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ON THE UNIVERSALITY AND ASYMPTOTIC BEHAVIOR OF THE MULTIPLICITY DISTRIBUTIONS OF CHARGED HADRONS IN THE COLLISIONS AT HIGH ENERGIES IN THE FRAMEWORK OF THE CLUSTER-CASCADING MODEL

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ABSTRACT. Multiplicity distributions of charged hadrons created in the e^+e^-, PP($\bar{P}$) and A_IA_t (nucleus-nucleus) collisions at high energies are analyzed in the framework of the cluster-cascading model (CCM). It is shown that the average numbers of clusters <N> reach corresponding plateaus (it depends on the type of collision e^+e^-, PP($\bar{P}$), A_iA_t). Multiplicity distributions of particles inside cluster do not depend on the type of colliding objects and have universal character.

ITRODUCTION

Cluster-cascading model (CCM) is based on the Negative Binomial Distribution (NBD) which is of the form [1]

$$P_n(<n>,K) = \frac{\Gamma(n+K)}{n!\Gamma(K)}\left(\frac{<n>}{<n>+K}\right)^n\left(\frac{K}{<n>+K}\right)^K , \qquad (1)$$

where $<n> \equiv <n_\pm>$ is average multiplicity of charged secondary hadrons . Parameter K determines the form of the distribution.

Quantities K and <n> are related to the dispersion as follows:

$$K = \frac{<n>^2}{D^2 - <n>} . \qquad (2)$$

Recurrence relation between P_n and P_{n+1} is of the form

$$\frac{(n+1)P_n + 1}{P_n} \equiv g(n) = \alpha + \beta\, n \tag{3}$$

α and β are related to <n> and K in the following way:

$$\alpha = \frac{K < n >}{K + < n >}, \qquad \beta = \frac{< n >}{K + < n >}\ . \tag{4}$$

According to CCM the process of multiple production is interpreted as follows: after the collision of high energy objects (leptons, hadrons, nuclei) an excited multiparticle compound system is created in the form of N-cluster state; Each cluster is produced from one particle (which is produced in the initial act of the collision) called “patriarch”. One “patriarch” forms one cluster. It is assumed that “patriarch” (clusters) can be considered as resonances which formed and decaying independently from each other.

<N>- average number of clusters is given by the expression

$$< N > = \frac{< n >}{< n_c >}\ , \tag{5}$$

where $<n_e>$ is the average number of particles in the cluster

$$< n_c > = -\frac{\beta}{(1-\beta)\ln(1-\beta)}\ . \tag{6}$$

Multiplicity distribution of particles inside the cluster is given by

$$F(n_c) = \frac{-(\beta)^{n_c}}{n_c \ln(1-\beta)}\ . \tag{7}$$

The dispersion of particles in clusters is given by the expression

$$\Big|, \qquad (8)$$

where

$$< n_c^2 >= -\frac{< n_c >}{(1-\beta)} \qquad (9)$$

THE ANALYSIS OF EXPERIMENTAL DATA

In Refs. [2,3] multiplicity distributions at charged secondaries P_n in e^+e^--annihilation (in the energy range $14 \le \sqrt{s} \le 130$ GeV), PP($\bar{P}$) - collisions (in the energy range $6 \le \sqrt{s} \le 900$ GeV) and A_iA_t- collisions (in the energy range $2.48 \le E_L \le 10$ AGeV) have been considered in the framework of the CCM.

It has been shown that the average number of produced clusters < N > practically does not depend on the energy (in the energy interval considered) and on the atomic weights of colliding nuclei and is approximately equal to ~ 4.5.

Analysis of data for A_iA_t- collisions at 200 AGeV shows that the number of created clusters practically remains the same (4.77±0.25 ÷ 5.98±0.35) (see Table 1).

It has been shown [2,3] that in e^+e^- - annihilation average number of cluster <N> reaches the plateau with the value ~ 15. Analysis of data at higher energies ($\sqrt{s}$ >130 GeV) confirms the assumption on the plateau in <N> (see Table 2).

