

BOUND qqq -SYSTEMS IN THE FRAMEWORK OF THE DIFFERENT VERSIONS OF THE THREE-DIMENSIONAL REDUCTIONS OF THE BETHE-SALPETER EQUATION

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ABSTRACT. Three different versions of 3D-reduction of the Bethe-Salpeter equation for bound qqq -systems taking into account both two- and three-body interaction potentials has been considered. The normalization condition for wave function is derived. Using the solutions of non-relativistic limit of above mentioned equations with oscillator type two-particle potential as a basis, system of algebraic equations for expansion coefficients are obtained. Next it will be investigated the dependence of existence of the stable solutions of obtained system of equations on Lorentz (spin) structure of two- and three-quarks confining potentials.

1. INTRODUCTION

The properties of baryons in the framework of the constituent quark model (bound qqq -systems) at first stage were studied in non-relativistic approach. Exhaustive review of such an approach is given in [1] and will not be discussed below. The necessity of the relativistic treatment of bound qqq -systems is apparent (well known) for baryons (N , Σ , Λ , Ξ , Δ , Ω) with constituent quarks from light sector (u , d , s). The natural basis of such investigation is Bethe-Salpeter (BS) equation for three-fermion bound systems which were used e.g. in [2-11]. The main approximation used in the BS-equation is instantaneous (static) approximation for the kernel or its Lorentz-invariant version (null-plane approximation). Below it will be used the instantaneous approximation for the kernel of the BS-equation and some additional approximation for the free two-particle Green functions in the 3-particle system for the formulation two different

versions of the 3D-reduction of the BS-equation. These versions then will be compared to the Salpeter type versions formulated in [11].

2. THE 3D-REDUCTION OF BS-EQUATION FOR BOUND qqq -SYSTEMS AND NORMALIZATION CONDITION FOR CORRESPONDING WAVE FUNCTION

The BS-equation for bound qqq -systems has well-known form:

$$\varphi_P(p, q) = G_0(P; p, q) \int \frac{d^4 p'}{(2\pi)^4} \frac{d^4 q'}{(2\pi)^4} K(P; p, q; p', q') \varphi_P(p', q'), \quad (1)$$

where $P = P(\sqrt{M^2 + \vec{P}^2}, \vec{P})$ is total 4-momenta of the system under consideration and (p, q) are one of three equivalent pairs of Jacoby variables

$$p_{ij} = \mu_{ji} p_i - \mu_{ij} p_j, \quad q_n = (\mu_i + \mu_j) p_n - \mu_n(p_i + p_j), \quad P = p_i + p_j + p_n,$$

$$\mu_i = \frac{m_i}{m_1 + m_2 + m_3}, \quad \mu_{ij} = \frac{m_i}{m_i + m_j}, \quad i, j, n = 1, 2, 3, \quad 123, 231, 312 \equiv \text{cycl}(ijn), \quad (2)$$

$$p_i = \mu_i P + p_{ij} - \mu_{ij} q_n, \quad p_j = \mu_j P - p_{ij} - \mu_{ji} q_n, \quad p_n = \mu_n P + q_n;$$

$$G_0(P; p, q) = G_0(p_1) \otimes G_0(p_2) \otimes G_0(p_3) = G_{0ij}(P; p_{ij}, q_n) \otimes S_n(p_n), \quad (3a)$$

$$S_l(p_l) = i(p_l \gamma_l - m_l)^{-1};$$

$$\begin{aligned} K(P; p, q; p', q') &= K^{(3)}(P; p, q; p', q') + [K^{(2)}(P; p, q; p', q') = \\ &= \sum_{\text{cycl}(ijn)} K_{ij}(p_i + p_j; p_{ij}, p'_{ij})(2\pi)^4 \delta(p_n - p'_n) (S_n(p_n))^{-1}]. \end{aligned} \quad (3b)$$

For the 3D-reduction of the equation (1) the following approximations are used:

$$K^{(3)}(P; p, q; p', q') \rightarrow \tilde{K}^{(3)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') \equiv -i\hat{V}^{(3)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}')$$

$$K_{ij}(p_i + p_j; p_{ij}, p'_{ij}) \rightarrow \tilde{K}_{ij}(\vec{p}_{ij}, \vec{p}'_{ij}) \equiv -i\hat{V}_{ij}(\vec{p}_{ij}, \vec{p}'_{ij}), \quad (4)$$

i.e. the instantaneous approximation for the kernels corresponding to two- and three-body forces, and (c.m., $\vec{P} = 0$)

$$G_{0ij}(M; p_{ij}, q_n) \rightarrow [G_{0ij}(M; p_{ij}, q_n) +$$

$$(G_{0ij}(M; p_i, p_j^{cr} = \mu_j M + p_{ij}^0 - \mu_{ji} q_n^0, \vec{p}_j) \equiv \quad (5)$$

$$\equiv G_{0ij}^{cr}(M; p_{ij}, q_n)]^{q_n^0 = \omega_n - \mu_n} \equiv G_{0ijn\text{eff}}(M; p_{ij}, \vec{q}_n),$$

i.e. in the ij free-particle Green function before two-body interaction potential the third (non-interacting) particle (n) is taken on mass-shell suggested in [12] and used in [10], where the 3D-reduction of BS-equation without three-body force and G_{0ij}^{cr} term was formulated. Below this version will be mentioned as the IKLR version. Then the additional term G_{0ij}^{cr} in (5) (suggested and used in [13] for the 3D-reduction of BS-equation for bound $q\bar{q}$ system) will be called as the IKLR+MW version.

