beta_1\beta_2)} \begin{pmatrix} 0 \\ 0 \\ 1 \\ \mu_2 \end{pmatrix} + \alpha_2\beta_2 E_{12}^{(\alpha_1-\alpha_2\beta_1-\beta_2)} \begin{pmatrix} 0 \\ 1 \\ 1 \\ \mu_1 \end{pmatrix} - \qquad (2.36)$$

$$-\alpha_1\alpha_2\beta_1\beta_2 E_{12}^{(-\alpha_1-\alpha_2-\beta_1-\beta_2)} \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}] \chi_M^{(\alpha_1\alpha_2)*}(\mathbf{p})\chi_M^{(\beta_1\beta_2)}(\mathbf{p}) = 2M, \begin{pmatrix} SAL \\ GR \\ MW \\ CJ, MNK \end{pmatrix},$$

where

$$E_{12}^{(\alpha_1\alpha_2\beta_1\beta_2)} = \sqrt{(1+\alpha_1\frac{m_1}{\omega_1})(1+\alpha_2\frac{m_2}{\omega_2})(1+\beta_1\frac{m_1}{\omega_1})(1+\beta_2\frac{m_2}{\omega_2})}. \quad (2.37)$$

Now we use the partial-wave expansion for the function $\chi_M^{(\alpha_1\alpha_2)}(\mathbf{p})$ [1]

$$\chi_M^{(\alpha_1\alpha_2)}(\mathbf{p}) = \sum_{LSJM_J} \langle \hat{\mathbf{n}} | LSJM_J \rangle R_{LSJ}^{(\alpha_1\alpha_2)}(p) \equiv \sum_{JM_J} \chi_{JM_J}^{(\alpha_1\alpha_2)}(\mathbf{p}); \left(\hat{\mathbf{n}} = \frac{\mathbf{p}}{p} \right).$$
(2.38)

where $R_{LSJ}^{(\alpha_1\alpha_2)}(p)$ are corresponding radial wave functions. And the potential functions $V_{OG}(\mathbf{p},\mathbf{p}')$, $V_C(\mathbf{p},\mathbf{p}')$ are represented in form (local potentials)

$$V(\mathbf{p}-\mathbf{p}') = (2\pi)^3 \sum_{\bar{L}\bar{S}\bar{J}\bar{M}_{\bar{J}}} V^{\bar{L}}(p,p') \langle \mathbf{n} | \bar{L}\bar{S}\bar{J}\bar{M}_{\bar{J}} \rangle \langle \bar{L}\bar{S}\bar{J}\bar{M}_{\bar{J}} | \mathbf{n}' \rangle, \quad (2.39)$$

where

$$V^{\bar{L}}(p,p') = \frac{2}{\pi}\int_0^\infty j_{\bar{L}}(pr)V(r)j_{\bar{L}}(p'r)r^2 dr, \tag{2.40}$$

$j_{\bar{L}}(x)$ being the spherical Bessel function. Then from the system of equations (2.17), the effective potentials of which are defined by the formulae (2.18)-(2.23) we obtain the following system of equations for the radial functions $R^{(\alpha_1\alpha_2)}_{LSJ}(p)$

SAL version

$$\left[M-(\alpha_1\omega_1+\alpha_2\omega_2)\right]R^{(\alpha_1\alpha_2)}_{J\binom{0}{1}J}(p) =$$

$$= \sum_{\alpha_1'\alpha_2'}\int_0^\infty p'^2 dp'[(N_{12}^{(\alpha_1\alpha_2)}(p)N_{12}^{(\alpha_1'\alpha_2')}(p') -$$

$$-\alpha_1\alpha_2\alpha_1'\alpha_2' N_{12}^{(-\alpha_1-\alpha_2)}(p)N_{12}^{(-\alpha_1'-\alpha_2')}(p'))V_1^J(p,p')]R^{(\alpha_1'\alpha_2')}_{J\binom{0}{1}J}(p') \tag{2.41}$$

$$\left[M-(\alpha_1\omega_1+\alpha_2\omega_2)\right]R^{(\alpha_1\alpha_2)}_{J+11J}(p) =$$

$$= \sum_{\alpha_1'\alpha_2'}\int_0^\infty p'^2 dp'\{[N_{12}^{(\alpha_1\alpha_2)}(p)N_{12}^{(\alpha_1'\alpha_2')}(p')V_1^{J\pm1}(p,p') - \tag{2.42}$$

$$-\alpha_1\alpha_2\alpha_1'\alpha_2' N_{12}^{(-\alpha_1-\alpha_2)}(p)N_{12}^{(-\alpha_1'-\alpha_2')}(p')V_{1(J\pm1)}(p,p')]R^{(\alpha_1'\alpha_2')}_{J\pm11J}(p') -$$

$$-[\alpha_1\alpha_2\alpha_1'\alpha_2' N_{12}^{(-\alpha_1-\alpha_2)}(p)N_{12}^{(-\alpha_1'-\alpha_2')}(p')\frac{2}{2J+1}V_{1(-)J}(p,p')]R^{(\alpha_1'\alpha_2')}_{J\mp11J}(p')\}$$