In PP($\bar{P}$) – collisions the plateau value <N> ≈ 8 is reached at $\sqrt{s}$ ≈30 Gev and this value practically does not change up to the energy $\sqrt{s}$ = 900 GeV [2,3].

Properties of the multiplicity distributions are studied usually by the analysis of the behavior of their momenta and first of all D(<n>). It is known that the dispersion increases linearly with <n>. The dependence is approximated by the formula

$$D = a + b \langle n \rangle, \qquad (10)$$

which is called Malhotra-Wroblewski formula [4]. Such a dependence takes place for different colliding objects ($\pi^{\pm}p$, $k^{\pm}p$, PP($\overline{P}$), A_iA_t) [4,5].

Parametrs in Equation (10) weakly depend on the of colliding objects. For PP($\overline{P}$) collisions in the energy interval ($3 \leq \sqrt{s} \leq 900$) GeV parameters in Eq (10) have the values: a = - 0.56 ± 0.02; b = 0.58 ± 0.01 (Table 3, Fig.1.) [4,5,6].

For the additional check the D (<n>) – dependence has been approximated by the expresion

$$D(\langle n \rangle) = a + b \langle n \rangle + c \langle n \rangle^2 \qquad (11)$$

It turned out that this fact does not make the discription better, the values of parameters a and b do not change, the value of the parameter c is practically zero (c = 0.00015 ± 0.00060). Thus the eviation from the linearity is not observed. This is in agreement with data at lower energies [4].

In Refs [2,3] the dependences D (< n >) and $D_c(\langle n_c \rangle)$ (D_c is the dispersion of the distribution of particles in the cluster, $\langle n_c \rangle$ is the average number of hadrons in the cluster) for e^+e^- , PP($\overline{P}$) and A_iA_t – collisions have been studied. It has been shown that in the energy interval studied $14 \leq \sqrt{s} \leq 130$ GeV for e^+e^- - collisions, for PP($\overline{P}$) – collisions $6 \leq \sqrt{s} \leq 900$ GeV, and for A_iA_t – collisions $E_L \leq 10$ AGeV) the dependence D (<n>) are placed on three different curves.
But the dependence $D_c(\langle n_c \rangle)$ is of the universal character – one curve for all types of interaction.

In the present paper the dependences D (< n >) and $D_c(\langle n_c \rangle)$ are analyzed in a more wide interval of energy and for more heavy nuclei – for e^+e^- -interaction three points are added ($\sqrt{s}$ =161, 189 and 200 Gev), for A_iA_t –collisions 8 points are added p(Ag, Ar, Xe, Au), S (S,Cu) and O(Cu,Au) at 200 AGeV [7-14].

Results of the approximation by the formula (10) are given in Table 3 (see Fig.1).

The D(<n>) – dependence is again presented by three curves for e^+e^-, PP($\overline{P}$) and A_lA_t–collisions (curves 1,2,3 respectively).

The values of the slope parameter b **Table 1**

Average characteristics of multyplicity distribution of secondary charged hadrons in A_iA_t – collisions at 200 AGeV

Characteristics	SS	SCu	OCu	OAu
<n>	75 ±4	97 ±5	70 ±3	98 ±5
D	72 ±3	94 ±4	69 ±4	96 ±5
K	1.1	1.08	1.05	1.05
$<n_c>$	12.53 ±0.06	19.76 ±0.10	12.53 ±0.06	20.53 ±0.11
D_c	21.66 ±0.90	37.49 ±1.59	21.67 ±1.26	39.05 ±2.03
<N>	5.98 ±0.35	4.91 ±0.30	5.56 ±0.36	4.77 ±0.25

Table 2

Characteristics of multiplicity distributions of secondary charged hadrons in e^+e^- - collisions in the energyinterval

