

**CHARGED LEPTON RADIATIVE AND B-MESON DOUBLE  
RADIATIVE DECAYS IN MODELS WITH UNIVERSAL  
EXTRA DIMENSIONS**

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**ABSTRACT.** This paper addresses the role of Large Extra Dimensions in some flavor changing neutral current (FCNC) driven processes. In particular we have investigated radiative decays of charged leptons within models with only one universal extra dimension (UED). Loop contributions with internal fermions and scalars of comparable mass would seem to yield sizeable amplitudes, since the generic quadratic suppression factor is changed into a linear one. Such scenarios can in principle be realized in models with universal extra space dimensions. Yet our calculations of the Kaluza-Klein (KK) contributions to these radiative decays show this expected relative enhancement to disappear due to the near mass degeneracy of the heavy neutrinos from the different generations. In this paper we estimate also the UED contribution to the  $B_{s,d} \rightarrow \gamma\gamma$  rate and find an enhancement of 3% and 6 %, respectively, over the SM prediction.

## 1. INTRODUCTION

The highly speculative idea of postulating extra dimensions to explain peculiar features observed in our world with 3+1 space-time dimensions has been revived for a novel reason, namely to provide an alternative approach to the gauge hierarchy problem [1,2]. An interesting feature of this novel insight into the hierarchy problem is that gravitational interactions become sizable not at the Planck scale, but already at the immensely lower scale  $\sim O(\text{TeV})$ , which in fact

must be considered as the only fundamental scale of nature. TeV scale dynamics in general will be explored directly by the LHC starting in 2007. The renaissance of multidimensional models is mainly due to superstring theories and their generalization, M-theory. It is the only consistent quantum theory known today that incorporates, at least in principle, all interactions including gravity. Both superstring and M-theory most naturally are formulated in  $d = 10$  and  $d = 11$  dimensions.

Since the extra dimensions can possess very different characteristics, models involving them lead to vastly different phenomenologies. Those characteristics refer not only to the size and other topological features of the extra dimensions (and whether they are of the space or time variety), but also to the fields that populate them. The options range from where only the graviton can propagate through the extra dimension(s) to where all fields can; in the latter case one talks about universal extra dimensions (UED) [3].

A remarkable feature of UED models [3] is the conservation of the so-called Kaluza-Klein (KK) parity, which leads to the absence of tree-level KK contributions to transitions at low energies, namely at scales  $\mu \ll 1/R$  with  $R$  denoting the compactification radius for extra dimensions. KK parity resembles  $R$  parity, which is conserved in many supersymmetric models. In particular KK parity prohibits the production of single KK modes off the interaction of ordinary particles.

Transitions driven by FCNC like  $K^0 - \bar{K}^0$ ,  $B^0 - \bar{B}^0$  oscillations and  $B_{s,d} \rightarrow \gamma\gamma$  are highly suppressed in the Standard Model (SM). Radiative  $\tau$  and  $\mu$  decays are even SM forbidden. New Physics contributions in general and KK ones in particular thus have (in principle) a considerably enhanced chance to make their presence felt in such processes.

In this paper we investigate lepton flavor violating radiative decays of charged leptons within models with only one universal extra dimension and also estimate their contributions to  $B_{s,d} \rightarrow \gamma\gamma$  transitions, which are allowed though suppressed in the SM. The article is organized as follows: after summarizing in Section 2 information about the UED model of Appelquist, Cheng and

Dobrescu (ACD) [3] relevant for our calculations we devote Sections 3 and 4 to the study of charged lepton decays  $l_i \rightarrow l_j \gamma$  and  $B_{s,d} \rightarrow \gamma \gamma$ , respectively, in the framework of the same model, before formulating our conclusions in Section 5. Some useful formulas are collected in the Appendix.

## 2. THE ACD MODEL

Modern models with extra space-time dimensions have received a great deal of attention because the scale at which extra dimensional effects become relevant could be around a few TeV [1,2,3]. If so, they could be searched for directly at the LHC. The first proposal for using large (TeV) extra dimensions in the SM with gauge fields in the bulk and matter localized on the orbifold fixed points was developed in Ref. [4]. The models with extra space-time dimensions can be built in several ways. Among them the following approaches are the most actively pursued ones: i) The ADD model of Arcani-Hammed, Dimopoulos and Dvali [1], where all elementary fields except the graviton are localized on a brane, while the graviton propagates in the whole bulk. ii) The RS model of Randall and Sundrum with warped 5-dimensional space-time and nonfactorized geometry [2]. iii) The ACD model of Appelquist, Cheng and Dobrescu (also referred to as model with Universal Extra Dimensions (UED)), where all fields can move in the whole bulk [3].