GR version

$$\left[M-(\alpha_1\omega_1+\alpha_2\omega_2)\right]R^{(\alpha_1\alpha_2)}_{J\binom{0}{1}J}(p) =$$

$$= \sum_{\alpha_1'\alpha_2'} \int_0^\infty p'^2 dp' \{[N_{12}^{(\alpha_1\alpha_2)}(p) N_{12}^{(\alpha_1'\alpha_2')}(p') V_1^J(p,p') +$$

$$+\alpha_2\alpha_2' N_{12}^{(\alpha_1-\alpha_2)}(p) N_{12}^{(\alpha_1'-\alpha_2')}(p') V_{2\oplus J}^{\binom{0}{1}}(x;p,p')] R_{J\binom{0}{1}J}^{(\alpha_1'\alpha_2')}(p') -$$

$$-[\alpha_2\alpha_2' N_{12}^{(\alpha_1-\alpha_2)}(p) N_{12}^{(\alpha_1'-\alpha_2')}(p') V_{2(-)J}^{\binom{0}{1}}(x;p,p')] R_{J\binom{1}{0}J}^{(\alpha_1'\alpha_2')}(p')\} \qquad (2.43)$$

$$[M - (\alpha_1\omega_1 + \alpha_2\omega_2)] R_{J\pm 11J}^{(\alpha_1\alpha_2)}(p) =$$

$$= \sum_{\alpha_1'\alpha_2'} \int_0^\infty p'^2 dp' [N_{12}^{(\alpha_1\alpha_2)}(p) N_{12}^{(\alpha_1'\alpha_2')}(p') V_1^{J\pm 1}(p,p') +$$

$$+\alpha_2\alpha_2' N_{12}^{(\alpha_1-\alpha_2)}(p) N_{12}^{(\alpha_1'-\alpha_2')}(p') V_2^J(x;p,p')] R_{J\pm 11J}^{(\alpha_1'\alpha_2')}(p') \qquad (2.44)$$

MW, CJ and MNK versions

$$[M - (\alpha_1\omega_1 + \alpha_2\omega_2)] R_{J\binom{0}{1}J}^{(\alpha_1\alpha_2)}(p) =$$

$$= \sum_{\alpha_1'\alpha_2'} \int_0^\infty p'^2 dp' \{[(N_{12}^{(\alpha_1\alpha_2)}(p) N_{12}^{(\alpha_1'\alpha_2')}(p') \begin{pmatrix} 1 \\ 1 \end{pmatrix} -$$

$$-\alpha_1\alpha_2\alpha_1'\alpha_2' N_{12}^{(-\alpha_1-\alpha_2)}(p) N_{12}^{(-\alpha_1'-\alpha_2')}(p') \begin{pmatrix} 1 \\ 0 \end{pmatrix}) V_1^J(p,p') +$$

$$+(\alpha_1\alpha_1' N_{12}^{(-\alpha_1\alpha_2)}(p) N_{12}^{(-\alpha_1'\alpha_2')}(p') \begin{pmatrix} 1 \\ \mu_2 \end{pmatrix} +$$

$$+\alpha_2\alpha_2' N_{12}^{(\alpha_1-\alpha_2)}(p) N_{12}^{(\alpha_1'-\alpha_2')}(p') \begin{pmatrix} 1 \\ \mu_1 \end{pmatrix}) V_{2\oplus J}^{\binom{0}{1}}(x;p,p')] R_{J\binom{0}{1}J}^{(\alpha_1'\alpha_2')}(p') +$$

$$+[(\alpha_1\alpha_1' N_{12}^{(-\alpha_1\alpha_2)}(p) N_{12}^{(-\alpha_1'\alpha_2')}(p') \begin{pmatrix} 1 \\ \mu_2 \end{pmatrix} -$$