Now using the approximation (4) and (5) after integrating over p_0 and q_0 the BS-equation (1) for the wave function

$$\tilde{\varphi}_M(\vec{p}, \vec{q}) = \int \frac{dp_0}{2\pi} \frac{dq_0}{2\pi} \varphi_0(p, q), \quad (6)$$

we obtain the equation

$$\begin{aligned}
\tilde{\phi}_M(\vec{p}, \vec{q}) = & \int \frac{d^3 \vec{p}'}{(2\pi)^3} \frac{d^3 \vec{q}'}{(2\pi)^3} \tilde{G}_0(M; \vec{p}, \vec{q}) \hat{V}^{(3)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') \tilde{\phi}_M(\vec{p}', \vec{q}') + \\
& + \sum_{cycl(ijn)} \int \frac{d\vec{p}'}{(2\pi)^3} \frac{d\vec{q}'}{(2\pi)^3} \tilde{G}_{0ijneff}(M; \vec{p}, \vec{q}) \hat{V}_{ij}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') \tilde{\phi}_M(\vec{p}', \vec{q}'),
\end{aligned} \tag{7}$$

where

$$\begin{aligned}
\tilde{G}_0(M; \vec{p}, \vec{q}) = & \left[\frac{\Lambda_{123}^{(+++)}(\vec{p}, \vec{q})}{M - \omega_1 - \omega_2 - \omega_3 + i0} + \right. \\
& \left. + \frac{\Lambda_{123}^{(---)}(\vec{p}, \vec{q})}{M + \omega_1 + \omega_2 + \omega_3} \right] \gamma_1^0 \otimes \gamma_2^0 \otimes \gamma_3^0,
\end{aligned} \tag{8}$$

$$\begin{aligned}
\tilde{G}_{0ijneff}(M; \vec{p}, \vec{q}) = & \left[\left(\frac{\Lambda_{ij}^{(++)}(\vec{p}, \vec{q})}{M - \omega_n - (\omega_i + \omega_j) + i0} - \right. \right. \\
& - \left. \frac{\Lambda_{ij}^{(--)}}{M - \omega_n + \omega_i + \omega_j - i0} \right) + \frac{\Lambda_{ij}^{(+-)}(\vec{p}, \vec{q})}{(\mu_{ji} - \mu_{ij})(M - \omega_n) + \omega_i + \omega_j + i0} + \\
& \left. + \frac{\Lambda_{ij}^{(++)}(\vec{p}, \vec{q})}{(\mu_{ij} - \mu_{ji})(M - \omega_n) + \omega_i + \omega_j - i0} \right] \gamma_i^0 \otimes \gamma_j^0 \otimes I_n,
\end{aligned} \tag{9}$$

$$\Lambda_{123}^{(\alpha_1 \alpha_2 \alpha_3)} = \Lambda_1^{(\alpha_1)} \otimes \Lambda_2^{(\alpha_2)} \otimes \Lambda_3^{(\alpha_3)},$$

$$\Lambda_{ij}^{(\alpha_i \alpha_j)} = \Lambda_i^{(\alpha_i)} \otimes \Lambda_j^{(\alpha_j)}, \quad \Lambda_i^{(\alpha_i)} = \frac{\omega_i + \alpha_i h_i}{2\omega_i}, \tag{10}$$

$$h_i = \gamma_i^0 (\vec{\gamma}_i \vec{p} + m_i).$$

Note that the first term in (9) corresponds to the IKLR version, as to the additional term in (9) appears in the IKLR+MW version.

For derivation of the normalization condition for the wave function $\tilde{\varphi}_M(\vec{p}, \vec{q})$ in the IKLR and IKLR+MW versions we use the procedure suggested in [10]. We begin with the equation for total Green operator

$$G(P) = G_0(P) + G_0(P)[K(P) = K^{(3)}(P) + K^{(2)}(P)]G(P), \quad (11)$$

from which in (4) and (5) approximations it follows the equation for the Green's operator in 3D-space (c.m.)

$$\begin{aligned} \tilde{G}(M) = i\tilde{G}_0(M) + \left[i\tilde{G}_0(M)\tilde{K}^{(3)}(M) + \right. \\ \left. + \sum_{cycl(ijn)} i\tilde{G}_{0ijneff}(M)\tilde{K}_{ij}^{(2)}(M) \right] \tilde{G}(M). \end{aligned} \quad (12)$$

This equation can be rewritten as

$$\tilde{G} = ig_0\Pi_+\Gamma^0 + g_0\tilde{U}\tilde{G}, \quad (13)$$

where

$$g_0 = [M - (h_1 + h_2 + h_3)]^{-1}, \quad \Gamma^0 = \gamma_1^0 \otimes \gamma_2^0 \otimes \gamma_3^0, \quad \Pi_+ = \Lambda_{123}^{(+++)} + \Lambda_{123}^{(---)}, \quad (14)$$

$$\tilde{U} = g_0^{-1} \left[\tilde{G}_0\hat{V}^{(3)} + \sum_{cycl(ijn)} \tilde{G}_{0ijneff}(M)\hat{V}_{ij} \right]. \quad (15)$$

Then the equation (7) can be written in the vector form (c.m.):

$$|\tilde{\varphi}_M\rangle = g_0\tilde{U}|\tilde{\varphi}_M\rangle. \quad (16)$$

Now it can be easily checked that if Green's operator satisfies the equation (13), then the following relation holds:

$$\tilde{G}\Gamma^0\Pi_+[g_0^{-1}-\tilde{U}]\tilde{G}\Gamma^0\Pi_+\Gamma^0=i\tilde{G}\Gamma^0\Pi_+\Gamma^0. \quad (17)$$

Noting that the equation (13) holds for arbitrary reference frame $P=(P_0, \vec{P})$, the Green operator $\tilde{G}(P)$ has the following representation:

$$\tilde{G}=i\sum_B\frac{|\tilde{\tilde{\varphi}}_M\rangle\langle\tilde{\tilde{\varphi}}_M|}{P_0^2-M_B^2+i0}+\tilde{R}, \quad \langle\tilde{\tilde{\varphi}}_M|=\langle\tilde{\varphi}_M|\Gamma^0. \quad (18)$$

Substituting this expression in the relation (17) after extracting the bound state poles in the operator \tilde{G} , the following condition can be obtained (c.m.):

$$\langle\tilde{\varphi}_M|\Pi_+[1-\frac{\partial}{\partial M}\tilde{U}(M)]|\tilde{\varphi}_M\rangle=2M, \quad (19)$$

which is normalization condition for the wave function $\tilde{\varphi}_M(\vec{p}, \vec{q})$ satisfying equation (7).