In UED scenarios the SM fields are thus described as nontrivial functions of all space-time coordinates. For bosonic fields one simply replaces all derivatives and fields in the SM Lagrangian by their 5-dimensional counterparts. These are the  $U(1)_Y$  and  $SU(2)_L$  gauge fields as well as the  $SU(3)_C$  gauge fields from the QCD sector. The Higgs doublet is chosen to be even under  $P_5$  ( $P_5$  is the parity operator in the five dimensional space) and possesses a zero mode. Note that all zero modes remain massless before the Higgs mechanism is applied. In addition we should note that as a result of action of the parity operator the fields receive additional masses  $\sim n/R$  after dimensional reduction and transition to the four dimensional Lagrangians;  $n$  is a positive integer denoting the KK mode.

In the five dimensional ACD model the same procedure for gauge fixing is possible as in the models in which fermions are localized on the 4-dimensional subspace. With the gauge fixed, one can diagonalize the kinetic terms of the bosons and finally derive expressions for the propagators. Compared to the SM, there are additional Kaluza-Klein (KK) mass terms. As they are common to all fields, their contributions to the gauge boson mass matrix is proportional to the unity matrix. As a consequence, the electroweak angle remains the same for all KK-modes and is the usual Weinberg angle  $\theta_W$ . Because of the KK-contribution to the mass matrix, charged and neutral Higgs components with  $n \neq 0$  no longer play the role of Goldstone bosons. Instead, they mix with  $W_5^\pm$  and  $\theta_W$  to form -- in addition to the Goldstone modes  $\theta_W$  and  $G_{(n)}^\pm$  -- three physical states  $a_{(n)}^0$  and  $a_{(n)}^\pm$ . Below we will study the impact of these new *charged* states.

The interactions of ordinary charged leptons with pairs of KK scalars and neutrinos ( $a_{(n)}$ ,  $\nu_{k(n)}$ ) is given by

$$L(l_j \nu_{i(n)} a_{(n)}^\pm) = \bar{l}_j (c_L P_L + c_R P_R) U_{ij} \nu_{i(n)} a_{(n)}^\pm + h.c. \quad (1)$$

with

$$P_{L,R} = \frac{1 \mp \gamma_5}{2}, \quad c_L = -(g_2 n m(l_j)) / (\sqrt{2} M_{W(n)}),$$

$$c_R = -(g_2 M_W) / (\sqrt{2} M_{W(n)}),$$

where  $g_2$  is the SU(2) coupling for the weak interactions,  $n$  labels the KK towers (e.g.  $M_{W(n)}$  is the mass for the  $n$ -th KK-mode:

$M_{W(n)}^2 = M_W^2 + n^2 / R^2$ ) and  $U_{ij}$  is an element in the MNS matrix, the leptonic analogue of the CKM matrix.

The complete list of Feynman rules for models with only one universal extra dimension has been given in [5].

The Lagrangian responsible for the interaction of the charged scalar KK towers  $a_{(n)}^*$  with the ordinary down quarks, is as follows

$$L = \frac{g_2}{\sqrt{2}M_{W(n)}} \bar{Q}_{i(n)} \left( C_L^{(1)} P_L + C_R^{(1)} P_R \right) a_{i(n)}^+ d_j + \frac{g_2}{\sqrt{2}M_{W(n)}} \bar{U}_{i(n)} \left( C_L^{(2)} P_L + C_R^{(2)} P_R \right) a_{i(n)}^+ d_j \quad (2)$$

with [5]

$$C_L^{(1)} = -m_3^{(i)} V_{ij}, \quad C_L^{(2)} = m_4^{(i)} V_{ij}, \quad C_R^{(1)} = M_3^{(i,j)} V_{ij},$$

$$C_R^{(2)} = -M_4^{(i,j)} V_{ij}$$

$$M_{W(n)}^2 = m^2(a_n^\pm) = M_W^2 + \frac{n^2}{R^2}; \quad (3)$$

here  $V_{ij}$  are elements of the CKM matrix. The mass parameters in (3) are defined as

$$m_3^{(i)} = -M_W c_{i(n)} + \frac{n}{R} \frac{m_i}{M_W} s_{i(n)}, \quad M_3^{(i,j)} = \frac{n}{R} \frac{m_j}{M_W} c_{i(n)},$$

$$m_4^{(i)} = M_W s_{i(n)} + \frac{n}{R} \frac{m_i}{M_W} c_{i(n)}, \quad M_4^{(i,j)} = \frac{n}{R} \frac{m_j}{M_W} s_{i(n)}; \quad (4)$$

here  $M_W$  and the masses of up (down)-quarks  $m_i(m_j)$  on the right hand side of (4) are zero mode masses and  $c_{i(n)}$ ,  $s_{i(n)}$  denote the sine and cosine of the fermion mixing angles, respectively

$$\operatorname{tg} 2\alpha_{f(n)} = \frac{m_f}{n/R}, \quad n \geq 1. \quad (5)$$

The masses for the fermions are calculated as

$$m_{f(n)} = \sqrt{\frac{n^2}{R^2} + m_f^2}. \quad (6)$$

In the phenomenological applications we use the restriction  $n/R > 250$  GeV and hence we assume that all the fermionic mixing angles except  $\alpha_{f(n)}$  are zero.

