Use of singularity points in method of reconstruction 3d surface from 2d shading image

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Annotation
Reconstruction of 3D geometrical shape from its 2D shading image is main problem in Computer Science. A new approach in solving of this problem is considered. There are complex surfaces, which have singularity points: fold, cusp, pinch points. We construct shading images of neighboring surfaces of these points. These images are used as geometrical primitives in segmentation process.

Keywords: shape from shading, singularity, cusp point, pinch point

Introduction

The construction of geometric model of three dimensional surface from its two dimensional shading image is one of the main problems in Computer Science. There are some approaches to solve this task. One approach is based on Berthold K.P. Horn research [1]:

An image of object will exhibit shading (gradations of reflected light intensity) which can be used to determine its shape, given only a picture from a single viewpoint. At each point in the image we know only the reflectivity at the corresponding object point. Let's consider 3d coordinate system. The z-axis is directed to the image, x and y-axis are paralleled of the image plane.

Method of reconstruction

It is possible to construct tangent plane on each point of smooth surface. We are interested in orientation of this plane to receive information about shape near this point. The normal vector, which is perpendicular of tangent plane, is used to determine this orientation. We can describe surface of object by means of z value. This value is depended on x and y value. We must define normal of surface through z and partial derivative z by x and y. We can express normal by way of small increment δx along x-axis from given point (x, y), z value is changed

$\delta z=(\delta z/\delta x) \cdot \delta x +e$.

The first partial derivative of z by x - p is incline of surface along x-axis, and by y - q is incline of surface along y-axis. The vector product of this two incline vector is normal vector n=(-p, -q, 1). The (p, q) is gradient vector of surface.

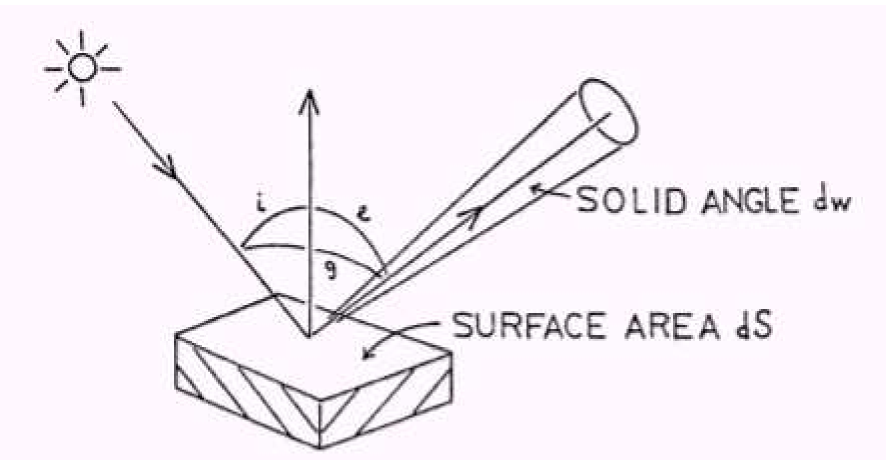

Fig. 1 Illustration of variables used in the definition of the reflectivity function.

Let's describe Reflectivity function: consider a surface of size ds. See fig1. Incident ray creates i angle and emitted ray created e angle at normal vector. Let the incident light intensity be I per unit area perpendicular to the incident ray. The amount of light falling on the surface element is then $I_1 \cdot \cos(i)$ ds. Let the emitted ray have intensity I_2 per unit solid angle per unit area perpendicular to the emitted ray. So the amount of light intercepted by an area subtending a solid angle dw at the surface element will be $I_2 \cos(e)$ ds dw. The reflectivity function is then defined to be I_2/I_1.

For many surfaces the fraction of the incident light which is scattered in a given direction is a smooth function of the angles involved. The shape can be obtained from the shading if we know the reflectivity function and the position of the light-source. The reflectivity and the gradient of the surface can be related by a non-linear first-order partial differential equation in two unknowns. The recipe for solving this equation is to set up an equivalent set of five ordinary differential equations (three for the coordinates and two for the components of the gradient).

A curve traced out by solving this set of equations for one set of starting values is called a characteristic strip. Starting one of these strips from each point on some initial curve will produce the whole solution surface. The initial curves can usually be constructed around so-called singular points, where the reflectivity uniquely determines the local normal. The singular points are brightest or darkest points on the image of the surface. Unfortunately, differential equations can not be solved from these points, because value of derivative of coordinates is zero. It is possible to build sphere around these points and starts solve equations from bound of this sphere. But the sphere is not enough to modeling difficult surfaces.

We consider more complex surfaces by means of singularity theorem [2].

Let p is a point on an image plane of object appropriate to a point **p**1 of object. We can draw a small circle in image a plane of object with the center in p point. If the piece of our object represented by this circle, itself looks like a circle, then this point p is regular point. Each point on the normal surface is center of spherical area. Surface within this area is represented as image of stable mapping of image plane to the object space. If the image of stable mapping looks as strongly curved disk at the given neighborhood point, then this is singular point. There are three kinds of singular points of stable mapping: The surface neighborhood of *double point* is represented as two cross sheet planes. The neighborhood of *triple point* is represented as three transversal cross sheets. Pinch point is another singular point. Every neighborhood of this point intersects itself. We have Whitney umbrella surface, which shading model was constructed. Red point is the pinch point. See Fig.2

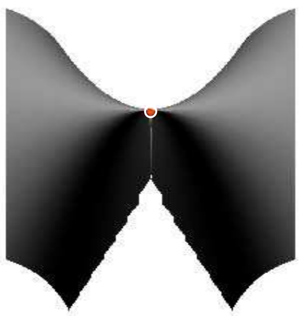

Fig. 2. Shading image of umbrella surface

The canonical form of umbrella is $(x, y, z)=(r \cdot t, r, t^2)$, where x, y and z are local coordinates of object space, r and t – coordinates of image plane. It is described geometrically how we get pinch point: when r crosses the origin of coordinates, parabola $r^2 \cdot z - x^2=0$ folds, passing through a double line, and opens again, as its plane goes forward along an y axis.