(14 ≤ $\sqrt{s}$ ≤ 200) GeV

$(s)^{1/2}$	14	22	34	91	130	161	189	200
<n>	9.40 ±0.40	11.3 ±0.4	13.50 ±0.50	20.74 ±0.81	23.46 ±0.70	24.46 ±0.80	27.47 ±0.50	27.58 ±0.64
D	3.22 ±0.15	3.77 ±0.16	4.47 ±0.20	6.28 ±0.35	7.55 ±0.50	7.68 ±0.71	8.71 ±0.25	8.64 ±0.33
$<n_c>$	1.12 ±0.05	1.14 ±0.04	1.22 ±0.04	1.41 ±0.05	1.59 ±0.07	1.60 ±0.05	1.74 ±0.07	1.72 ±0.04

D_c	0.30 ±0.01	0.37 ±0.01	0.56 ±0.05	0.82 ±0.05	1.12 ±0.07	1.14 ±0.10	1.35 ±0.04	1.31 ±0.05
<N>	8.40 ±0.36	9.65 ±0.34	11.06 ±0.41	13.06 ±0.48	14.99 ±0.63	15.29 ±0.44	15.79 ±0.29	16.12 ±0.37
K	90.91 ±3.4	43.48 ±2.1	27.78 ±1.2	22.73 ±1	17.13 ±0.80	17.33 ±0.85	15.26 ±0.60	16.16 ±0.65

Table 3

Results of the approximation of D (< n >) – dependence be eq. (10) D =a + b <n >

Reaction	Energy Interval [GeV]	a	b
PP($\overline{P}$)	$3\leq \sqrt{s} \leq 900$	-0.56±0.02	0.58±0.01
π^+p	$7\leq E_L\leq 100$	-0.61±0.06	0.55±0.02
K^+P	$8\leq E_L\leq 100$	-0.46±0.14	0.53±0.04
e^+e^-	$14\leq \sqrt{s} \leq 200$	0.41±0.05	0.03±0.01
A_iA_t	$<n>\leq 25$	-1.22±0.04	0.79±0.01

Results of the approximation of $D_c(<n_c>)$ – dependence be eq. (10) $D_c = a + b < n_c >$

pp(p), e^+e^-, A_iA_t	Summary Data	-1.62±0.02	1.72±0.02

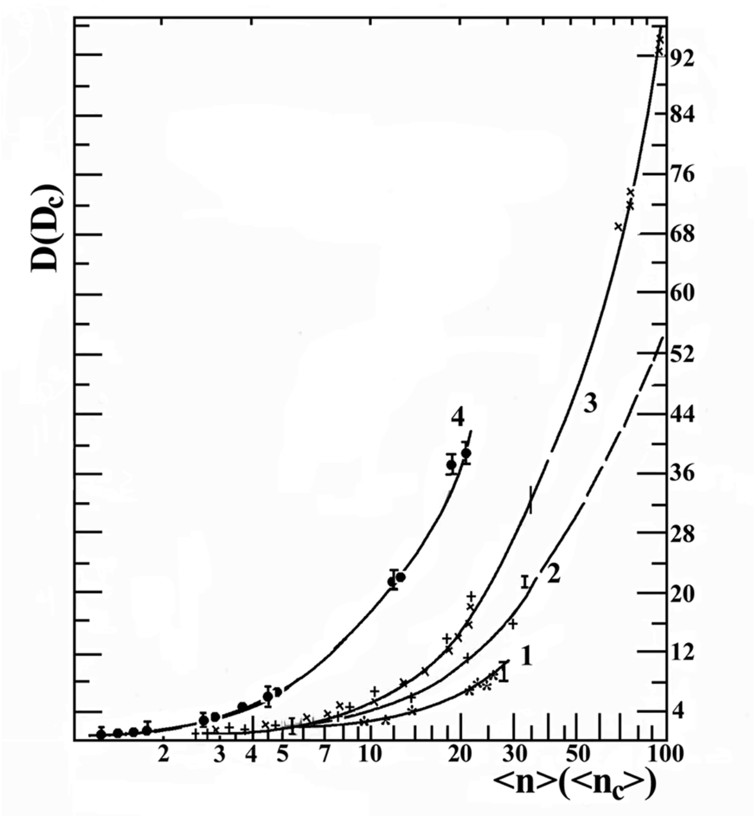

Fig.1. The dependence of the dispersion – D, on the <n> -average multiplicity - D(<n>)