3. 3D-EQUATIONS FOR THE “FREQUENCY COMPONENTS” OF THE WAVE FUNCTION AND CORRESPONDING NORMALIZATION CONDITION

Introducing the “frequency components” of wave function

$$\tilde{\varphi}_M^{(\alpha_1\alpha_2\alpha_3)}(\vec{p}, \vec{q})\equiv\Lambda_{123}^{(\alpha_1\alpha_2\alpha_3)}\tilde{\varphi}_M(\vec{p}, \vec{q}), \quad (20a)$$

and taking into account the relation

$$\sum_{\alpha_1 \alpha_2 \alpha_3} \tilde{\varphi}_M^{(\alpha_1 \alpha_2 \alpha_3)}(\vec{p}, \vec{q}) = \tilde{\varphi}_M(\vec{p}, \vec{q}), \quad (20b)$$

from (7) equation it can be obtained the following system of equations

$$\begin{aligned} & [M \mp (\omega_1 + \omega_2 + \omega_3)] \tilde{\varphi}_M^{(\pm \pm \pm)}(\vec{p}, \vec{q}) = \\ & = \sum_{\alpha'_1 \alpha'_2 \alpha'_3} \int \frac{d^3 \vec{p}'}{(2\pi)^3} \frac{d^3 \vec{q}'}{(2\pi)^3} \Lambda_{123}^{(\pm \pm \pm)}(\vec{p}, \vec{q}) \{ \gamma_1^0 \otimes \gamma_2^0 \otimes \gamma_3^0 \hat{V}^{(3)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') + \\ & + \sum_{cycl(ijn)} \left[-1 + \frac{2\omega_n}{M + \omega_i + \omega_j - \omega_n} \equiv f_{ijn}(M; \vec{p}, \vec{q}) \right] \times \\ & \times \gamma_i^0 \otimes \gamma_j^0 \hat{V}_{ij}^{(2)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') \} \tilde{\varphi}_M^{(\alpha'_1 \alpha'_2 \alpha'_3)}(\vec{p}', \vec{q}'); \end{aligned} \quad (21a)$$

$$\begin{aligned} & [M \mp (\omega_1 + \omega_2) - \omega_3] \tilde{\varphi}_M^{(\pm \pm \mp)}(\vec{p}, \vec{q}) = \\ & = \sum_{\alpha'_1 \alpha'_2 \alpha'_3} \int \frac{d^3 \vec{p}'}{(2\pi)^3} \frac{d^3 \vec{q}'}{(2\pi)^3} \Lambda_{123}^{(\pm \pm \mp)}(\vec{p}, \vec{q}) (\pm 1) \left\{ \gamma_1^0 \otimes \gamma_2^0 \hat{V}_{12}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') + \right. \\ & \left. [M \mp (\omega_1 + \omega_2) - \omega_3] \times \left[\frac{\gamma_2^0 \otimes \gamma_3^0 \hat{V}_{23}(\vec{p}, \vec{q}; \vec{p}', \vec{q}')}{(\mu_{32} - \mu_{23})(M - \omega_1) \pm (\omega_2 + \omega_3) + i0} + \right. \right. \\ & \left. \left. \times \frac{\gamma_3^0 \otimes \gamma_1^0 \hat{V}_{31}(\vec{p}, \vec{q}; \vec{p}', \vec{q}')}{(\mu_{31} - \mu_{13})(M - \omega_2) \pm (\omega_3 + \omega_1) - i0} \right] \right\} \tilde{\varphi}_M^{(\alpha'_1 \alpha'_2 \alpha'_3)}(\vec{p}', \vec{q}'). \end{aligned} \quad (21b)$$

$$[M \mp (\omega_2 + \omega_3) - \omega_1] \tilde{\varphi}_M^{(\mp \mp \pm)}(\vec{p}, \vec{q}) =$$

$$\begin{aligned}
&= \sum_{\alpha'_1 \alpha'_2 \alpha'_3} \int \frac{d^3 \vec{p}'}{(2\pi)^3} \frac{d^3 \vec{q}'}{(2\pi)^3} \Lambda_{123}^{(\mp\pm\pm)}(\vec{p}, \vec{q})(\pm 1) \{ \gamma_2^0 \otimes \gamma_3^0 \hat{V}_{23}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') + \\
&\quad + [M \mp (\omega_2 + \omega_3) - \omega_1] \times \left[\frac{\gamma_3^0 \otimes \gamma_1^0 \hat{V}_{31}(\vec{p}, \vec{q}; \vec{p}', \vec{q}')}{(\mu_{13} - \mu_{31})(M - \omega_2) \pm (\omega_3 + \omega_1) + i0} + \right. \\
&\quad \left. + \frac{\gamma_1^0 \otimes \gamma_2^0 \hat{V}_{12}(\vec{p}, \vec{q}; \vec{p}', \vec{q}')}{(\mu_{12} - \mu_{21})(M - \omega_3) \pm (\omega_1 + \omega_2) - i0} \right] \Big\} \tilde{\phi}_M^{(\alpha'_1 \alpha'_2 \alpha'_3)}(\vec{p}', \vec{q}'). \quad (21c)
\end{aligned}$$