An important visibility feature of the object seen from a large distance along a certain direction is its contour generator, also known as outline, or profile.

The apparent contour is the projection of the contour generator onto a plane perpendicular to the view direction. In many cases, drawing just the visible part of the apparent contour gives a good impression of the shape of the object. Generally, this apparent contour is a smooth curve, with some isolated singularities.

The first singularity is Whitney fold [3]. This singularity takes place at plane projection of sphere in points of equator. This mapping is expressed in coordinates as $y_1=x_1^2$, $y_2=x_2$. Where y_1 y_2 is coordinate system of image plane, x_1 x_2 is coordinate system of object space.

The second singularity is Whitney cusp.

Canonical form of cusp surface is $z=y^3-x \cdot y$, see fig 3.

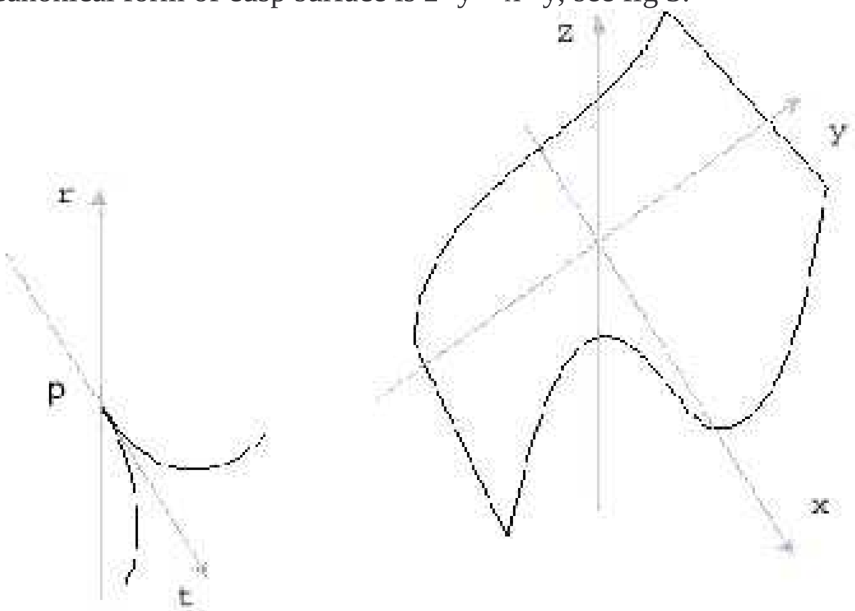

Fig 3. Cusp surface

We project cusp surface on image plane (r, t). This surface has fold points, where tangent plane is perpendicular of z-axis. The projection of these points on image plane has semicubical parabola, which has cusp point P at origin. A cusp is a point on a continuous curve where the tangent vector reverses sign as the curve is traversed. It is described geometrically how we get cusp point: when cubic curve goes forward along x-axis, x value is changed: if $x>0$, we have two critical point; at origin of coordinate system - $x=0$ and we have cusp point; if $x<0$, we have not critical point.

The shading image of contour was constructed, see fig 4.

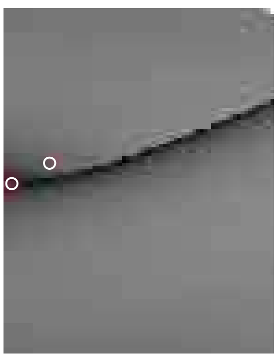

Fig 4. Shading image of cusp contour

Shading surface was constructed, see fig 5. Cusp point is marked as red point.

Fig 5.

Marking the feature points of surface and receiving shading image from several angles is used in second approach. In case of complex surface, for example- human face, it is not possible to mark points everywhere.

Conclusion

Our method takes into account both positive sides of these approaches, but instead of differential equations, we use algebraic equations of predetermined sorted surfaces.

On the first step we define outline of the surface and receive rough model. On the second step we mark feature points and these neighbor areas. Then we compare them with predetermined sorted surfaces such as cusp surface or sphere by means of least squares fitting method.

Then we step by step change geometrical parameters of already defined model until we will receive real shading model.

Appendix

Algorithm:

Let p be a critical point on (x, y) image plane

Let be a_1, a_2, a_3 parameters of deformation

Let $(x_1 y_1, x_2 y_2 .... x_n y_n)$ coordinates of fold critical points near cusp point p

Repeat

Find p:

Intensity (p) =max

Analyze neighborhood of p:

Move origin of coordinate system of image plane on critical point

If D (p) = 0 (derivative), there is degenerate singular point - cusp point

Find deformation: F(x) = $a_1 \cdot x^3 + a_2 \cdot x + a_3$

Let A is matrix of values of basis functions

$$A = \begin{pmatrix} 1 & x_1 & x_1^{\;3} \\ 2 & x_2 & x_2^{\;3} \\ 3 & x_n & x_n^{\;3} \end{pmatrix}$$

Solve deformation parameters $\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = ((A^T A)^{-1} A^T) y$

If D (p) ≠ 0 there is Morse critical point – fold point

Find deformation: F(x) = $a_1 \cdot x^2 + a_2$

$$A = \begin{pmatrix} 1 & x_1 & x_1^{\;2} \\ 2 & x_2 & x_2^{\;2} \\ 3 & x_n & x_n^{\;2} \end{pmatrix}$$

Solve deformation parameters $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = ((A^T A)^{-1} A^T) y$

Until no other critical points.

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