Curve 1 – e^+e^- collision

Curve 2 – PP($\overline{P}$) collision

Curve 3 – A_iA_t collision

Curve 4 – the dependence $D_c(\langle n_c \rangle)$ for
e^+e^- , $PP(\bar{P})$ and A_iA_t – collisions (summary data)

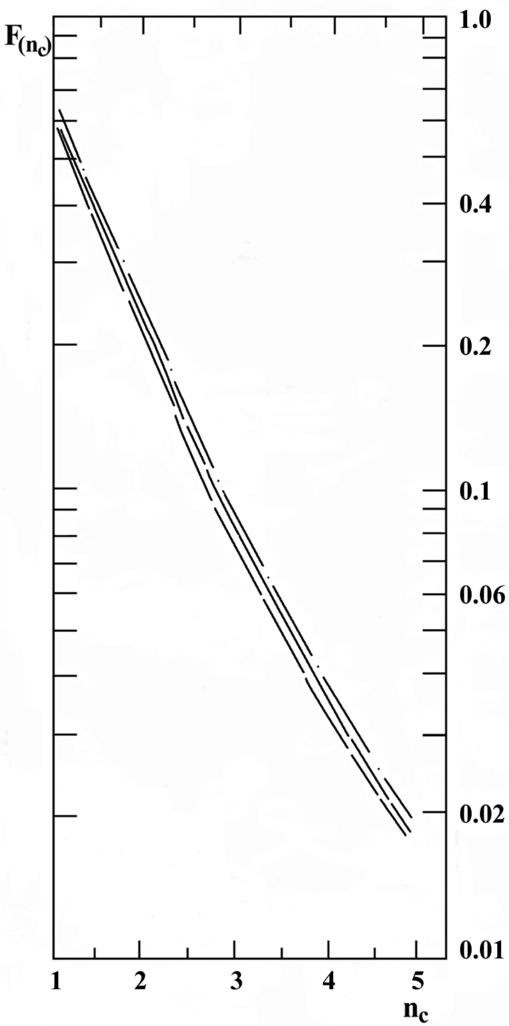

Fig.2. $F(n_c)$ – multiplicity distribution of particles (hadrons) inside clusters for fixed $<n_c>$

HeTa($E_L = 2.48$AGeV-·-); (PP ($\sqrt{s}$ = 62.2GeV- - -); (e^+e^-($\sqrt{s}$ = 200GeV —)). $<n_c> = 1.70$; $D_c = 1.24$

(for D(<n>)-dependence) are = 0.30±0.01 for e^+e^- - collisions; b = 0.58 ± 0.01 for pp(p)-collision; b = 0.79±0.01 for A_iA_t – collisions.

Thus at fixed <n> - average multiplicity of charged hadrons the dispersion in PP($\bar{P}$) – collisions is narrower than in A_iA_t – collisions and in e^+e^- - collisions is narrower than in PP($\bar{P}$) – collisions.

This fact seemes to be related to the leading hadrons (in PP($\bar{P}$) – collisions) and striping hadrons (in A_iA_t – collisions); or, intensification of intranuclear movement of nucleus.

In A_iA_t – collisions the dependence D(<n>) is approximated by Eq. (10) with parameters given in Table 3 (a = -1.22±0.04; b = 0.79 ± 0.01).

These parameters describe the data satisfactory up to the multiplicity <n> ≤ 25. At higher values of <n> (data on S(S,Cu), O (Cu, Au) at 200 AGeV) the approximation deviates from the data. Thus in the area of multiplicities higher than 25 the linearity of the D (<n>) – dependence is broken. It should be mentioned that there may arise multi nucleon interactions in these areas.