$$\begin{aligned}
&[M \mp (\omega_1 + \omega_3) - \omega_2] \tilde{\phi}_M^{(\pm\mp\pm)}(\vec{p}, \vec{q}) = \\
&= \sum_{\alpha'_1 \alpha'_2 \alpha'_3} \int \frac{d^3 \vec{p}'}{(2\pi)^3} \frac{d^3 \vec{q}'}{(2\pi)^3} \Lambda_{123}^{(\pm\mp\pm)}(\vec{p}, \vec{q})(\pm 1) \{ \gamma_3^0 \otimes \gamma_1^0 \hat{V}_{31}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') \\
&\quad + [M \mp (\omega_1 + \omega_3) - \omega_2] \times \left[\frac{\gamma_1^0 \otimes \gamma_2^0 \hat{V}_{12}(\vec{p}, \vec{q}; \vec{p}', \vec{q}')}{(\mu_{21} - \mu_{12})(M - \omega_3) \pm (\omega_1 + \omega_2) + i0} + \right. \\
&\quad \left. + \frac{\gamma_2^0 \otimes \gamma_3^0 \hat{V}_{23}(\vec{p}, \vec{q}; \vec{p}', \vec{q}')}{(\mu_{23} - \mu_{32})(M - \omega_1) \pm (\omega_2 + \omega_3) - i0} \right] \Big\} \tilde{\phi}_M^{(\alpha'_1 \alpha'_2 \alpha'_3)}(\vec{p}', \vec{q}'). \quad (21d)
\end{aligned}$$

Now the normalization condition (19) takes the form:

$$\begin{aligned}
&\int \frac{d^3 \vec{p}}{(2\pi)^3} \frac{d^3 \vec{q}}{(2\pi)^3} \left[\left| \tilde{\phi}_M^{(++++)}(\vec{p}, \vec{q}) \right|^2 + \left| \tilde{\phi}_M^{(----)}(\vec{p}, \vec{q}) \right|^2 \right] - \\
&- \int \frac{d^3 \vec{p}}{(2\pi)^3} \frac{d^3 \vec{q}}{(2\pi)^3} \frac{d^3 \vec{p}'}{(2\pi)^3} \frac{d^3 \vec{q}'}{(2\pi)^3} \sum_{\text{cycl}(ijn)} \left[\frac{\partial f_{ijn}(M; \vec{p}, \vec{q})}{\partial M} \times \right. \\
&\quad \times \tilde{\phi}_M^{+(---)}(\vec{p}, \vec{q}) \Lambda_{123}^{(---)}(\vec{p}, \vec{q}) \hat{V}_{ij}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') \Big] \sum_{\alpha'_1 \alpha'_2 \alpha'_3} \tilde{\phi}_M^{(\alpha'_1 \alpha'_2 \alpha'_3)}(\vec{p}', \vec{q}') = \\
&\quad = 2M. \quad (22)
\end{aligned}$$

Note that the Salpeter type equation derived in the [11] from 3D-reduction of BS-equation (LKMP version) can be obtained from the equation (21a) if $f_{ijn} \rightarrow 1$ and put $(\alpha'_1 \alpha'_2 \alpha'_3) = (\pm \pm \pm)$, i.e. the components $\tilde{\varphi}_M^{(\pm \pm \mp)}$, $\tilde{\varphi}_M^{(\pm \mp \pm)}$, $\tilde{\varphi}_M^{(\mp \pm \pm)}$ are zeros. As to corresponding normalization condition for the wave function it follows from (22) putting $f_{ijn} = 1$.

Now we represent function $\tilde{\varphi}_M^{(\alpha_1 \alpha_2 \alpha_3)}(\vec{p}, \vec{q})$ as

$$\tilde{\varphi}_M^{(\alpha_1 \alpha_2 \alpha_3)}(\vec{p}, \vec{q}) = N_{123}^{(\alpha_1 \alpha_2 \alpha_3)}(\vec{p}, \vec{q}) \prod_{i=1}^3 \left(\frac{1}{\alpha_i \vec{\sigma}_i \vec{p}_i} \right) \chi_M^{(\alpha_1 \alpha_2 \alpha_3)}(\vec{p}, \vec{q}), \quad (23)$$

$$N_{123}^{(\alpha_1 \alpha_2 \alpha_3)}(\vec{p}, \vec{q}) = \prod_{i=1}^3 \sqrt{\frac{\omega_i + \alpha_i m_i}{2\omega_i}}$$

and the Lorentz structure of the three-body confining interaction potential we take the form [14]:

$${}_c \hat{V}^{(3)} = v_0^{(3)} L_+ + \left\{ \begin{matrix} L_- \\ L_+ \end{matrix} \right\} v_c^{(3)}, \quad \begin{array}{ll} \text{model A} \\ \text{model B} \end{array} \quad (24a)$$

where

$$v_0^{(3)} = 3a, \quad L_{\mp} = \left\{ \begin{matrix} 1/2 \\ 1/4 \end{matrix} \right\} (\mp I_1 \otimes I_2 \otimes I_3 + \sum_{cycl(ijn)} \gamma_i^0 \otimes \gamma_j^0 \otimes I_n)$$

and a is negative constant, $v_c^{(3)}$ is increasing positive function of quarks coordinates.

As to the two-body interaction operator it is taken in the form given in the review paper [15] concerning bound $q\bar{q}$ -systems problem:

$$\begin{aligned}\hat{V}_{ij} = & \gamma_i^0 \otimes \gamma_j^0 V_{ij}^{og} + [x \gamma_i^0 \otimes \gamma_j^0 + (1-x) I_i \otimes I_j] V_{ij}^c + \\ & + [I_i \otimes I_j + \gamma_i^5 \otimes \gamma_j^5] V_{ij}^T, \quad 0 \leq x \leq 1,\end{aligned}\quad (24b)$$

where “og”, “c” and “T” means “one gluon”, “confinement” and “t-Hooft”, accordingly.

