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On the boundary value problems of the dynamical theory of the generalized two-dimensional couple-stress thermoelasticity solved in quadratures

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Abstract:

We have considered the basic dynamical homogeneous system of partial differential equations of generalized Green-Lindsay couple-stress thermoelasticity on the plane for homogeneous, izotropic elastic media with the centre of symmetry. We have constructed regular solution of the boundary problems on the line.In the works are obtained in quadrates the solutions of the following boundary-value problems of the generalized Green-Lindsay theory of couple-stress thermoelasticity :

1. *On border of area are given: the component of normal of displacement vector, the component of touching of voltage vector, rotations and flow of heat.*
2. *On border of area are given: the components of normal voltage vector and the couple-stress, the component of touching of displacement vector and heat.*

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The basic dynamical homogeneous system of partial differential equations of generalized Green-Lindsay couple-stress thermoelasticity on the plane for homogeneous, izotropic elastic media with the centre of symmetry has the form [1]

$$\begin{cases} (\mu+\alpha)\Delta\upsilon+(\lambda+\mu-\alpha)graddv\upsilon+2\alpha rot\upsilon_3-\gamma grad\upsilon_4-\gamma\tau_1\dfrac{\partial}{\partial t}grad\upsilon_4=\varsigma\dfrac{\partial^2\upsilon}{\partial t^2} \\ (\nu+\beta)\Delta\upsilon_3+2\alpha rot\upsilon-4\alpha\upsilon_3=I\dfrac{\partial^2\upsilon_3}{\partial t^2} \\ \Delta\upsilon_4-\dfrac{1}{\partial\ell}\dfrac{\partial\upsilon_4}{\partial t}-\dfrac{\tau_0}{\partial\ell}\dfrac{\partial^2\upsilon_4}{\partial t^2}-\eta\dfrac{\partial}{\partial t}div\upsilon=0 \end{cases} \qquad (1)$$

Where, $\upsilon=(\upsilon_1,\upsilon_2)$ is displacement vector, υ_3 _is characteristic of the rotation; _υ_4 is the temperature variation; $x=(x_1,x_2)$ is the point of the twodimensional Euclidean space _R^2, t is the time, Δ-is the twodimensional Laplacian operator. $\varsigma,\mu,\lambda,\alpha,\partial\ell,I,\nu,\beta,\gamma,\eta$ -Constants whoch satisfy the following condition [1]:

$$\varsigma>0,\ \mu>0,\quad 3\lambda+2\mu>0,\ \alpha>0,\ \partial\ell>0,\ I>0,\ \nu>0,\ \beta>0,\ \gamma/\eta>0.$$

τ_0,τ_1-The constants of relaxations [1]: $\tau_1\geq\tau_0>0$.

With the first system we discuss (interesting from the practical point of vew) the following case about which depend on the t (time).

Along with (1) we shall consider some possible (nteresting from practical point of view) cases when $\upsilon_k(x,t),\quad k=1,\overline{4}$ depends on the time t:

$$\upsilon_k(x,t)=\frac{1}{2\pi i}\int_{\sigma-i\infty}^{\sigma+i\infty}e^{\pi}u_k(x,\tau)d\tau\ \ ,\tau=\sigma+iq,\ \varsigma>0-$$ the general dynamical case (representation by the Laplace-Mellin integral). The system (1) in both cases towards the vector $U=(u,u_3,u_4)^T=\|u_k\|_{4x1}-$comes to the following form:

$$(\mu+\alpha)\Delta u+(\lambda+\mu-\alpha)graddivu+2\alpha rotu_3-\gamma_\tau gradu_4-\zeta\tau^2 u=0$$
$$(\nu+\beta)\Delta u_3+2\alpha rotu-4\alpha u_3=0 \qquad (2)$$
$$\Delta u_4-\frac{\tau}{\partial\ell_\tau}u_4-\eta\tau divu=0$$

where, $\gamma_\tau=\gamma(1+\tau_1\tau)$, $\frac{1}{\partial\ell_\tau}=\frac{1}{\partial\ell}(1+\tau_0\tau)$, $rotu_3=\left(\frac{\partial u_3}{\partial x_2},-\frac{\partial u_3}{\partial x_1}\right)^T$ – is vector,

$rotu=\left(\frac{\partial u_2}{\partial x_1}-\frac{\partial u_1}{\partial x_2}\right)$ is scalar.