### 3. RADIATIVE DECAYS $L_I \rightarrow L_J \gamma$ OF CHARGED LEPTONS

In [6] a mainly model independent analysis of  $\mu \rightarrow e \gamma$  and  $\tau \rightarrow \mu \gamma$  on the one-loop level has been given. An important statement is that when the masses of the internal scalar and fermion masses are comparable, the transition amplitude becomes considerably enhanced over what one would usually expect. The authors state that such a situation arises in UED theories. We will analyze the situation in more detail specifically in theories with only one extra dimension [3].

Real decay processes, where the photon has to be on-shell represent a magnetic transition described by two form factors:

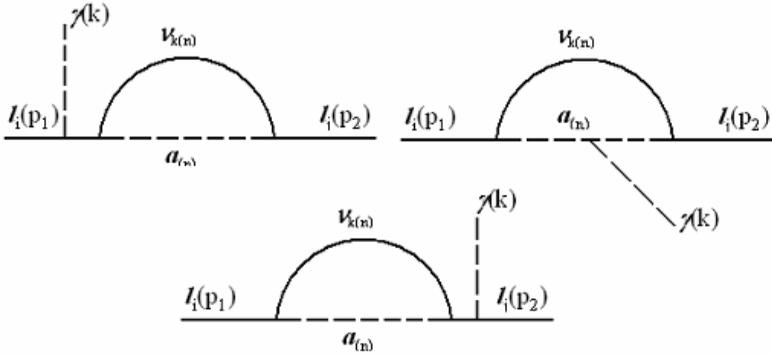
$$A(l_i \rightarrow l_j \gamma) = \varepsilon^\mu(k) \bar{u}_j(p_2) i \sigma_{\mu\nu} k^\nu (F_{2V} + F_{2A} \gamma_5) u_i(p_1) \quad (7)$$

$u_{i,j}$  denote the lepton spinors with momenta  $p_i$  and  $k = p_1 - p_2$  and  $\varepsilon^\mu(k)$  the photon polarization vector.

For completeness we give also the amplitude for such transitions with an *off* shell photon (or  $Z^0$ ), which contains four additional form factors:

$$\begin{aligned}
A(l_i \rightarrow l_j \gamma) = & \varepsilon^\mu(k) \bar{u}_j(p_2) [(F_{1V} + F_{1A} \gamma_5) \gamma_\mu + \\
& + i \sigma_{\mu\nu} k^\nu (F_{2V} + F_{2A} \gamma_5) + k_\mu (F_{3V} + F_{3A} \gamma_5)] u_i(p_1) \quad (8)
\end{aligned}$$

The specifics of the underlying dynamics then determine the form factors  $F_{iV}$  and  $F_{iA}$ . 'Switching on' one universal extra dimension expands the particle content of the model. In particular, KK-modes of the charged scalar boson are real particles in this case [3,5]. The relevant Feynman diagrams are shown in Fig. 1. Their contributions to  $l_i \rightarrow l_j \gamma$  are explicitly calculated in this section.



**Fig.1.**  $l_i \rightarrow l_j \gamma$  decay in the model with Universal Extra Dimensions (the dashed line in the loops correspond to the charged KK towers  $a_{(n)}^*$ , while the solid lines in the loops are for neutrino KK towers).

One can conclude from the expressions for the masses of the neutrino and physical scalar 'towers'

$$m^2(\nu_{k(n)}) = m^2(\nu_k) + \frac{n^2}{R^2}, \quad m^2(a_{(n)}) = M_W^2 + \frac{n^2}{R^2} \quad (9)$$

that they are comparable already for the first excited mode ( $n=1$ ), because very rough estimates for the compactification radius tell us:  $1/R > 250$  GeV.

Explicit calculations yield for the form factors  $F_{2V,2A}$  – relevant for  $l_i \rightarrow l_j \gamma$  – and the other four form factors  $F_{1V,1A}$  and  $F_{3V,3A}$  (which become relevant for  $l_i \rightarrow l_j \bar{l}$ )

$$F_{1V} = -\frac{i}{(4\pi)^2} e g_2^2 \frac{M_W^2}{M_{W(n)}^2} \frac{k^2}{m^2(a_{(n)})} U_{ik} U_{jk}^* \times$$

$$\times \left\{ \frac{n}{RM_W} \frac{m(v_{k(n)})}{M_W} \frac{(m(l_i) + m(l_j))^2}{2m^2(a_{(n)})} f_1(x_k) + \right. \quad (10a)$$