$$-\alpha_2\alpha_2' N_{12}^{(\alpha_1-\alpha_2)}(p) N_{12}^{(\alpha_1'-\alpha_2')}(p') \begin{pmatrix} 1 \\ \mu_1 \end{pmatrix}) V_{2(-)J}(x;p,p')] R_{J\binom{1}{0}J}^{(\alpha_1'\alpha_2')}(p')\} \qquad (2.45)$$

$$\left[M-(\alpha_1\omega_1+\alpha_2\omega_2)\right]R^{(\alpha_1\alpha_2)}_{J\pm 11J}(p)=$$

$$=\sum_{\alpha_1'\alpha_2'}\int_0^\infty p'^2dp'\{[N^{(\alpha_1\alpha_2)}_{12}(p)N^{(\alpha_1'\alpha_2')}_{12}(p')\begin{pmatrix}1\\1\end{pmatrix}V_1^{J\pm 1}(p,p')-$$

$$-\alpha_1\alpha_2\alpha_1'\alpha_2'N^{(-\alpha_1-\alpha_2)}_{12}(p)N^{(-\alpha_1'-\alpha_2')}_{12}(p')\begin{pmatrix}1\\0\end{pmatrix}V_{1(J\pm 1)}(p,p')+$$

$$+(\alpha_1\alpha_1'N^{(-\alpha_1\alpha_2)}_{12}(p)N^{(-\alpha_1'\alpha_2')}_{12}(p')\begin{pmatrix}1\\\mu_2\end{pmatrix}+ \qquad (2.46)$$

$$+\alpha_2\alpha_2'N^{(\alpha_1-\alpha_2)}_{12}(p)N^{(\alpha_1'-\alpha_2')}_{12}(p')\begin{pmatrix}1\\\mu_1\end{pmatrix})V_2^J(x;p,p')]R^{(\alpha_1'\alpha_2')}_{J\pm 11J}(p')+$$

$$+[\alpha_1\alpha_2\alpha_1'\alpha_2'N^{(-\alpha_1-\alpha_2)}_{12}(p)N^{(-\alpha_1'-\alpha_2')}_{12}(p')\frac{2}{2J+1}V_{1(-)J}(p,p')]R^{(\alpha_1'\alpha_2')}_{J_\mu 11J}(p')\},$$

where

$$V^{\binom{0}{1}}_{n\oplus J}=\frac{1}{2J+1}\left[\begin{pmatrix}J+1\\J\end{pmatrix}V_n^{J+1}+\begin{pmatrix}J\\J+1\end{pmatrix}V_n^{J-1}\right],$$

$$V_{n(-)J}=\frac{\sqrt{J(J+1)}}{2J+1}\left[V_n^{J+1}-V_n^{J-1}\right],\quad n=1,2, \qquad (2.47)$$

$$V_{n(J\pm 1)}=\frac{1}{(2J+1)^2}\left[V_n^{J\pm 1}+4J(J+1)V_n^{J\mp 1}\right]$$

Note that if only confining potential (2.14) and parameter $x=\frac{1}{2}$ are taken into account, then equations for MW (2.45), (2.46) and SAL versions (2.41), (2.42) versions are the same. As to versions GR and CJ, MNK equations (2.43), (2.44) and (2.45), (2.46) coincide with each other and what is more, dependence of these equations on total spin (S) disappears. Dependence of spin (S) appears only if V_{OG} is taken into account, what can be seen from formulae (2.20), (2.22) too.

It is interesting to compare the system of equations (2.41)-(2.46) with the system of equations obtained from (2.1) without the approximation (2.4), (2.8) (see eqs. (4.16, 17) in [1], ignoring the terms corresponding to t'Hooft interaction)