For the component voltage vector and the couple-stress we have:

$$\sigma_{jk}=[\lambda divu-\gamma_\tau u_4]\sigma_{jk}+(\mu+\alpha)\frac{\partial u_k}{\partial x_j}+(\mu-\alpha)\frac{\partial u_j}{\partial x_k}-2\alpha\varepsilon jku_3$$
$$\mu_k=(\nu+\beta)\frac{\partial u_3}{\partial x_k} \qquad j,k=1,\overline{2}$$

where, $\varepsilon_{jk}=\begin{cases}0, & j=0\\ 1, & j<k\\ -1, & j>k\end{cases}$, δ_{jk} – symbol of Kronikery.

Let now:

$D\equiv\{x=(x_1,x_2)\in R^2,\ x_2\geq 0\}$ – is hemi- plane .

Now, we consider the regular value $U=(u,u_3,u_4)^T=\|u_k\|_{4x1}$ in the D of the boundary problems for the system(2), on the line $x_2=0$:

problem IV: On border of area are given: the component of normal of displacement vector, the component of touching of voltage vector, rotations and flow of heat:

$$\sigma_{12}=\varphi_1(x_1), \qquad u_3=\varphi_3(x_1)$$
$$u_2=\varphi_2(x_1), \qquad \frac{\partial u_4}{\partial x_2}=\varphi_4(x_1)$$

problem V: On border of area are given: the components of normal voltage vector and the couple-stress , the component of touching of displacement vector and heat:

$$u_1=\psi_1(x_1) \qquad \frac{\partial u_3}{\partial x_2}=\psi_3(x_1)$$
$$\sigma_{22}=\psi_2(x_1) \qquad u_4=\psi_4(x_1)$$

Where, $\varphi_j(x_1)$, $\psi_j(x_1)$ $j=1,\overline{4}$ -is the function given on the line $x_2=0$.

With the condition of the problem IV on the line $x_2=0$, we get: $rotu$, $\frac{\partial}{\partial x_2}divu$ – from the following equations:

$$\sigma_{12} = (\mu-\alpha)\frac{\partial u_1}{\partial x_2} + (\mu+\beta)\frac{\partial u_2}{\partial x_1} - 2\alpha u_3 = \varphi_1(x_1),$$

$$\left.\frac{\partial u_1}{\partial x_2}\right|_{x_2=0} = \frac{1}{\mu-\alpha}\left(\varphi_1 - (\mu+\alpha)\frac{\partial \varphi_2}{\partial x_1} + 2\alpha\varphi_3\right),$$

$$\left. rotu\right|_{x_2=0} = \left(\frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2}\right)\Bigg|_{x_2=0} \equiv \varphi_5(x_1)$$

from the equations (3) of the system (2) ,we get:

$\frac{\partial}{\partial x_2}\left(\Delta - \frac{\tau}{\partial \ell_\tau}\right)u_4 = \eta\tau\frac{\partial}{\partial x_2}divu$ after the limit in this equal ,when $x_2 = 0$, we obtain:

$$\left.\frac{\partial}{\partial x_2}divu\right|_{x_2=0} = \frac{1}{\eta\tau}\left(\Delta - \frac{\tau}{\partial \ell_\tau}\right)\varphi_4 = \varphi_6(x_1)$$

where, φ_5, φ_6 _ $x_2 = 0$ - is given of function on the line $x_2 = 0$.