$$\left. \left( 1 + \frac{n^2}{R^2 M_W^2} \frac{m(l_i)m(l_j)}{M_W^2} \right) \left[ \frac{1}{4} f_2(x_k) + \right. \right.$$

$$\left. \left. + \frac{m^2(l_i) + m(l_i)m(l_j) + m^2(l_j)}{m^2(a_{(n)})} f_3(x_k) + \frac{1}{6} \frac{k^2}{m^2(a_{(n)})} f_4(x_k) \right] \right\},$$

$$F_{2V} = -\frac{1}{2} \frac{i}{(4\pi)^2} e g_2^2 \frac{M_W^2}{M_{W(n)}^2} \frac{m(l_i) + m(l_j)}{m^2(a_{(n)})} U_{ik} U_{jk}^* \times$$

$$\times \left\{ \frac{n}{RM_W} \frac{m(v_{k(n)})}{M_W} \left( -f_5(x_k) + \frac{k^2}{m^2(a_{(n)})} f_1(x_k) - \right. \right.$$

$$\left. \left. - \frac{m^2(l_i) + m^2(l_j)}{m^2(a_{(n)})} f_6(x_k) \right) + \left( 1 + \frac{n^2}{R^2 M_W^2} \frac{m(l_i)m(l_j)}{M_W^2} \right) \times \right.$$

$$\times \left\{ -\frac{1}{2} f_7(x_k) + \frac{k^2}{m^2(a_{(n)})} f_3(x_k) - \frac{2m^2(l_i) + 2m^2(l_j) + 3m(l_i)m(l_j)}{3m^2(a_{(n)})} f_4(x_k) \right\} \quad (10b)$$

$$F_{3V} = -\frac{1}{2} \frac{i}{(4\pi)^2} e g_2^2 \frac{M_W^2}{M_{W(n)}^2} \frac{m^2(l_i) - m^2(l_j)}{m^2(a_{(n)})} U_{ik} U_{jk}^* \times \left\{ \frac{n}{RM_W} \frac{m(v_{k(n)})}{M_W} \frac{m(l_i) + m(l_j)}{m^2(a_{(n)})} f_1(x_k) + \right. \quad (10c)$$

$$\left. + \left( 1 + \frac{n^2}{R^2 M_W^2} \frac{m(l_i)m(l_j)}{M_W^2} \right) \left[ \frac{1}{2} f_2(x_k) + \frac{2(m^2(l_i) + m(l_i)m(l_j) + m^2(l_j))}{m^2(a_{(n)})} f_3(x_k) + \frac{1}{3} \frac{k^2}{m^2(a_{(n)})} f_4(x_k) \right] \right\},$$

where

$$x_k = \frac{m^2(v_{k(n)})}{m^2(a_{(n)})} \quad (11)$$

and summation over the tower indices  $n$  is assumed in Eqs.(10a-10c); the functions  $f(x_n)$  are given in the Appendix. The axial form-factors are related to the corresponding vector ones by:

$$F_A(m(l_i), m(l_j)) = F_V(m(l_i), -m(l_j)). \quad (12)$$

Eqs.(10a-10c) demonstrate explicitly the general relation between form factors:

$$(m(l_i) - m(l_j))F_{1V} = -k^2 F_{3V} \quad (13)$$

Using Eq.(7) we obtain for the decay width:

$$\Gamma(l_i \rightarrow l_j \gamma) = \frac{|F_{2V}|^2 + |F_{2A}|^2}{8\pi} \left( \frac{m^2(l_i) - m^2(l_j)}{m(l_i)} \right)^3. \quad (14)$$

Let us note that the ratio  $x_n = m^2(\nu_{k(n)})/m^2(a_{(n)})$  is close to unity for all  $n$ , namely  $0.9 < x_k < 1$ :

$$x_k = \frac{m^2(\nu_{k(n)})}{m^2(a_{(n)})} = \frac{m^2(\nu_k) + \frac{n^2}{R^2}}{M_W^2 + \frac{n^2}{R^2}}. \quad (15)$$

With the rough estimate for the compactification radius ( $1/R > 250\text{GeV}$ ) we have already for the first KK-mode  $x_k > 0.9$ . Noting that the masses of the scalar and fermion towers are close to each other for the same  $n$ , we can simplify Eq.(10b):

$$F_{2V} = \frac{1}{2} \frac{i}{(4\pi)^2} e g_2^2 \frac{M_W^2}{M_{W(n)}^2} \frac{m(l_i)}{m^2(a_{(n)})} U_{ik} U_{jk}^* \times \left\{ \frac{n}{RM_W} \frac{m(\nu_{k(n)})}{M_W} \left( -\frac{1}{4} + \frac{x_k}{12} \right) + \frac{x_k}{60} \right\} \quad (16)$$

and thus

$$\text{Br}(l_i \rightarrow l_j \gamma) = \frac{6\alpha}{\pi} \frac{M_W^8}{M_{W(n)}^8} \left( U_{ik} U_{jk}^* f(x_k) \right)^2, \quad (17)$$