Then the system of equations for wave functions $\chi_M^{(\alpha_1 \alpha_2 \alpha_3)}(\vec{p}, \vec{q})$ can be obtained from the equations (21) and has the form:

$$\begin{aligned}[M \mp (\omega_1 + \omega_2 + \omega_3)] \chi_M^{(\pm \pm \pm)}(\vec{p}, \vec{q}) = \\ = \int \frac{d^3 \vec{p}'}{(2\pi)^3} \frac{d^3 \vec{q}'}{(2\pi)^3} \left[\sum_{\alpha'_1 \alpha'_2 \alpha'_3} c V_{eff}^{(3)(\pm \pm \pm, \alpha'_1 \alpha'_2 \alpha'_3)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') \chi_M^{(\alpha'_1 \alpha'_2 \alpha'_3)}(\vec{p}', \vec{q}') + \right. \\ \left. + \sum_{cycl(ijn)} \left[\frac{1}{-f_{ijn}(M; \vec{p}, \vec{q})} \right] \times \right. \\ \left. \times \sum_{\alpha'_i \alpha'_j} V_{ieff}^{(\pm \pm, \alpha'_i \alpha'_j)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') \chi_M^{(\alpha'_i \alpha'_j \alpha'_n = \pm)}(\vec{p}', \vec{q}') \right]; \quad (25a)\end{aligned}$$

$$\begin{aligned}[M \mp (\omega_1 + \omega_2) - \omega_3] \chi_M^{(\pm \pm \mp)}(\vec{p}, \vec{q}) = \\ = \int \frac{d^3 \vec{p}'}{(2\pi)^3} \frac{d^3 \vec{q}'}{(2\pi)^3} (\pm 1) \left\{ \sum_{\alpha'_1 \alpha'_2} V_{12eff}^{(\pm \pm, \alpha'_1 \alpha'_2)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') \chi_M^{(\alpha'_1 \alpha'_2 \mp)}(\vec{p}', \vec{q}') + \right. \\ \left. + [M \mp (\omega_1 + \omega_2) - \omega_3] \left[\sum_{\alpha'_2 \alpha'_3} \frac{V_{23eff}^{(\pm \mp, \alpha'_2 \alpha'_3)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') \chi_M^{(\pm \alpha'_2 \alpha'_3)}(\vec{p}', \vec{q}')}{(\mu_{32} - \mu_{23})(M - \omega_1) \pm (\omega_2 + \omega_3) - i0} + \right. \right. \\ \left. \left. + \sum_{\alpha'_3 \alpha'_1} \frac{V_{31eff}^{(\mp \pm, \alpha'_3 \alpha'_1)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') \chi_M^{(\alpha'_1 \pm \alpha'_3)}(\vec{p}', \vec{q}')}{(\mu_{31} - \mu_{13})(M - \omega_2) \pm (\omega_3 + \omega_1) + i0} \right] \right\}; \quad (25b)\end{aligned}$$

$$\begin{aligned}
& [M \mp (\omega_2 + \omega_3) - \omega_1] \chi_M^{(\mp\pm\pm)}(\vec{p}, \vec{q}) = \\
& = \int \frac{d^3 \vec{p}'}{(2\pi)^3} \frac{d^3 \vec{q}'}{(2\pi)^3} (\pm 1) \left\{ \sum_{\alpha'_2 \alpha'_3} V_{23eff}^{(\pm\pm, \alpha'_2 \alpha'_3)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') \chi_M^{(\mp\pm \alpha'_2 \alpha'_3)}(\vec{p}', \vec{q}') + \right. \\
& + [M \mp (\omega_2 + \omega_3) - \omega_1] \left[\sum_{\alpha'_1 \alpha'_2} \frac{V_{12eff}^{(\mp\pm, \alpha'_1 \alpha'_2)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') \chi_M^{(\alpha'_1 \alpha'_2 \pm)}(\vec{p}', \vec{q}')}{(\mu_{12} - \mu_{21})(M - \omega_3) \pm (\omega_1 + \omega_2) + i0} + \right. \\
& \left. \left. + \sum_{\alpha'_3 \alpha'_1} \frac{V_{31eff}^{(\pm\mp, \alpha'_3 \alpha'_1)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') \chi_M^{(\alpha'_1 \pm \alpha'_3)}(\vec{p}', \vec{q}')}{(\mu_{13} - \mu_{31})(M - \omega_2) \pm (\omega_3 + \omega_1) - i0} \right] \right\}; \quad (25c)
\end{aligned}$$

$$\begin{aligned}
& [M \mp (\omega_1 + \omega_3) - \omega_2] \chi_M^{(\pm\mp\pm)}(\vec{p}, \vec{q}) = \\
& = \int \frac{d^3 \vec{p}'}{(2\pi)^3} \frac{d^3 \vec{q}'}{(2\pi)^3} (\pm 1) \left\{ \sum_{\alpha'_3 \alpha'_1} V_{31eff}^{(\pm\pm, \alpha'_3 \alpha'_1)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') \chi_M^{(\alpha'_1 \pm \alpha'_3)}(\vec{p}', \vec{q}') + \right. \\
& + [M \mp (\omega_1 + \omega_3) - \omega_2] \left[\sum_{\alpha'_1 \alpha'_2} \frac{V_{12eff}^{(\mp\pm, \alpha'_1 \alpha'_2)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') \chi_M^{(\alpha'_1 \alpha'_2 \pm)}(\vec{p}', \vec{q}')}{(\mu_{12} - \mu_{21})(M - \omega_3) \pm (\omega_1 + \omega_2) - i0} + \right. \\
& \left. \left. + \sum_{\alpha'_2 \alpha'_3} \frac{V_{23eff}^{(\mp\pm, \alpha'_2 \alpha'_3)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') \chi_M^{(\pm \alpha'_2 \alpha'_3)}(\vec{p}', \vec{q}')}{(\mu_{23} - \mu_{32})(M - \omega_1) \pm (\omega_2 + \omega_3) - i0} \right] \right\}, \quad (25d)
\end{aligned}$$

where

$$\begin{aligned}
& {}_c V_{eff}^{(3)(\pm\pm\pm, \alpha'_1 \alpha'_2 \alpha'_3)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') = \\
& = N_{123}^{(\pm\pm\pm)}(\vec{p}, \vec{q}) [v_0^{(3)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') B_+^{(\pm\pm\pm, \alpha'_1 \alpha'_2 \alpha'_3)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') +
\end{aligned}$$