With the condition of the problem V on the line $x_2 = 0$, we get: $\frac{\partial}{\partial x_2}[rotu_3]$, $\frac{\partial u}{\partial x_2}$, $divu$ –

from the following equations:

$$\sigma_{22} = \lambda divu + 2\mu\frac{\partial u_2}{\partial x_2} - \gamma_\tau u_4 = \psi_2(x_1)$$

$$\lambda\frac{\partial u_1}{\partial x_1} + (\lambda+2\mu)\frac{\partial u_2}{\partial x_2} - \gamma_\tau = \psi_2(x_1)$$

$$\left.\frac{\partial u_2}{\partial x_2}\right|_{x_2=0} = \frac{1}{\lambda+2\mu}\left\{\psi_2(x_1) + \gamma_\tau\xi_4(x_1) - \lambda\frac{\partial\psi_1}{\partial x_1}\right\} = \psi_5(x_1)$$

$$\left. divu\right|_{x_2=0} = \frac{\partial\psi_1}{\partial x_1} + \frac{\partial\psi_5}{\partial x_2} = \psi_6(x_1)$$

$$\left.\frac{\partial}{\partial x_2}[rotu_3]_1\right|_{x_2=0} = \left.\frac{\partial^2 u_3}{\partial x_2{}^2}\right|_{x_2=0} = \frac{\partial\psi_3}{\partial x_2} = \psi_7(x_1)$$

$$\left.\frac{\partial}{\partial x_2}[rotu_3]_2\right|_{x_2=0} = -\frac{\partial}{\partial x_2}\left(\frac{\partial u_3}{\partial x_1}\right)\Bigg|_{x_2=0} = -\frac{\partial\psi_3}{\partial x_1} = \psi_8(x_1)$$

$$\left.\frac{\partial[u]_1}{\partial x_2}\right|_{x_2=0} = \left.\frac{\partial u_1}{\partial x_2}\right|_{x_2=0} = \frac{\partial\psi_1}{\partial x_2} = \psi_9(x_1)$$

$$\left.\frac{\partial[u]_2}{\partial x_2}\right|_{x_2=0} = \left.\frac{\partial u_2}{\partial x_2}\right|_{x_2=0} = \frac{\partial\psi_5}{\partial x_2} = \psi_{10}(x_1)$$

where, $\psi_j(x_1)$, $(j = 5,1\ddot{0})$ _ is given of function on the line $x_2 = 0$.

Same time , with (2) force , the vector $(divu, u_4)$-is the solution for the Neiman's problem in the space D :

$$\begin{cases}\left[(\lambda+2\mu)\Delta-\varsigma\tau^2\right]divu-\gamma_\tau\Delta u_4=0\\ \left(\Delta-\dfrac{\tau}{\partial\ell_\tau}\right)u_4-\eta\tau dvu=0\end{cases}\quad(4)\qquad \frac{\partial}{\partial x_2}divu\bigg|_{x_2=0}=\varphi_6(x_1),\quad \frac{\partial}{\partial x_2}u_4\bigg|_{x_2=0}=\varphi_4(x_1)\qquad(4')$$

The problem is solved in quadratures [2].
the vector -$(rotu, u_3)$ is the solution for the Dirixle's problem in the space D :

$$\begin{cases}\left[(\mu+\alpha)\Delta-\varsigma\tau^2\right]rotu-2\alpha\Delta u_3=0\\ 2\alpha rotu+\left[(\nu+\beta)\Delta-\left(4\alpha+I\tau^2\right)\right]u_3=0\end{cases}\quad(5)\quad rotu=\varphi_5(x_1),\ u_3=\varphi_3(x_1)\qquad(5')$$

The problem is solved in quadratures[1].
Thus, the boundary value problem IV comes to the boundary value problems for the systems (4) and (5) , in the following form,on the line $x_2=0$:

problem A: $\frac{\partial}{\partial x_2}divu\Big|_{x_2=0}=\varphi_6(x_1),\quad \frac{\partial}{\partial x_2}u_4\Big|_{x_2=0}=\varphi_4(x_1)$

problem B: $rotu=\varphi_5(x_1),\ u_3=\varphi_3(x_1)$

with the solutions of the problems A and B it is possible to construct the solution of the problem IV in quadratures.
Really, let the problems A and B have the solutions
Let's discuss the plots to the first equation of the system (2) on the axes: x_1 and x_2 , we have:

$\Delta u_1-\frac{\varsigma\tau^2}{\mu+\alpha}u_1=F_1(x_1,x_2),\quad \frac{\partial u_1}{\partial x_2}=\varphi_{12}\quad(6)-$ The Neimani's problem for the Gelgomci's equation. Where,

$F_1(x_1,x_2)=-\frac{\lambda+\mu-\alpha}{\mu+\alpha}\frac{\partial}{\partial x_1}dvu-\frac{2\alpha}{\mu+\alpha}\frac{\partial u_3}{\partial x_2}+\frac{\gamma_\tau}{\mu+\alpha}\frac{\partial u_4}{\partial x_1}-$ The function given.