If the D(<n>) – dependence is approximated by Eq.(11) the experiment is satisfactory described with the parameters a = -0.94 ± 0.06; b = 0.70±0.01; c = 0.003±0.0004. The small value of the parameter c indicates that the deviation from the linearity is small.

$D_c(<n_c>)$ – dependence is again universal and the growth of dispersion is much faster than for D(<n>) –dependence (b = 1.72±0.02 Fig.1, Table 3). Here again the nonlinear dependence (11) works better. The values of parameters are the following: a = -1.49±0.03; b = 1.60±0.02; c = 0.02± 0.0003. Again the deviation from linearity is small.

The more direct indication on the universality of intracluster dynamics can be obtained by comparison of fixed <n> - average multiplicities, with multiplicity distribution of charged secondaries P_n (or with D - dispersion) for e^+e^-, pp(p) and A_iA_t - collisions . The results obtained have to be compared with the behavior of $F(n_c)$ – multiplicity distributions of particles inside clusters.

Experimental data show [2,3,7,8] that multiplicity distributions P_n (or dispersions D) for e^+e^-, PP($\bar{P}$) and A_iA_t – collisions at fixed <n>, for example $< n > \approx 20$ (for e^+e^- at $\sqrt{s}$ = 91 GeV, <n> = 20.71±0.80; D = 6.98±0.35; for PP($\bar{P}$) at $\sqrt{s}$ = 200 GeV, <n> =21.30 ±0.80; D = 10.90±0.40; for CT_a – interactions at E_L = 4.3AGeV, <n> = 19.75 ± 0.40; D = 15.71±0.35) significantly different from each other. The narrowest P_n - distribution (dispersion) is observed for e^+e^-collision, the widest – for CT_a- nucleus collision (see Fig.1).

Significantly another picture arises when considering the multiplicity distribution (dispersions) inside clusters $F(n_c)$, at fixed $<n_c>$ average multiplicity of particles in clusters (e^+e^- - collision at $\sqrt{s}$ = 91GeV ; $<n_c>$ = 1.41±0.05; D_c = 0.82±0.03. PP, $\sqrt{s}$ = 30.4 GeV; <nc> = 1.43 ±0.06; D_c = 0.87±0.04. (e^+e^-, $\sqrt{s}$ = 200GeV; $<n_c>$ = 1.72±0.04. D_c = 1.31±0.05; PP, $\sqrt{s}$ = 62.2GeV; $<n_c>$ = 1.69±0.07; D_c = 1.26±0.05; HeTa, E_L = 2.48 AGeV; $<n_c>$ = 1.67±0.05; D_c = 1.22 ± 0.04. P$\bar{P}$ -collision at $\sqrt{s}$ = 900 GeV, $<n_c>$ = 4.55±0.12; D_c = 6.01± 0.14; CTa – collisions at E_L = 4.3 AGeV, $<n_c>$ = 4.50 ±0.07; D_c = 6.00 ± 0.02 (Fig.2).

As is seen, distributions of hadrons inside clusters $F(n_c)$, at fixed $<n_c>$ are practically identical for different types of colliding objects.

Interesting point of view of the above stated results is mentioned in [13]. It is shown that charged hadron multiplicity distribution per jet for e^+e^- - collision practically coincides with the same distributions in pp collisions (if we assume that mean multiplicities in e^+e^- and pp-collisions are the same).

Thus according to CCM the process of multiple production at high energies can be considered as consisting of two stages: first stage- creation and formation of clusters (patriarchs) depends on the type of colliding objects; second stage – formation of hadrons inside clusters and decay of clusters is of the universal character and does not depend on the type of colliding objects [13].

It can be said, that the picture obtained is in accordance with the existing chromodinamical models of particle production at high energies.

According to this models the interaction process at high energy can be considered of consisting of two stages – first: partons are created (this stage depends on the type of interaction (strong, weak, electromagnetic); second stage - hadronization of partons occurs [9,10].

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