$$+ \left\{ \begin{array}{l} B_-^{(\pm\pm\pm, \alpha'_1 \alpha'_2 \alpha'_3)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') \\ B_+^{(\pm\pm\pm, \alpha'_1 \alpha'_2 \alpha'_3)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') \end{array} \right\} v_c^{(3)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}')] N_{123}^{(\alpha'_1 \alpha'_2 \alpha'_3)}(\vec{p}', \vec{q}'), \quad \begin{array}{l} \text{A} \\ \text{B} \end{array} \quad (26a)$$

$$\begin{aligned} V_{ij,eff}^{(\alpha_i \alpha_j, \alpha'_i \alpha'_j)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') = \\ = N_{ij}^{(\alpha_i \alpha_j)}(\vec{p}, \vec{q}) [V_{ij}(\vec{p}, \vec{q}; \vec{p}', \vec{q}')]_1 B_{ij}^{(\alpha_i \alpha_j, \alpha'_i \alpha'_j)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') + \\ + {}_2 V_{ij}(x; \vec{p}, \vec{q}; \vec{p}', \vec{q}') {}_2 B_{ij}^{(\alpha_i \alpha_j, \alpha'_i \alpha'_j)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') + \end{aligned} \quad (26b)$$

$$+ {}_T V_{ij}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') {}_T B_{ij}^{(\alpha_i \alpha_j, \alpha'_i \alpha'_j)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}')] N_{ij}^{\alpha'_i \alpha'_j}(\vec{p}', \vec{q}'),$$

$${}_1 V = V^{og} + V^c, \quad {}_2 V(x) = V^{og} + (2x - 1)V^c,$$

$$\begin{aligned} B_-^{(\pm\pm\pm, \alpha'_1 \alpha'_2 \alpha'_3)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') = \\ = 1 + \sum_{i=1}^3 f_i^{(\pm)} f_i^{(\alpha'_i)'} - \sum_{i < j=1}^3 f_i^{(\pm)} f_j^{(\pm)} f_i^{(\alpha'_i)'} f_j^{(\alpha'_j)'} - \prod_{i=1}^3 f_i^{(\pm)} f_i^{(\alpha'_i)'}, \end{aligned}$$

$$B_+^{(\pm\pm\pm, \alpha'_1 \alpha'_2 \alpha'_3)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') = \quad (26c)$$

$$= 1 - \prod_{i=1}^3 f_i^{(\pm)} f_i^{(\alpha'_i)'}, \quad f_i^{(\alpha_i)} = \frac{\alpha_i \vec{\sigma}_i \vec{p}_i}{\omega_i + \alpha_i m_i}, \quad f_i^{(\alpha'_i)'} = \frac{\alpha'_i \vec{\sigma}_i \vec{p}'_i}{\omega'_i + \alpha'_i m_i},$$

$${}_1 B_{ij}^{(\alpha_i \alpha_j, \alpha'_i \alpha'_j)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') = 1 + f_i^{(\alpha_i)} f_j^{(\alpha_j)} f_i^{(\alpha'_i)'} f_j^{(\alpha'_j)'},$$

$${}_2 B_{ij}^{(\alpha_i \alpha_j, \alpha'_i \alpha'_j)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') = f_i^{(\alpha_i)} f_i^{(\alpha'_i)'} + f_j^{(\alpha_j)} f_j^{(\alpha'_j)'}, \quad (26d)$$

$${}_T B_{ij}^{(\alpha_i \alpha_j, \alpha'_i \alpha'_j)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') =$$

$$= (1 + f_i^{(\alpha_i)} f_j^{(\alpha_j)}) (1 + f_i^{(\alpha_i)'} f_j^{(\alpha_j)'}) - (f_i^{(\alpha_i)} + f_j^{(\alpha_j)}) (f_i^{(\alpha_i)'} + f_j^{(\alpha_j)'}).$$

As to the normalization condition for the functions $\chi_M^{(\alpha_1\alpha_2\alpha_3)}(\vec{p}, \vec{q})$ it can be obtained from the condition (22) substituting (23) expression. As a result we have:

$$\begin{aligned} & \int \frac{d^3\vec{p}}{(2\pi)^3} \frac{d^3\vec{q}}{(2\pi)^3} \sum_{\alpha=+,-} \left| \chi_M^{(\alpha\alpha\alpha)}(\vec{p}, \vec{q}) \right|^2 - \\ & - \sum_{\alpha'_1\alpha'_2\alpha'_3} \sum_{cycl(ijn)} \int \frac{d^3\vec{p}'}{(2\pi)^3} \frac{d^3\vec{q}'}{(2\pi)^3} \frac{d^3\vec{p}}{(2\pi)^3} \frac{d^3\vec{q}}{(2\pi)^3} [\chi_M^{*(---)}(\vec{p}, \vec{q}) \frac{\partial f_{ijn}(M; \vec{p}, \vec{q})}{\partial M} \\ & \times V_{ij,eff}^{(---,\alpha'_1\alpha'_2\alpha'_3)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') \chi_M^{(\alpha'_1\alpha'_2\alpha'_3)}(\vec{p}', \vec{q}')] = 2M, \end{aligned} \quad (27)$$

from which it follows the normalization condition for the LKMP version of 3D-reduction of BS-equation if $f_{ijn} \rightarrow 1$.