$\Delta u_2-\frac{\varsigma\tau^2}{\mu+\alpha}u_2=F_2(x_1,x_2),\quad u_2=\varphi_2\qquad(7)-$ The Dirichle's problem for the Gelgomci's equation. Where,

$F_2(x_1,x_2)=-\frac{\lambda+\mu-\alpha}{\mu+\alpha}\frac{\partial}{\partial x_2}dvu+\frac{2\alpha}{\mu+\alpha}\frac{\partial u_3}{\partial x_!}+\frac{\gamma_\tau}{\mu+\alpha}\frac{\partial u_4}{\partial x_2}-$ The function given.

From equation 2 of the system (2), we have:
,

$\Delta u_3-\frac{I\tau^2+4\alpha}{\nu+\beta}u_3=F_3(x_1,x_2),\quad u_3=\varphi_3\qquad(8)-$ The Dirichle's problem for the Gelgomci's equation. Where,

$F_3(x_1,x_2)=-\frac{2\alpha}{\nu+\beta}rotu-$ The function given.

From equation 3 of the system (2), we have:
,

$\left(\Delta-\frac{\tau}{\partial\ell_\tau}\right)u_4=F_4(x_1,x_2),\quad \frac{\partial u_4}{\partial x_2}=\varphi_4(x_1)\quad(9)-$ The Neimani's problem for the Gelgomci's equation. Where,

$F_4(x_1,x_2)=\eta\tau divu-$ The function given.

The problems (6)-(9) are solved in quadratures.

The formula of the solution for to problem (6) will give the following form:

$$u_1 = \frac{1}{\pi}\int_{-\infty}^{+\infty} H\left(i\tau\sqrt{\varsigma/(\mu+\alpha)}\,|x-y|\right)\varphi_{12}(y)dy - \frac{1}{2\pi}\iint_{x_2>0} G_1(x,z)F_1(z)dz \qquad (6')$$

The formula, of the solution for to problem (7) will give the following form:

$$u_2 = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{\partial}{\partial n(y)} H\left(i\tau\sqrt{\varsigma/(\mu+\alpha)}\,|x-y|\right)\varphi_2(y)dy - \frac{1}{2\pi}\iint_{x_2>0} G_2(x,z)F_2(z)dz \qquad (7')$$

The formula of the solution for to problem (8) will give the following form:

$$u_3 = \frac{1}{\pi}\int_{-\infty}^{+\infty} \frac{\partial}{\partial n(y)} H\left(i\tau\sqrt{(I\tau^2+4\alpha)/(\nu+\beta)}\,|x-y|\right)\varphi_3(y)dy - \frac{1}{2\pi}\iint_{x_2>0} G_3(x,z)F_3(z)dz \qquad (8')$$

The formula of the solution for to problem (9) will give the following form:

$$u_4 = \frac{1}{\pi}\int_{-\infty}^{+\infty} H\left(i\tau\sqrt{\tau/\partial\ell_\tau}\,|x-y|\right)\varphi_4(y)dy - \frac{1}{2\pi}\iint_{x_2>0} G_4(x,z)F_4(z)dz \qquad (9')$$

The vector $(u, rotu_3, u_4)$-is solution the following system:

$$\begin{cases} \left[(\mu+\alpha)\Delta - \varsigma\tau^2\right]u + 2\alpha rotu_3 = \gamma_\tau gradu_4 - (\lambda+\mu-\alpha)graddivuu \\ -2\alpha\Delta u + \left[(\nu+\beta)\Delta - (4\alpha + I\tau^2)\right]rotu_3 = 0 \end{cases} \qquad (10)$$

Where, the vector $(divu, u_4)$- is the solution for the Dirixle's problem:

$$\begin{cases} \left[(\lambda+2\mu)\Delta - \varsigma\tau^2\right]divu - \gamma_\tau\Delta u_4 = 0 \\ \left(\Delta - \dfrac{\tau}{\partial\ell_\tau}\right)u_4 - \eta\tau divu = 0 \end{cases} \qquad (11)$$

$divu = \psi_6(x_1), u_4 = \psi_4(x_1)$ $(11')$. The problem is solved in quadratures[2].