$$
\begin{aligned}
&\left[M-(\alpha_1\omega_1+\alpha_2\omega_2)\right]R^{(\alpha_1\alpha_2)}_{J\binom{0}{1}J}(p)= \\
&=A^{(\alpha_1\alpha_2)}(M;p)\sum_{\alpha_1'\alpha_2'}\int_0^\infty p'^2dp'\{[(N_{12}^{(\alpha_1\alpha_2)}(p)N_{12}^{(\alpha_1'\alpha_2')}(p')+ \\
&+\alpha_1\alpha_2\alpha_1'\alpha_2'N_{12}^{(-\alpha_1-\alpha_2)}(p)N_{12}^{(-\alpha_1'-\alpha_2')}(p'))V_1^J(p,p')+ \\
&+(\alpha_1\alpha_1'N_{12}^{(-\alpha_1\alpha_2)}(p)N_{12}^{(-\alpha_1'\alpha_2')}(p')+ \\
&+\alpha_2\alpha_2'N_{12}^{(\alpha_1-\alpha_2)}(p)N_{12}^{(\alpha_1'-\alpha_2')}(p'))V^{\binom{0}{1}}_{2\oplus J}(x;p,p')]R^{(\alpha_1'\alpha_2')}_{J\binom{0}{1}J}(p')- \\
&-[(\alpha_1\alpha_1'N_{12}^{(-\alpha_1\alpha_2)}(p)N_{12}^{(-\alpha_1'\alpha_2')}(p')- \\
&-\alpha_2\alpha_2'N_{12}^{(\alpha_1-\alpha_2)}(p)N_{12}^{(\alpha_1'-\alpha_2')}(p'))V_{2(-)J}(x;p,p')]R^{(\alpha_1'\alpha_2')}_{J\binom{1}{0}J}(p')\}
\end{aligned}
\qquad (2.48)
$$

$$
\begin{aligned}
&\left[M-(\alpha_1\omega_1+\alpha_2\omega_2)\right]R^{(\alpha_1\alpha_2)}_{J\pm 11J}(p)= \\
&=A^{(\alpha_1\alpha_2)}(M;p)\sum_{\alpha_1'\alpha_2'}\int_0^\infty p'^2dp'\{[N_{12}^{(\alpha_1\alpha_2)}(p)N_{12}^{(\alpha_1'\alpha_2')}(p')V_1^{J\pm 1}(p,p')+ \\
&+\alpha_1\alpha_2\alpha_1'\alpha_2'N_{12}^{(-\alpha_1-\alpha_2)}(p)N_{12}^{(-\alpha_1'-\alpha_2')}(p')V_{1(J\pm 1)}(p,p')+ \\
&+(\alpha_1\alpha_1'N_{12}^{(-\alpha_1\alpha_2)}(p)N_{12}^{(-\alpha_1'\alpha_2')}(p')+ \qquad (2.49) \\
&+\alpha_2\alpha_2'N_{12}^{(\alpha_1-\alpha_2)}(p)N_{12}^{(\alpha_1'-\alpha_2')}(p'))V_2^J(x;p,p')]R^{(\alpha_1'\alpha_2')}_{J\pm 11J}(p')+ \\
&+[\alpha_1\alpha_2\alpha_1'\alpha_2'N_{12}^{(-\alpha_1-\alpha_2)}(p)N_{12}^{(-\alpha_1'-\alpha_2')}(p')\frac{2}{2J+1}V_{1(-)J}(p,p')]R^{(\alpha_1'\alpha_2')}_{J\mp 11J}(p')\},
\end{aligned}
$$

where

$$A^{(\pm\pm)} = \pm 1, \ \ A^{(\pm\mp)} = 0, \ \ (SAL); \ \ A^{(+\pm)} = \pm 1, \ \ A^{(-\pm)} = 0, \ \ (GR);$$

$$A^{(\pm\pm)} = \pm 1, \ \ A^{(\pm\mp)} = \frac{M}{\omega_1 + \omega_2}, \ \ (MW);$$

$$A^{(\alpha_1\alpha_2)} = \frac{M + (\alpha_1\omega_1 + \alpha_2\omega_2)}{2(\omega_1 + \omega_2)}, \ \ (CJ); \qquad (2.50)$$

$$A^{(\alpha_1\alpha_2)} = \frac{1}{2BR}\left[M(a - p_0^{(+)2}) - p_0^{(+)}M(\alpha_1\omega_1 - \alpha_2\omega_2) + B(\alpha_1\omega_1 + \alpha_2\omega_2)\right], \ \ (MNK).$$

Note that the last expression in (2.50) is obtained from (3.62) in [1] after some transformation.

Main difference between the system of equations (2.41)-(2.46) and (2.48), (2.49) with the expression (2.50) is following: 1) In the wave functions $R_{LSJ}^{(\alpha_1\alpha_2)}(p)$ satisfying the system of equations (2.48), (2.49) for (SAL) and (GR) versions the nonzero functions are only $R_{LSJ}^{(\pm\pm)}(p)$ and $R_{LSJ}^{(+\pm)}(p)$ respectively (about this fact was mentioned above), whereas in the corresponding system of equations (2.41), (2.42), (2.43), (2.44) all components of wave functions $R_{LSJ}^{(\alpha_1\alpha_2)}(p)$ are nonzero; 2) The system of equations (2.48), (2.49) for the MW, CJ and MNK versions are nonlinear over *M* whereas the system of equations (2.45), (2.46) are linear one; 3) Dirac equations (2.41), (2.42) obtained from the Salpeter equation do not depend on mixture parameter *x*, what can be seen directly from (2.13), (2.14).