4. THE SOLUTION PROCEDURE FOR OBTAINED EQUATION

Main problem which must be considered below is dependence of existence of the stable solution of the system of equations (25) on Lorentz structure of the two and three quarks confining interaction potentials. For this (ij) quarks confining interaction potential is taken in the form used in [16] for investigation mass spectra of bound $q\bar{q}$ systems

$$cV_{ij}(\vec{r}_i, \vec{r}_j) = -a_{ij} + b_{ij} \left| r_i - r_j \right|^2, \quad (28a)$$

$$a_{ij} = \frac{2}{3} \alpha_S(m_{ij}^2) V_0, \quad b_{ij} = \frac{2}{3} \alpha_S(m_{ij}^2) \frac{m_i m_j}{2m_{ij}} \omega_0^2, \quad \alpha_S(m_{ij}^2) =$$

$$= \frac{12\pi}{33-2n_f} \left(\ln \frac{m_{ij}^2}{\Lambda^2} \right)^{-1}, \quad m_{ij} = m_i + m_j \quad (28)$$

with free parameters: ω_0 , Λ and n_f (number of flavours). Note that appearing the 2/3 in a_{ij} and b_{ij} instead of 4/3 for $q\bar{q}$ systems is related to color depending part of wave function. As to three-quark confining interaction potential $v_c^{(3)}$ we take the form suggested in [14], but instead of linear type interaction we use oscillator type interaction

$$v_c^{(3)}(\vec{r}_1, \vec{r}_2, \vec{r}_3) = b(|\vec{r}_1 - \vec{r}_2|^2 + |\vec{r}_2 - \vec{r}_3|^2 + |\vec{r}_3 - \vec{r}_1|^2), \quad (29)$$

which is practically more easily solvable and compatible with two-body model.

Below we consider bound qqq -systems with $m_1 = m_2 \equiv m$, which included all observable baryons mentioned in the introduction $N, \Sigma, \Lambda, \Xi, \Delta, \Omega$ and use variables $\vec{p} \equiv \vec{p}_{12}$, $\vec{q} \equiv \vec{q}_3 = \vec{p}_3$ (c.m.). Then in the momentum space from (28a) and (29) we have:

$$\begin{aligned} {}_cV_{ij}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') = & -(2\pi)^6 \left\{ a_{ij} + b_{ij} \left(\left(\frac{1}{4} + \frac{3}{4} \delta_{(ij)}^{(12)} \right) \frac{\partial}{\partial \vec{p}'} \frac{\partial}{\partial \vec{q}'} + \right. \right. \\ & + (1 - \delta_{(ij)}^{(12)}) \left[\frac{\partial}{\partial \vec{q}'} \frac{\partial}{\partial \vec{q}'} + \frac{1}{2} (-)^{i+j+1} \left(\frac{\partial}{\partial \vec{p}'} \frac{\partial}{\partial \vec{q}'} + \right. \right. \\ & \left. \left. \left. + \frac{\partial}{\partial \vec{q}'} \frac{\partial}{\partial \vec{p}'} \right) \right] \right\} \delta^{(3)}(\vec{p} - \vec{p}') \delta^{(3)}(\vec{q} - \vec{q}'), \quad (30a) \end{aligned}$$

$$v_c^{(3)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') =$$

$$= -(2\pi)^6 b \left(\frac{3}{2} \frac{\partial}{\partial \vec{p}'} \frac{\partial}{\partial \vec{p}'} + 2 \frac{\partial}{\partial \vec{q}'} \frac{\partial}{\partial \vec{q}'} \right) \delta^{(3)}(\vec{p} - \vec{p}') \delta^{(3)}(\vec{q} - \vec{q}'), \quad (30b)$$

$$v_0^{(3)}(\vec{p}, \vec{q}; \vec{p}', \vec{q}') = (2\pi)^6 3a \delta^{(3)}(\vec{p} - \vec{p}') \delta^{(3)}(\vec{q} - \vec{q}').$$

If we put the expressions (30a,b) in the right side of the systems of equations (25) then there will appear the following type of integrals:

$$I(\vec{c}, \vec{d}) = \int \frac{\partial}{\partial \vec{c}'} \frac{\partial}{\partial \vec{d}'} \{ \delta^{(3)}(\vec{c} - \vec{c}') \delta^{(3)}(\vec{d} - \vec{d}') B(\vec{c}, \vec{d}; \vec{c}', \vec{d}') \times \\ \times N(\vec{c}', \vec{d}') \chi_M(\vec{c}', \vec{d}') \} d^3 \vec{c}' d^3 \vec{d}', \quad (31a)$$

which taking into account the boundary condition for the bound state wave functions $\chi(c_{x,y,z}, d_{x,y,z} = \pm\infty) = 0$ are reduced expression

$$I(\vec{c}, \vec{d}) = \frac{\partial}{\partial \vec{c}'} \frac{\partial}{\partial \vec{d}'} B(\vec{c}, \vec{d}; \vec{c}', \vec{d}') N(\vec{c}', \vec{d}') \chi_M(\vec{c}', \vec{d}') \Big|_{\vec{d}' = \vec{d}}^{\vec{c}' = \vec{c}}. \quad (31b)$$

As a result the system of equation (25) will be reduced to the system of the second order differential equations (nonlinear over M) for wave functions $\chi_M^{(\alpha_1' \alpha_2' \alpha_3')}(\vec{p}', \vec{q}')$.

For the wave functions we use the partial wave expansion:

$$\chi_M^{(\alpha_1 \alpha_2 \alpha_3)}(\vec{p}, \vec{q}) = \\ = \sum_{l_p l_q L S_{12} S J M_J} \left\langle \hat{\vec{p}}, \hat{\vec{q}} \left| (l_p l_q) L (S_{12} \frac{1}{2}) S J M_J \right. \right\rangle_M R_{l_p l_q S_{12}}^{(\alpha_1 \alpha_2 \alpha_3) L S J}(p, q), \quad (32)$$

where l_p and l_q are orbital momenta corresponding to moments \vec{p} and \vec{q} , respectively, S_{12} is total spin for the two particles system

(12), L and S are total orbital momenta and spin of the system (123). As a result from above mentioned system of differential equations for the function $\chi_M^{(\alpha_1\alpha_2\alpha_3)}(\vec{p}, \vec{q})$ it can be obtained the system of differential equations for radial wave functions ${}_M R_{l_p l_q S_{12}}^{(\alpha_1\alpha_2\alpha_3)LSJ}(p, q)$.