Thus, the boundar y value problem V comes to the boundary value problems for the systems (11) and (10) , in the following form,on the line $x_2 = 0$:

problem C: $divu = \psi_6(x_1), \quad u_4 = \psi_4(x_1)$

problem D: $\dfrac{\partial}{\partial x_2} rotu = \varphi_7(x_1), \quad \dfrac{\partial u_3}{\partial x_2} = \varphi_8(x_1)$

To solve the problem IV (the problem V-solving similarly):

Let's rewrite the system (4) in the following form:

$$L(\partial x, \tau)\upsilon = 0 \qquad (12)$$

where, $\upsilon = (\upsilon_1, \upsilon_2)$ is vector, $\upsilon_1 = divu, \quad \upsilon_2 = u_4$,

$$L(\partial x, \tau) = \left\| \begin{matrix} (\lambda+2\mu)\Delta - \varsigma\tau^2 & -\gamma_\tau\Delta \\ -\eta\tau & \Delta - \dfrac{\tau}{\partial\ell_\tau} \end{matrix} \right\| \qquad (12')$$

For the determinant of the Matrix $L(\partial x, \tau)$, : we have:

$$\left[(\lambda+2\mu)\Delta - \varsigma\tau^2\right]\left[\Delta - \frac{\tau}{\partial\ell_\tau}\right] - \gamma_\tau\eta\tau\Delta = (\lambda+2\mu)\Delta^2 - \frac{\tau}{\partial\ell_\tau}(\lambda+2\mu)\Delta - \varsigma\tau^2 + \frac{\tau^3\varsigma}{\partial\ell_\tau} - \gamma_\tau\eta\tau\Delta =$$

$$= (\lambda+2\mu)\Delta^2 - \left[\frac{\tau}{\partial\ell_\tau}(\lambda+2\mu) + \varsigma\tau^2 + \gamma_\tau\eta\tau\right]\Delta + \frac{\tau^3\varsigma}{\partial\ell_\tau} = (\lambda+2\mu)(\Delta+\lambda_1^2)(\Delta+\lambda_2^2)$$

Where,

$$\lambda_1^{\ 2}+\lambda_2^{\ 2}=\frac{\tau}{\partial\ell_\tau}+\frac{\varsigma\tau^2}{\lambda+2\mu}+\frac{\eta\tau\gamma_\tau}{\lambda+2\mu},$$

$$\lambda_1^{\ 2}\cdot\lambda_2^{\ 2}=\frac{\varsigma\tau 3}{\partial\ell_\tau(\lambda+2\mu)} \qquad (13)$$

Let's discuss the vectors: $\upsilon^{(1)}=\left(\upsilon_1^{(1)},\upsilon_2^{(1)}\right),\quad \upsilon^{(2)}=\left(\upsilon_1^{(2)},\upsilon_2^{(2)}\right)\quad (14)$

Where,

$$\upsilon_1^{(1)}=\left(\Delta-\frac{\tau}{\partial\ell_\tau}\right)\varphi,\qquad \upsilon_2^{(1)}=-\eta\tau\varphi$$

$$\upsilon_1^{(2)}=-\gamma_\tau\Delta\varphi,\qquad \upsilon_2^{(2)}=\left[(\lambda+2\mu)\Delta-\varsigma\tau^2\right]\varphi \qquad (15)$$

φ-is researching scalar function.

If scalar φ- satisfy the following condition:

$$\left(\Delta+\lambda_1^{\ 2}\right)\left(\Delta+\lambda_2^{\ 2}\right)\varphi=0 \qquad (16)$$

then, the vectors: $\upsilon^{(1)}=\left(\upsilon_1^{(1)},\upsilon_2^{(1)}\right),\quad \upsilon^{(2)}=\left(\upsilon_1^{(2)},\upsilon_2^{(2)}\right)$ – are the solution of the system (12).