3. PROCEDURE FOR SOLVING THE OBTAINED 4 EQUATIONS

For solving bound-state equations (2.41)-(2.46) or (2.48), (2.49), we need to specify the interaction potentials V_{OG} and V_C (2.14). Below for $V_C(r)$ we use the following form [1], [8]

$$V_C(r) = \frac{4}{3}\alpha_S(m_{12}^2)\left(\frac{\mu_{12}\omega_0^2 r^2}{2\sqrt{1 + A_0 m_1 m_2 r^2}} - V_0\right) \tag{3.1}$$

$$\alpha_S(Q^2) = \frac{12\pi}{33 - 2n_f}\left[\ln\frac{Q^2}{\Lambda^2}\right]^{-1}, m_{12} = m_1 + m_2, \mu_{12} = \frac{m_1 m_2}{m_{12}}, \tag{3.2}$$

where Q^2 is the momentum transferred and the 4/3 comes from the color-dependent part of the $q\bar{q}$ interaction, n_f is the number of flavors (n_f =3 for *u, d, s* quarks; n_f =4 for *u, d, s, c*; n_f =5 for *u, d, s, c, b*). ω_0, A_0, V_0 and Λ are considered to be the free parameters of the model. The potential given by expression (3.1) effectively reduces to the harmonic oscillator potential for the light quarks *u, d, s* and to the linear potential to the heavy *c, b* quarks is the dimensionless parameter A_0 is chosen small enough. Moreover, asymptotically, for a large *r* it is linear and almost flavor-independent. The one-gluon exchange potential is given by standard expression [1], [8]

$$V_{OG}(r) = -\frac{4}{3}\frac{\alpha_S(m_{12}^2)}{r^2}. \tag{3.3}$$

Now we have to specify the numerical procedure for solution of the systems of radial equations (2.41)-(2.46), (2.48), (2.49). A possible algorithm looks as follows: we choose the known basic functions by $R_{nL}(p)$. The unknown radial wave functions are expanded in the linear combination of the basic functions

$$R_{LSJ}^{(\alpha_1\alpha_2)}(p) = \sqrt{2M(2\pi)^3}\sum_{n=0}^{\infty} C_{nLSJ}^{(\alpha_1\alpha_2)} R_{nL}(p), \tag{3.4}$$

where $C_{nLSJ}^{(\alpha_1\alpha_2)}$ are the coefficients of the expansion. The integral equation for the radial wave functions is then transformed into the system of linear equations for these coefficients. If the transaction is carried out the finite system of equations is obtained that can be solved by using conventional numerical methods. The convergence of the whole procedure, with more terms taken into account in the expansion (3.4) depends on the successful choice of the basis. In case of the confining potential of form (3.1) it is natural to take as a basis the functions corresponding to oscillator potential, which is obtained from (3.1) at $A_0 = 0$, in non-relativistic limit of the system of equations obtained from (2.41)-2.46), (2.48),(2.49). The radial wave functions in this case have the form [1] (the formula (4.52)).

$$p_0 = \sqrt{\mu_{12}\omega_0\sqrt{\frac{4}{3}\alpha_S(m_{12}^2)}},$$

$$z = \frac{p}{p_0} \quad R_{nL}(z) = c_{nL} z^L \exp(-\frac{z^2}{2})\, {}_1F_1(-n, L+\frac{3}{2}, z^2),$$

$$c_{nL} = \sqrt{\frac{2\Gamma(n+L+\frac{3}{2})}{\Gamma(n+1)}}\frac{1}{\Gamma(L+\frac{3}{2})},$$

where ${}_1F_1$ denotes the confluent hypergeometric function.

Now, satisfying the expression (3.4) into the system of equations (2.41)–(2.46) gebraic equations for the coefficients $C_{nLSJ}^{(\alpha_1\alpha_2)}$ can be obtained

$$MC_{nLSJ}^{(\alpha_1\alpha_2)} = \sum_{\alpha_1'\alpha_2'}\sum_{n'L'S'} H_{nLSJ;n'L'S'J'}^{(\alpha_1\alpha_2;\alpha_1'\alpha_2')}(M) C_{nLSJ}^{(\alpha_1'\alpha_2')} . \qquad (3.5)$$

It is necessary to note that the matrix $H_{\alpha\beta}(M)$ depends on meson mass *M* only for MW, CJ and MNK versions as it can been seen from equations (3.6) for *M* is not linear one and therefore should be solved, e.g. by iteration. As to the system of Dirac equations (2.41)–(2.46) such a problem does not exist.