In four-dimensional models with small Dirac neutrino masses the ratio of neutrino mass square differences to the  $W$ -boson square mass

serves as a highly efficient *suppression* factor for  $l_i \rightarrow l_j \gamma$ . Eq. 16 exhibits an apparently *linear* dependence on the neutrino to  $W$  mass ratio for the exchange of KK towers in the loops. This might be seen at first as leading to a very considerable enhancement of the  $l_i \rightarrow l_j \gamma$  amplitude. This conclusion, however, would be fallacious. For upon explicit substitution of Eq.9 for the KK masses into the form factor expression in Eq.16 the *quadratic* dependence re-emerges:

$$\begin{aligned} \frac{n}{RM_W} U_{ik} U_{jk}^* \frac{m(\nu_{k(n)})}{M_W} &= \\ &= \frac{n}{RM_W} U_{ik} U_{jk}^* \left( \frac{n}{R} + \frac{rm^2(\nu_k)}{2n} \right) \frac{1}{M_W} = U_{ik} U_{jk}^* \frac{m^2(\nu_k)}{2M_W^2}. \end{aligned} \quad (18)$$

The initial appearance of a merely linear suppression thus disappears due to the near-degeneracy of the masses for neutrino KK-towers from different generations. For example, we have for two neutrino generations:

$$\begin{aligned} m(\nu_{\mu(n)}) - m(\nu_{e(n)}) &= \\ &= (m(\nu_{\mu}) - m(\nu_e)) \frac{m(\nu_{\mu}) + m(\nu_e)}{2n/R} \ll m(\nu_{\mu}) - m(\nu_e) \end{aligned} \quad (19)$$

In the end the following expression emerges for the branching ratio:

$$(l_i \rightarrow l_j \gamma) = \frac{3\alpha}{32\pi} \frac{M_W^8}{M_{W(n)}^8} \left( U_{ik} U_{jk}^* \frac{m^2(\nu_k)}{M_W^2} \right)^2, \quad (20)$$

This expression shows that it cannot enhance the SM result [7-12] significantly:

$$\text{Br}(l_i \rightarrow l_j \gamma)_{CM} = \frac{3\alpha}{32\pi} \left( U_{ik} U_{jk}^* \frac{m^2(\nu_k)}{M_W^2} \right)^2. \quad (21)$$

In the slightly extended SM with massive left-handed neutrinos lepton flavor violating processes like of  $\mu \rightarrow e\gamma$  and  $\tau \rightarrow \mu\gamma$  are extremely suppressed. For example, taking into consideration the data from modern neutrino experiments [13], the branching ratio for  $\mu \rightarrow e\gamma$  is predicted to be as low as  $7 \cdot 10^{-57}$  [7-12] in the SM.

A planned experiment is expected to lower the existing upper bound  $\text{Br}(\mu \rightarrow e\gamma) < 1.2 \cdot 10^{-11}$  to the  $10^{-13} - 10^{-14}$  levels [14].

#### 4. $B_{S,D} \rightarrow \gamma\gamma$ DECAY IN MODELS WITH ONE UNIVERSAL EXTRA DIMENSION

Detailed studies of the decays of strange hadrons were instrumental in developing the Standard Model (SM). Recent findings on B decays – in particular the CP asymmetry observed in  $B \rightarrow \psi K_S$  by the BABAR and BELLE collaborations at the B factories – represent a striking confirmation of the SM [38]. Yet they do not invalidate the various theoretical arguments pointing to its incompleteness, i.e. the existence of physics beyond the SM (BSMP). If anything they even strengthen the case for a new paradigm. History might well repeat itself in the sense that future detailed studies of the decays of beauty and charm hadrons and tau leptons will reveal the intervention of BSMP.

The BABAR and BELLE experiments running at the two B factories at SLAC in the USA and at KEK in Japan are producing the huge high-quality data sets required for such searches for BSMP. There is even a proposal in Japan for building a Super-B factory with much higher luminosity; similar plans are being pursued in Italy. Further information will be gained from the tau-charm factories at Cornell University in the US and at Beijing in China.

Some experimental evidence for an incompleteness of the CKM description has actually merged in the CP asymmetry in  $B \rightarrow \phi K_S$  and

similar channels. It also points towards radiative and related B decays as promising areas for search for BSMP.

Exploration of B-physics, including  $B$ -meson rare decays is one of the central issues of the physical programs at accelerator facilities operating now or soon getting online. The process  $B_{s,d} \rightarrow \gamma\gamma$ , which is the subject of this section, has an unusual experimental signature that can be searched for at least at  $B$  factories. It should be noted that the two photon final states produced in this process can be CP even as well as CP odd. This feature might allow searches for nontraditional sources of CP violation in  $B$ -physics. In general the process  $B_{s,d} \rightarrow \gamma\gamma$  is sensitive to BSMP effects. The experimental feasibility has stimulated efforts of theoretical groups as well [15-32].  $B_{s,d} \rightarrow \gamma\gamma$  rates have been calculated within the SM with and without QCD corrections, in multi-Higgs doublet as well as supersymmetric models.