From the system(16) we get:

$$\varphi(x,\tau)=\sum_{k=1}^{2}a_k H_0^{(1)}\left(\lambda_k|x|\right),$$

where, $H_0^{(1)}\left(\lambda_k|x|\right)$ are Hunkel functions (the first knd the zero row), a_k – are constants which satisfies the following conditions:

$$\begin{cases}\sum_{k=1}^{2}a_k=0\\ \sum_{k=1}^{2}a_k\lambda_k^{\ 2}=\dfrac{i}{2\pi(\lambda+2\mu)}\end{cases}$$

Then, 2th rows $\varphi(x,\tau)$ – partial production close to zero area have speciality of $\ell n|x|$ – form:

$$\varphi(x,\tau)=\frac{2i}{\pi(\lambda+2\mu)\left(\lambda_1^{\ 2}-\lambda_2^{\ 2}\right)}\left\{H_0^{(1)}\left(\lambda_1|x|\right)-H_0^{(1)}\left(\lambda_2|x|\right)\right\} \qquad (17)$$

where, $|x|=\sqrt{x_1^{\ 2}+x_2^{\ 2}}$

After the putting system (17) in the system(15),we get:

$$\upsilon_1^{(1)}=\left(\Delta-\frac{\tau}{\partial\ell_\tau}\right)\cdot\frac{2i}{\pi(\lambda+2\mu)\left(\lambda_1^{\ 2}-\lambda_2^{\ 2}\right)}\left\{H_0^{(1)}\left(\lambda_1|x|\right)-H_0^{(1)}\left(\lambda_2|x|\right)\right\}=$$

$$\frac{2i}{\pi(\lambda+2\mu)\left(\lambda_1^{\ 2}-\lambda_2^{\ 2}\right)}\left\{-\left(\lambda_1^{\ 2}+\frac{\tau}{\partial\ell_\tau}\right)H_0^{(1)}\left(\lambda_1|x|\right)+\left(\lambda_2^{\ 2}+\frac{\tau}{\partial\ell_\tau}\right)H_0^{(1)}\left(\lambda_2|x|\right)\right\}$$

so, we obtain:

$$\upsilon_1^{(1)} = \frac{2i}{\pi(\lambda+2\mu)\left(\lambda_1^{\,2}-\lambda_2^{\,2}\right)}\left\{\left(\lambda_2^{\,2}+\frac{\tau}{\partial\ell_\tau}\right)H_0^{(1)}\left(\lambda_2|x|\right)-\left(\lambda_1^{\,2}+\frac{\tau}{\partial\ell_\tau}\right)H_0^{(1)}\left(\lambda_1|x|\right)\right\}$$

$$\upsilon_2^{(1)} = \eta\tau\cdot\frac{2i}{\pi(\lambda+2\mu)\left(\lambda_1^{\,2}-\lambda_2^{\,2}\right)}\left\{H_0^{(1)}\left(\lambda_2|x|\right)-H_0^{(1)}\left(\lambda_1|x|\right)\right\}$$

$$\upsilon_1^{(2)} = \gamma_\tau\Delta\cdot\frac{2i}{\pi(\lambda+2\mu)\left(\lambda_1^{\,2}-\lambda_2^{\,2}\right)}\left\{H_0^{(1)}\left(\lambda_2|x|\right)-H_0^{(1)}\left(\lambda_1|x|\right)\right\}=$$

$$=\gamma_\tau\cdot\frac{2i}{\pi(\lambda+2\mu)\left(\lambda_1^{\,2}-\lambda_2^{\,2}\right)}\left\{\lambda_1^{\,2}H_0^{(1)}\left(\lambda_1|x|\right)-\lambda_2^{\,2}H_0^{(1)}\left(\lambda_2^{\,2}|x|\right)\right\}$$