4. THE NUMERICAL RESULTS AND ADDITIONAL 5 CONCLUSIONS

The main problem we have investigated at first stage is dependence of the existence of stable solutions of the eq. (3.6) i.e. the equations (2.41)–(2.46), (2.48), (2.49) on Lorentz (spin) structure of the confining $q\bar{q}$ interaction potential, i.e. on the parameter *x*. This will be done taking as examples the $u\bar{s}$, $c\bar{u}$ and $c\bar{s}$, bound states with constituent quark masses , , and the free parameters of the confining potential (3.1), (3.2) - , , $A_0 = 0.0270$, $\Lambda = 120\text{MeV}$

Note, that in [8] only the SAL version of 3D-reduction of Bethe-Salpeter equation was considered as to MW, CJ and MNK without additional approximation (2.4) with oscillator like potential ($A_0 = 0$ in (3.1)) were considered in Refs.7, 9.

The results of the calculations are given for states ${}^{2S+1}L_J$ (note, that for cases ${}^3S_1, {}^3P_2$ are ignored additional corresponding terms ${}^3D_1, {}^3F_2$ because they give small contribution in the calculated mass).

The additional conclusions (to pure theoretical results obtained in the section 2), which can be obtained from the Tables 1, 2 are the following:

1. The area of changing of parameters *x*, for which stable solutions of two-body Dirac equations exist, is a little enlarged compared for all versions of 3D-equations.

2. For $c\bar{u}$ and $c\bar{s}$ bound systems Gross version works better what is related to the large difference of constituent masses.
3. The area of existence of the stable solutions is enlarged with increasing of the constituent masses which is theoretically understandable.
4. Masses of the bound $q\bar{q}$ systems obtained from solutions of Dirac equations are bigger then masses corresponding to 3D-equations obtained from BS equation for all versions except GR version case.
5. It is very important, that for *x*=0.5 (i.e. the equal mixture of scalar and 4th-component of vector confining $q\bar{q}$-interaction potential) the stable solutions of considered relativistic equations always exist and can be used for investigations the mass spectrum and properties of bound $q\bar{q}$ systems.

Table 1. The dependence of the $q\bar{q}$ system mass for light constituent quarks on the mixing parameter *x* in the different 3D-reductions of Bethe-Salpeter equations and corresponding Dirac equations. "*" denotes the absence of the stable solutions. Masses are given in MeV.

x	0.0	0.1	0.3	0.5	0.7	0.9	1.0
	$u\bar{s}$ $^3S_1$ (892)						
SAL	*	812	870	914	950	980	993
SALD	979						
GR	839	859	897	934	967	*	*
GRD	944	947	954	962	975	*	*
MW	863	877	907	943	983	*	*
MWD	879	887	905	928	957	*	*
CJ	878	893	924	959	998	*	*
CJD	924	930	942	955	972	*	*
MNK	814	830	861	891	*	*	*
MNKD	923	929	941	955	972	*	*

	$u\bar{s}$ (1350)						
SAL	1189	1204	1213	1210	1202	1189	1182
SALD	1349						
GR	1233	1232	1229	1223	1218	*	*
GRD	1304	1302	1300	1298	1298	*	*
MW	1255	1253	1249	1250	*	*	*
MWD	1278	1274	1267	1260	1257	*	*
CJ	1268	1267	1263	1260	1264	*	*
CJD	1296	1294	1290	1287	1284	1285	*
MNK	1217	1212	1202	1190	*	*	*
MNKD	1295	1293	1289	1286	1284	*	*
	$u\bar{s}$ $^3P_2$ (1430)						
SAL	*	*	1189	1289	1367	1430	1458
SALD	1318						
GR	1119	1159	1237	1310	1381	*	*
GRD	1278	1284	1297	1314	1336	*	*
MW	1184	1209	1262	1326	1384	*	*
MWD	1185	1200	1234	1275	1326	*	*
CJ	1181	1211	1276	1345	1421	*	*
CJD	1254	1264	1286	1310	1337	1369	1388
MNK	1137		1223	1282	1344	1408	*
MNKD	1254	1264	1285	1309	1388	1372	*