In the SM  $B_{s,d} \rightarrow \gamma\gamma$  first arise at the one loop level with the exchange in the loops by up-type quarks and W-bosons [15-19]. SM predictions for the branching ratios are of the order of  $10^{-7}$  ( $10^{-9}$ ).

It has been shown that in extended versions of the SM (multi-Higgs doublet and supersymmetric models) one could reach branching ratios as large as  $Br(B_{s,d} \rightarrow \gamma\gamma) \sim 10^{-6}$  depending on the parameters of the models. This enhancement was achieved mainly due to exchange of charged scalar Higgs particles within the loop. There exists an analogous possibility in other exotic models as well for the scalar particle exchange inside the loop, which could enhance this process. For example, the Appelquist, Cheng and Dobrescu (ACD) model with only one universal extra dimension [3] presents us with such an opportunity. One should note that in the above approach towers of charged Higgs particles arise as real objects with definite masses, not as fictitious (ghost) fields.

In this Section we calculate the contributions from these real scalars to  $B_{s,d} \rightarrow \gamma\gamma$ . The Feynman graphs, describing the contributions of scalar physical states to process under consideration, are shown in Fig.2.

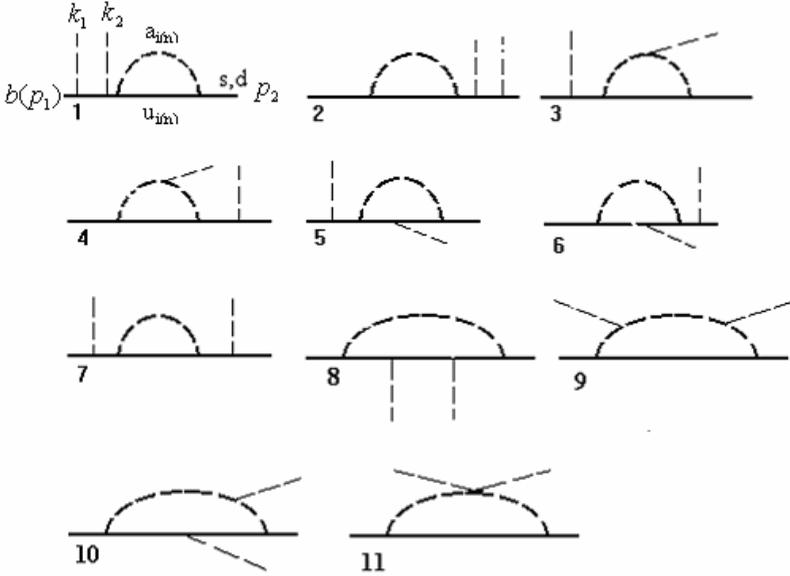
The amplitude for the decay  $B_{s,d} \rightarrow \gamma\gamma$  has the form

$$T(B \rightarrow \gamma\gamma) = \varepsilon_1^\mu(k_1) \varepsilon_2^\nu(k_2) [A g_{\mu\nu} + iB \varepsilon_{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta] \quad (22)$$

This equation holds after gauge fixing for the final photons which we have chosen as

$$\varepsilon_1 k_1 = \varepsilon_2 k_2 = \varepsilon_1 k_2 = \varepsilon_2 k_1 = 0. \quad (23)$$

This is possible because in the CM frame both final photons are collinear, where  $\varepsilon_1$  and  $\varepsilon_2$  are photon polarization vectors, respectively.



**Fig.2.** Double radiative B-meson decay  $B_{s,d} \rightarrow \gamma\gamma$  in the theory with only one extra universal dimension (the dashed line in the loops correspond to the charged KK towers  $a_{(n)}^*$ , while the solid lines in the loops are for up-quark KK towers).

The gauge condition of Eq.(23) together with energy-momentum conservation leads to

$$\varepsilon_i P = \varepsilon_i p_1 = \varepsilon_i p_2 = 0, \quad (24)$$

where

$$P = k_1 + k_2 \quad \text{and} \quad p_1 = p_2 + k_1 + k_2. \quad (25)$$

Let us write down some useful kinematical relations resulting from Eqs.(23-25):

$$\begin{aligned} (k_1 k_2) &= (P k_1) = (P k_2) = \frac{1}{2} M_B^2, \quad (p_1 p_2) = -m_b m_{s(d)}, \quad (P p_1) = m_b M_B, \\ (p_1 k_1) &= (p_1 k_2) = \frac{1}{2} m_b M_B, \quad (p_2 k_1) = (p_2 k_2) = -\frac{1}{2} m_{s(d)} M_B, \\ (P p_2) &= -m_{s(d)} M_B. \end{aligned} \quad (26)$$