$${}_\tau\upsilon_2^{(2)} = \left[(\lambda+2\mu)\Delta-\varsigma\tau^2\right]\cdot\frac{2i}{\pi(\lambda+2\mu)\left(\lambda_1^{\,2}-\lambda_2^{\,2}\right)}\left\{H_0^{(1)}\left(\lambda_1|x|\right)-H_0^{(1)}\left(\lambda_{21}|x|\right)\right\}=$$

$$=\frac{2i}{\pi(\lambda+2\mu)^2\left(\lambda_1^{\,2}-\lambda_2^{\,2}\right)}\left\{\left(\lambda_2^{\,2}+\frac{\varsigma\tau^2}{\lambda+2\mu}\right)H_0^{(1)}\left(\lambda_2|x|\right)-\left(\lambda_1^{\,2}+\frac{\varsigma\tau^2}{\lambda+2\mu}\right)H_0^{(1)}\left(\lambda_1|x|\right)\right\}$$

Let's construct matrixes of fundamental solutions for system (12):

$$\Phi(x,\tau)=\begin{Vmatrix}\upsilon_1^{(1)} & \upsilon_1^{(2)}\\ \upsilon_2^{(1)} & \upsilon_2^{(2)}\end{Vmatrix}$$

where,

$$\Phi(x,\tau)=\frac{2i}{\pi(\lambda+2\mu)\left(\lambda_1^{\,2}-\lambda_2^{\,2}\right)}\begin{Vmatrix}\left(\lambda_2^{\,2}+\frac{\tau}{\partial\ell_\tau}\right)\beta_2-\left(\lambda_1^{\,2}+\frac{\tau}{\partial\ell_\tau}\right)\beta_1 & \gamma_\tau\left(\lambda_1^{\,2}\beta_1-\lambda_2^{\,2}\beta_2\right)\\ -\eta\tau(\beta_1-\beta_2) & \left(\lambda_2^{\,2}+\frac{\varsigma\tau^2}{\lambda+2\mu}\right)\beta_2-\left(\lambda_1^{\,2}+\frac{\varsigma\tau^2}{\lambda+2\mu}\right)\beta_1\end{Vmatrix} \qquad (18)$$

where, $\beta_k = H_0^{(1)}\left(\lambda_k|x|\right),\quad k=1,2$

Let's construct the conjugate operator :

$$\tilde{L}(\partial x,\tau)=\begin{Vmatrix}(\lambda+2\mu)\Delta-\varsigma\tau^2 & -\eta\tau\\ -\gamma_\tau\Delta & \Delta-\frac{\tau}{\partial\ell_\tau}\end{Vmatrix} \qquad (19)$$

(19) Is identical of the transpose matrixe $L(\partial x,\tau)$.

Let, $\upsilon=(\upsilon_1,\upsilon_2)$ and $\tilde{\upsilon}=(\tilde{\upsilon}_1,\tilde{\upsilon}_2)$ – are regular vectors in the space $D^+\subset R^2$ bounded by closed line ℓ.

Let's use Green's formula:

$$\int_{D^+}\left(\upsilon^T\tilde{L}\,\tilde{\upsilon}-\tilde{\upsilon}^{Tt}L\,\upsilon\right)dx=\int_\ell\left(\upsilon^T\tilde{T}\tilde{\upsilon}-\tilde{\upsilon}^{Tt}T\upsilon\right)d\ell \qquad (20)$$

where,

$$T\upsilon=\left((\lambda+2\mu)\frac{\partial\upsilon_1}{\partial n}-\gamma_\tau\frac{\partial\upsilon_2}{\partial n},\ \frac{\partial\upsilon_2}{\partial n}\right),\quad (21)$$

$$\tilde{T}\tilde{\upsilon}=\left((\lambda+2\mu)\frac{\partial\tilde{\upsilon}_1}{\partial n},-\gamma_\tau\frac{\partial\tilde{\upsilon}_1}{\partial n}+\frac{\partial\tilde{\upsilon}_2}{\partial n}\right) \quad (22)$$