For calculation of the mass spectrum of bound (u, d, s) quark systems ($N, \Sigma, \Lambda, \Xi, \Delta, \Omega$ baryons) we will use the solutions of the differential equations obtained from system of equations (21a,b) in the non-relativistic limit. For this reason it must be used the approximations:

$$\omega_i \rightarrow m_i + \frac{\vec{p}_i^2}{2m_i}; \quad (\text{in left side})$$

$$\omega_i \rightarrow m_i, \gamma_i^0 \rightarrow 1, h_i \rightarrow m_i, \gamma_i^5 \rightarrow 0; \quad (\text{in right side}) \quad (33)$$

$\tilde{\varphi}_M^{(+++)} \rightarrow \chi_{MNR}^{(+++)}$, other components are equal to zero.

As a result taking into account only qq -interaction confinement potential we obtain well known equations ($m_1 = m_2 = m$)

$$\begin{aligned} [(\varepsilon_{B_p} + \varepsilon_{B_q}) - (\frac{\vec{p}^2}{2m_p} + \frac{\vec{q}^2}{2m_q}) + \frac{\mu_p \omega_p^2}{2} \Delta_p + \frac{2}{3} \alpha_S (4m^2) V_0 + \\ + \frac{\mu_q \omega_q^2}{2} \Delta_q + \frac{4}{3} \alpha_S ((m + m_3)^2) V_0] \chi_{MNR}(\vec{p}, \vec{q}) = 0, \end{aligned} \quad (34)$$

where

$$M - (2m + m_3) \equiv \varepsilon_{B_p} + \varepsilon_{B_q}, \quad \mu_p = \frac{m}{2}, \quad \mu_q = \frac{2mm_3}{2m + m_3}, \quad (35)$$

$$\omega_p = \omega_0 \sqrt{\frac{2}{3} [\alpha_S (4m^2) + \alpha_S ((m + m_3)^2) \frac{m_3}{m + m_3}]},$$

$$\omega_q = \omega_0 \sqrt{\frac{2}{3} \frac{2m+m_3}{m+m_3} \alpha_S ((m+m_3)^2)}.$$

The wave function $\chi_{MNR}(\vec{p}, \vec{q})$ is represented analogously to (32)

$$\chi_{MNR}(\vec{p}, \vec{q}) = \sum_{n_p n_q l_p l_q m_p m_q} \langle \hat{\vec{p}} | l_p m_p \rangle \langle \hat{\vec{q}} | l_q m_q \rangle R_{n_p l_p}(p) R_{n_q l_q}(q), \quad (36)$$

$$R_{n_p l_p}(p) = p_0^{-3/2} R_{n_p l_p}(x), \quad x = \frac{p}{p_0}, \quad p_0 = \sqrt{\mu_p \omega_p}, \quad (37a)$$

$$R_{n_q l_q}(q) = q_0^{-3/2} R_{n_q l_q}(y), \quad y = \frac{q}{q_0}, \quad q_0 = \sqrt{\mu_q \omega_q};$$

$$R_{nl}(u) = \frac{1}{\Gamma(l + \frac{3}{2})} \sqrt{\frac{2\Gamma(n + l + \frac{3}{2})}{\Gamma(n + 1)}} u^l \exp(-\frac{1}{2}u^2) {}_1F_1(-n, l + \frac{3}{2}; u^2); \quad (37b)$$

$$\begin{aligned} \varepsilon_{n_p l_p} + \frac{2}{3} \alpha_S (4m^2) V_0 &= (2n_p + l_p + \frac{3}{2}) \omega_p, \quad \varepsilon_{n_q l_q} + \\ &+ \frac{2}{3} \alpha_S ((m + m_3)^2) V_0 = (2n_q + l_q + \frac{3}{2}) \omega_q. \end{aligned} \quad (37c)$$

Now using the radial wave functions (37a) as a basis functions the unknown functions ${}_M R_{l_p l_q S_{12}}^{(\alpha_1 \alpha_2 \alpha_3) LSJ}(p, q)$ can be written as

$${}_M R_{l_p l_q S_{12}}^{(\alpha_1 \alpha_2 \alpha_3) LSJ}(p, q) = \sum_{n_p n_q} C_{n_p n_q S_{12}}^{(\alpha_1 \alpha_2 \alpha_3) LSJ}(M) R_{n_p l_p}(p) R_{n_q l_q}(q), \quad (38)$$

and putting it in the system of equation mentioned above we obtain the system of algebraic equation (nonlinear over M) for coefficients $C_{n_p n_q S_{12}}^{(\alpha_1 \alpha_2 \alpha_3)LSJ}(M)$, solution of which gives mass we are looking for.

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**ბმული qqq-სისტემებისათვის ბეტე-სოლპიტერის
განტოლების სამგანზომილებიან განტოლებაზე დაყვანის
სამი სხვადასხვა ვარიანტი**

გასკენა

განხილულია ბმული *qqq*-სისტემისათვის ბეტე-სოლპიტერის განტოლების სამგანზომილებიან განტოლებაზე დაყვანის სამი სხვადასხვა ვარიანტი ორ- და სამნაწილაკოვანი ურთიერთქმედების პოტენციალის გათვალისწინებით. მიღებულია ტალღური ფუნქციისათვის ნორმირების პირობა. მიღებული განტოლების არარელატივისტური ზღვრის ორნაწილაკოვანი კონფაინმენტური ურთიერთქმედების ოსცილატორული პოტენციალისათვის ამოხსნების (როგორც საბაზისო ფუნქციების) გამოყენებით დადგენილია ალგებრულ განტოლებათა სისტემა გაშლის კოეფიციენტებისათვის. შემდეგში გამოკვლეული იქნება ბმული მდგომარეობის არსებობის გამოკიდებულება ორ- და სამნაწილაკოვანი კონფაინმენტის (დატყვევების) პოტენციალების ლორენც (სპინურ) სტრუქტურაზე.