Table 2. The dependence of the $q\bar{q}$ system mass for heavy constituent quarks on the mixing parameter *x* in the different 3D-reductions of Bethe-Salpeter equations and corresponding Dirac equations. "*" denotes the absence of the stable solutions. Masses are given in MeV.

x	0.0	0.1	0.3	0.5	0.7	0.9	1.0

	$c\bar{u}\ {}^1S_0$ (1863)						
SAL	1881	1895	1920	1943	1965	1985	1994
SALD	1983						
GR	1883	1896	1921	1944	1966	1986	1995
GRD	1979	1979	1978	1978	1978	1978	1979
MW	1915	1922	1935	1951	1972	*	*
MWD	1924	1929	1942	1958	1979	*	*
CJ	1921	1928	1943	1960	1982	*	*
CJD	1932	1937	1948	1961	1978	2003	*
MNK	1928	1934	1946	1961	1978	*	*
MNKD	1927	1932	1944	1958	1977	*	*
	$c\bar{u}\ {}^3S_1$ (2010)						
SAL	1883	1897	1922	1946	1968	1988	1988
SALD	1981						
GR	1886	1899	1924	1947	1969	1989	1999
GRD	1977	1977	1978	1979	1981	1982	1983
MW	1918	1924	1938	1955	1977	*	*
MWD	1921	1926	1939	1955	1975	*	*
CJ	1923	1930	1948	1963	1981	*	*
CJD	1932	1937	1948	1961	1978	2003	*
MNK	1930	1935	1948	1963	1981	*	*
MNKD	1927	1932	1944	1958	1977	*	*
	$c\bar{s}\ {}^1S_0$ (1971)						
SAL	2020	2031	2055	2070	2088	2105	2113
SALD	2106						
GR	2023	2033	2052	2071	2089	2106	2114
GRD	2106	2100	2100	2100	2100	2100	2100
MW	2044	2050	2062	2077	2094	2118	*
MWD	2052	2058	2070	2084	2101	2126	*
CJ	2051	2057	2071	2087	2105	2126	*

CJD	2063	2067	2077	2087	2100	2116	2127
MNK	2052	2057	2069	2082	2097	2116	*
MNKD	2059	2063	2073	2085	2100	2120	*
	$c\bar{s}\ {}^3S_1$ (2107)						
SAL	2023	2033	2054	2073	2091	2108	2116
SALD	2104						
GR	2025	2035	2055	2074	2092	2110	2118
GRD	2098	2099	2100	2102	2103	2105	2106
MW	2047	2053	2065	2080	2098	2124	*
MWD	2049	2054	2066	2080	2097	2121	*
CJ	2053	2060	2074	2089	2108	*	*
CJD	2063	2067	2077	2089	2108	2116	2127
MNK	2054	2059	2071	2084	2100	2119	*
MNKD	2059	2063	2073	2085	2100	2120	*

6

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ÁÌÖËÉ $q\bar{q}$ ÓÉÓÔÄÌÄÁÉ ÏÒ-ÓáÄÖËÏÅÀÍ ÃÉÒÀÊÉÓ ÂÀÍÔÏËÄÁÄÁÉÓ ×ÀÒÂËÄÁÛÉ, ÒÏÌËÄÁÉÝ ÌÉÙÄÁÖËÉÀ ÁÄÔÄ-ÓÏËÐÉÔÄÒÉÓ ÂÀÍÔÏËÄÁÉÓ 3-ÂÀÍÆÏÌÉËÄÁÉÀÍÉ ÒÄÃÖØÝÉÉÓ ÛÄÃÄÂÀÃ ÓáÅÀÃÀÓáÅÀ ÅÀÒÉÀÍÔÄÁÛÉ ÌÉÙÄÁÖË ÂÀÍÔÏËÄÁÄÁÉÃÀÍ