The total contributions into CP even (A) and CP odd (B) amplitudes from Eq.(22) are calculated as sums of the appropriate contributions of the diagrams in Fig.2 corresponding to a tower of scalar particle contributions in the ACD model with only one extra dimension. Let us note that we used the following formula for the hadronic matrix elements:

$$\langle 0 | \bar{s} \gamma_\mu \gamma_5 b | B_s(P) \rangle = -i f_B P_\mu. \quad (27)$$

Apart from one particle reducible (1PR) diagrams, one particle irreducible (1PI) ones contribute to the amplitudes, and hence, to their CP even (A) and CP odd (B) parts. We should note that each of the 1PI contributions is finite. Let us discuss these contributions in more details. In the SM only one 1PI diagram (the one with the W-boson exchange in the loop, when both photons are emitted by virtual up-type quarks) gives a contribution of order  $\sim 1/M_W^2$ . In Ref.[33] it was

observed that diagrams with light quark exchanges contribute as  $\sim 1/M_W^2$ , while diagrams containing the heavy quarks are of order  $\sim 1/M_W^4$ . In the ACD model the contributions of such diagrams are also of order  $\sim 1/M_W^4$  because the estimate for all KK-tower masses, including the ones exchanged in the loops, is given in units of  $1/R > 250\text{GeV}$  [34]. Similar considerations show that all the 1PI diagrams existing in the ACD model also are of order  $\sim 1/M_W^4$ . Thus, the leading 1PI diagrams are negligible and we do not consider them.

The total contributions to the  $B \rightarrow \gamma\gamma$  decay amplitudes are:

$$\begin{aligned}
A &= \frac{i}{(4\pi)^2} \frac{1}{4} e^2 g_2^2 f_B \frac{Q_d}{M_{W(n)}^2} \frac{1}{m^2(a_{(n)}^\pm)} \frac{m_b^3}{m_{s(d)}} V_{is(d)}^* V_{ib} \times \\
&\quad \times \left\{ \frac{n}{RM_W} m_3^{(i)} m_{i(n)} c_{i(n)} f_8(x_i) + \right. \\
&\quad \left. + (m_3^{(i)2} - \frac{n^2}{R^2 M_W^2} m_b m_{s(d)} c_{i(n)}^2) \frac{1}{2} f_7(x_i) \right\}. \\
B &= \frac{i}{(4\pi)^2} \frac{1}{2} e^2 g_2^2 f_B \frac{Q_d}{M_{W(n)}^2} \frac{1}{m^2(a_{(n)}^\pm)} \frac{m_b}{m_{s(d)}} V_{is(d)}^* V_{ib} \times \\
&\quad \times \left\{ \frac{n}{RM_W} m_3^{(i)} m_{i(n)} c_{i(n)} f_8(x_i) + \right. \\
&\quad \left. + (m_3^{(i)2} + \frac{n^2}{R^2 M_W^2} m_b m_{s(d)} c_{i(n)}^2) \frac{1}{2} f_7(x_i) \right\}, \quad (28)
\end{aligned}$$

where

$$f_8(x) = \frac{-5x^2 + 8x - 3 + 2(3x - 2)\ln x}{6(1-x)^3},$$

$$f_7(x) = \frac{-2x^3 - 3x^2 + 6x - 1 + 6x^2 \ln x}{6(1-x)^4},$$

$$x_i = \frac{m^2(u_{i(n)})}{m^2(a_{(n)}^*)}. \quad (29)$$

As is obvious from Fig.2, the correct calculation must include the crossed diagrams (not shown on Fig.2). In the kinematics we use, cf. Eqs.(23)-(26), this leads to a factor 2 for all amplitudes, except for the one given by diagram 11. However, diagram 11 belongs to the class of 1PI diagrams. As it was stated above, those contributions are order  $\sim 1/M_W^4$  and thus negligible compared to those from the 1PR diagrams.

On the other hand, using the unitarity of the CKM matrix, the amplitude for double radiative B-meson decay can be rewritten as:

$$T = \sum_{i=u,c,t} \lambda_i T_i = \lambda_t \left\{ T_t - T_c + \frac{\lambda_u}{\lambda_t} (T_u - T_c) \right\}. \quad (30)$$

Let us note that we restricted ourselves to calculating the leading order terms of  $\sim 1/M_W^2$  from the up-quark KK-towers. In this approximation it turns out that the  $u_{(n)}$  and the  $c_{(n)}$  towers have equal contributions. Therefore, the expressions for the amplitudes have a simpler form than before:

$$A = \lambda_t (A_{t(n)} - A_{c(n)}), \quad B = \lambda_t (B_{t(n)} - B_{c(n)}) \quad (31)$$

Furthermore, it is easy to obtain from Eq.(22) the expression for the  $B \rightarrow \gamma\gamma$  decay partial width:

$$\Gamma(B \rightarrow \gamma\gamma) = \frac{1}{32\pi M_B} \left[ 4|A|^2 + \frac{1}{2} M_B^4 |B|^2 \right]. \quad (32)$$

Now we are in the position to compare the ACD contribution to the decay with that of the SM.