where, T-is transpose operator,

$$T(\partial n,\gamma_\tau)=\begin{Vmatrix}(\lambda+2\mu)\dfrac{\partial}{\partial n} & -\gamma_\tau\dfrac{\partial}{\partial n}\\ 0 & \dfrac{\partial}{\partial n}\end{Vmatrix},\qquad \tilde T(\partial n,\gamma_\tau)=\begin{Vmatrix}(\lambda+2\mu)\dfrac{\partial}{\partial n} & 0\\ -\gamma_\tau\dfrac{\partial}{\partial n} & \dfrac{\partial}{\partial n}\end{Vmatrix}$$

we get the formula of representation of regular solution $\upsilon(x)$:

$$2\pi(\lambda+2\mu)\upsilon(x)=\int_\ell \Phi(x-y,\tau)T\upsilon(y)d_y\ell-\int_\ell\left[\tilde T\tilde\Phi(x-y,\tau)\right]^T\upsilon(y)d_y\ell-\int_{D^+}\Phi(x-y,\tau)L\upsilon(y)dy \qquad (23)$$

where, $\tilde\Phi(x,\tau)$ is fundamental solutions for operator $\tilde L(\partial x,\tau)$,
We have:

$$\tilde\Phi^T(x-y,\tau)=\Phi(x-y,\tau)$$

Let's consider the potentials:

$V(x;\varphi)=\int_\ell \Phi(x-y;\tau)\varphi(y)d_y\ell-$ the potential of the simple layer,

$W(x;\psi)=\int_\ell\left[\tilde T(\partial n,\gamma)\tilde\Phi(x-y;\tau)\right]^T\psi(y)d_y\ell-$ the potential of the double layer .

Problem A. Construct the regular solution of system $L(\partial x,\tau)\upsilon(x)=0$ on the plane $X_2\geq 0$, which satisfy on the line $X_2=0$ the folloving condition: $\dfrac{\partial\upsilon(z)}{\partial x_2}=f(z)$,

where, $f=(f_1,f_2)$-is vector-function bounded in the infinite.

Let's represent the solution of problem by the simple potential:

$$V(x;\varphi)=\int_{-\infty}^{+\infty}\Phi(x-y;\tau)\varphi(y)dy$$

For $\varphi(y)-$ we get:

$$\pi(\lambda+2\mu)\varphi(z)+\int_{-\infty}^{+\infty}T(\partial n,\gamma_\tau)\Phi(z-y;\tau)\varphi(y)dy=F(z)\equiv\begin{Vmatrix}(\lambda+2\mu)f_1-\lambda_\tau f_2\\ f_2\end{Vmatrix} \qquad (24)$$

On the plane we heve: $T(\partial n,\gamma_\tau)\Phi(z-y;\tau)=0 \qquad (24')$

With (24') force in the (24) we get:

$$\varphi(z)=\frac{F(z)}{\pi(\lambda+2\mu)}$$

and we get:

$$\upsilon(x)=\frac{1}{\pi(\lambda+2\mu)}\int_{-\infty}^{+\infty}\Phi(x-y;\tau)F(y)dy \qquad (25)$$

The recived solution is unique.

Really, let the similarity problem A have the solution $\upsilon^{(0)}(x)$,then with (23) force we heve:

$$2\pi(\lambda+2\mu)\upsilon^{(0)}(x)=\int_{-\infty}^{+\infty}\Phi(x-y;\tau)T\upsilon^{(0)}(y)dy \qquad (26)$$

$$\pi(\lambda+2\mu)T\upsilon^{(0)}(z)=\int_{-\infty}^{+\infty}T(\partial_{n(z)},\gamma_\tau)\Phi(z-y;\tau)T\upsilon^{(0)}(y)dy$$

With (24') force in the (26) we get: $\upsilon^{(0)}(x)=0$

Let's rewrite the system (5) in the following form:

$$M(\partial x, \tau)h = 0 \qquad (27)$$

where, $h = (h_1, h_2)$ is vector, $h_1 = rotu$, $h_2 = u_3$,

$$M(\partial x, \tau) = \begin{Vmatrix} (\mu + \alpha)\Delta - \varsigma\tau^2 & -2\alpha\Delta \\ 2\alpha & (\nu + \beta)\Delta - (4\alpha + I\tau^2) \end{Vmatrix} \qquad (27')$$

The system(27)-solving similarly of system (12).

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