ÊÅÀÒÊ-ÀÍÔÉÊÅÀÒÊÉÓ ($q\bar{q}$) ÓÉÓÔÄÌÄÁÉÓÀÈÅÉÓ ÌÉÙÄÁÖËÉÀ ÏÒ-ÓáÄÖËÏÅÀÍÉ ÃÉÒÀÊÉÓ ÂÀÍÔÏËÄÁÄÁÉ, ÁÄÔÄ-ÓÏËÐÉÔÄÒÉÓ (ÁÓ) ÂÀÍÔÏËÄÁÉÃÀÍ 3-ÂÀÍÆÏÌÉËÄÁÉÀÍÉ ÒÄÃÖØÝÉÉÓ ÛÄÃÄÂÀÃ ÌÉÙÄÁÖËÉ ÂÀÍÔÏËÄÁÄÁÉÃÀÍ áÖÈ ÓáÅÀÃÀÓáÅÀ ÅÀÒÉÀÍÔÛÉ, ÒÏÝÀ ÁÓ-ÂÀÍÔÏËÄÁÉÓ ÂÖËÉ ÂÀÍáÉËÖËÉÀ ÄÒÈÃÒÏÖË (ÓÔÀÔÉÊÖÒ) ÆÙÅÀÒÛÉ ÉÌÐÖË-ÓÖÒ ßÀÒÌÏÃÂÄÍÀÛÉ, ÒÉÓÈÅÉÓÀÝ ÂÀÌÏÚÄÍÄÁÖËÉÀ ÃÀÌÀ-ÔÄÁÉÈÉ ÁÖÍÄÁÒÉÅÉ ÌÉÀáËÏÄÁÄÁÉ. ×ÏÒÌÖËÉÒÄÁÖËÉÀ ÍÉÒÌÉÒÄÁÉÓ ÐÉÒÏÁÄÁÉ ÔÀËÙÖÒÉ ×ÖÍØÝÉÄÁÉÓÀÈÅÉÓ, ÒÏÌËÄÁÉÝ ÀÊÌÀÚÏ×ÉËÄÁÄÍ ÌÉÙÄÁÖË ÏÒ-ÓáÄÖËÏÅÀÍ ÃÉÒÀÊÉÓ ÂÀÍÔÏËÄÁÄÁÓ. $q\bar{q}$ ÖÒÈÉÄÒÈØÌÄÃÄÁÉÓ ÐÏÔÄÍÝÉÀ-

ËÉÓ ÊÏÍ×ÀÉÍÌÄÍÔÉÓ ÍÀßÉËÉÓÀÈÅÉÓ ËÏÒÄÍÝ (ÓÐÉÍÖÒÉ) ÓÔÒÖØÔÖÒÀ ÀÙÄÁÖËÉÀ ÛÄÌÃÄÂÉ ×ÏÒÌÉÈ $x\gamma_1^0 \otimes \gamma_2^0 +$ $+(1-x)I_1 \otimes I_2$, ÓÀÃÀÝ $0 \le x \le 1$. ÍÀÜÅÄÍÄÁÉÀ, ÒÏÌ ÓÏËÐÉÔÄ-ÒÉÓ ÂÀÍÔÏËÄÁიÃÀÍ ÌÉÙÄÁÖËÉ ÃÉÒÀÊÉÓ ÏÒ-ÓáÄÖËÏÅÀÍÉ ÂÀÍÔÏËÄÁÄÁÉ ÀÒÀÀ ÃÀÌÏÊÉÃÄÁÖËÉ *á*-ÆÄ. ÒÀÝ ÛÄÄáÄÁÀ ÓáÅÀ ÅÀÒÉÀÍÔÄÁÓ, ÀÓÄÈÉ ÃÀÌÏÊÉÃÄÁÖËÄÁÀ ÞÀËÀÛÉ ÒÜÄÁÀ. ÁÌÖËÉ ÓÉÓÔÄÌÄÁÉÓÀÈÅÉÓ $u\bar{s}$, $c\bar{u}$, $c\bar{s}$ ÂÀÌÏÊ-ÅËÄÖËÉÀ ÂÀÍÔÏËÄÁÄÁÉÓ ÓÔÀÁÉËÖÒÉ ÀÌÏáÓÍÄÁÉÓ ÀÒ-ÓÄÁÏÁÉÓ ÃÀÌÏÊÉÃÄÁÖËÄÁÀ *á*-ÆÄ ÃÀ ÛÄÃÀÒÄÁÖËÉÀ ÉÌ ÛÄÃÄÂÄÁÈÀÍ, ÒÏÌÄËÉÝ ÌÉÙÄÁÀ ÁÓ-ÂÀÍÔÏËÄÁÉÓ 3-ÂÀÍÆÏ-ÌÉËÄÁÉÀÍÉ ÒÄÃÖØÝÉÉÓ ÛÄÃÄÂÀÃ ÃÀÌÀÔÄÁÉÈÉ ÌÉÀáËÏÄ-ÁÉÓ ÂÀÒÄÛÄ ÃÀ ÌÉÙÄÁÖËÉÀ ÆÏÂÉÄÒÈÉ ÀáÀËÉ ÃÀÓÊÅÍა.