Namely, let us consider the ratio

$$\frac{\Gamma(B_{s(d)} \rightarrow \gamma\gamma)_{ACD}}{\Gamma(B_{s(d)} \rightarrow \gamma\gamma)_{CM}} = \frac{24}{Q_d^2} \frac{M_W^4}{M_{W(n)}^4} \frac{M_W^2 n^2 / R^2}{m^4 (a_n^\pm)} \times$$

$$\times \frac{\left\{ \frac{m_3^{(l)} m_{l(n)}}{M_W^2} c_{l(n)} f_8(x_{l(n)}) + \frac{n}{RM_W} f_8(x_{c(n)}) \right\}^2}{4 \left( C(x_l) + \frac{23}{3} \right)^2 + 2 \left( C(x_l) + \frac{23}{3} + 16 \frac{m_{s(d)}}{m_b} \right)^2}. \quad (33)$$

where

$$C(x) = \frac{22x^3 - 153x^2 + 159x - 46}{6(1-x)^3} + \frac{3(2-3x)}{(1-x)^4} \ln x,$$

$$x_l = \frac{m_l^2}{M_W^2}. \quad (34)$$

Rough numerical estimates show that the pure UED contributions to  $B_s \rightarrow \gamma\gamma$  and  $B_d \rightarrow \gamma\gamma$  enhance the SM estimate by about 3% and 6%, respectively.

## CONCLUSION

In this paper we have investigated lepton flavor violating radiative decays of charged leptons  $l_i \rightarrow l_j \gamma$  within models with only one universal extra dimension and have estimated also their contributions to the highly suppressed SM rates for  $B_{s,d} \rightarrow \gamma\gamma$ .

Planned experiment are expected to lower the existing upper bound  $Br(\mu \rightarrow e\gamma) < 1.2 \cdot 10^{-11}$  to the  $10^{-13} - 10^{-14}$  levels [14]. There are bad news and good news from our analysis:

- The bad news are that UED models with only one additional spatial dimension cannot raise  $Br(\mu \rightarrow e\gamma)$  into a range, where it could ever be observed.
- The good news are that if  $\mu \rightarrow e\gamma$  is ever observed, it must have a completely different origin.

The pure UED contribution to the  $B_s \rightarrow \gamma\gamma$  [ $B_d \rightarrow \gamma\gamma$ ] rate is 3% [6%] of the SM estimates of  $Br(B_s \rightarrow \gamma\gamma) \sim 10^{-7}$  and  $Br(B_d \rightarrow \gamma\gamma) \sim 10^{-9}$ , i.e. rather small. It is quite possible that the as yet uncalculated radiative QCD corrections could enhance these rates further and that they become observable at a Super- $B$  factory. Then they might be relevant for the central goal of  $B$  physics studies to not only discover New Physics, but also identify its salient features.

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## APPENDIX

The functions  $f(x_i)$  appearing in Eqs.(10a-10c,16) are given by:

$$f_1(x) = \frac{1 - 9x - 9x^2 + 17x^3 - 6x^2(3+x)\ln x}{36(1-x)^5},$$

$$f_2(x) = \frac{2 - 9x + 18x^2 - 11x^3 + 6x^3 \ln x}{18(1-x)^4},$$

$$f_3(x) = \frac{1 - 8x + 36x^2 + 8x^3 - 37x^4 + 12x^3(4+x)\ln x}{144(1-x)^6},$$

$$f_4(x) = \frac{3 - 20x + 60x^2 - 120x^3 + 65x^4 + 12x^5 - 60x^4 \ln x}{240(1-x)^6},$$

$$f_5(x) = \frac{x^2 - 1 - 2x \ln x}{2(1-x)^3}, \quad f_6(x) = \frac{-1 - 9x + 9x^2 + x^3 - 6x(1+x)\ln x}{6(1-x)^5},$$

$$f_7(x) = \frac{-1 + 6x - 3x^2 - 2x^3 + 6x^2 \ln x}{6(1-x)^4},$$

$$f(x_k) = \frac{n}{RM_W} \frac{m(v_{k(n)})}{M_W} \left( -\frac{1}{4} + \frac{x_k}{12} \right) + \frac{x_k}{60}.$$

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**დამუხტული ლეპტონების რადიაციული და B-მეზონების  
ორმაგი რადიაციული დაშლები უნივერსალური  
დამატებითი განზომილებების მოდელში**

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