The Evaluation of the Time Delay and Defasing of the Seismic Image by an adapted Wavelet

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Abstract
In this work, we suggest a method based on the concept of the wavelet transform to valuate the time delay and defasing of seismic waves; in fact, the seismic wave propagation takes place in a complex field, which generates delays and defasing. To assess the performance of our approach, we draw comparisons of the results of this approach and those given by the method of the gradient.
keywords: Seismic trace, wavelet transform, time-delay, defasing, adapted wavelet.

1 Introduction

The evaluation of the delay time and defasing of seismic signals is a fundamental step in various applications in the treatment of multidimensional signals (particularly marine seismic, acoustics, biomedicine,...).

In geophysics, the propagation of waves takes place in a complex field, which generates delayed waves (velocity), defasing, and sometimes dispersal (surface waves); therefore, the estimation of the delay and defasing is a step of the seismic profile pre-treatment, and usually a means to align and separate the waves [1], to correct the statics, to denoise, and to filter or analyze the waves propagation velocities.

Traditionally, the technics of calculating the delay and defasing of the seismic profiles are based on the crossing correlation see (see [1] and [2]). Other alternatives lie in the statistics of the Higher than Two order in the spectral domain (see [3] and [4]).

We propose, in this work, a method for estimating the time delay and defasing of the seismic signals based on the concept of the continuous wavelet transform and we demonstrate that through modelling the seismic signal, choosing a wavelet analysis (a seismic wavelet) and a scale factor-all of which should be adapted-it will be possible to do the denoising and meanwhile give an estimation of the delay time and the defasing of the seismic waves, which enable to achieve a fine wave alignment, that is more adapted to a propagation in a complex field.

Moreover, we suggest a method of calculating an adapted wavelet relying on a seismic profile. We also demonstrate, basing ourselves on synthetic signals and in the presence of noise of diverse powers, the interest and performance of this approach by drawing comparisons of the obtained results of our method and also of classical wavelets and those given by the method of gradient (see [5]).

2 Presentation of the problem

The seismic exploration methods consist of provoking a shock in the substratum through an explosion, vibrating truck, weight fall,...etc, and observing superficially the reflected waves on the geologic or refracted layers along certain interfaces (see [7] and [8]).

A seismic signal, also called a seismic trace, is obtained by the reflexion of a seismic wave on the different layers of rock; the noted seismic trace $f(t)$ is the reponse registered by a sensor (geophone or hydrophone).

If we note by $s(t)$ the seismic impulsion (emitted by a provoked shock), a modelling of a seismic trace will be given by an equation of the type (see [8]):

$$f(t) = G(t)[r(t) * s(t)] + b(t) \tag{1}$$

with

- $G(t)$ a gain curve.
- $r(t)$ log of reflexing or impulsive trace, which is the registered if the seismic impulsion is a Dirac and without noise.
- $b(t)$ is the additive noise supposed to be uncertain.

Generally speaking, for N sources emitting $s_k(t)$ signals (impulsions) with $k = 1,..,N$ and after a propagation in a supposedly homogeneous and isotropic complex field (there is no dissipation of energy and there is a presence of delays and defasings), the received seismic on a given sensor can b written as the following (see [2]):

$$f(t) = \sum_{k=1}^{N} a_k \delta(t - \tau_k) e^{i\varphi_k} * s_k(t) + b_k(t) \tag{2}$$

with: τ_k and φ_k are respectively the delay and the defasing of the $k^r d$ wave on the sensor, a_k represents the gain in amplitude and b_k the additive noise.

To simplify, we consider the case of a single wave received on two different sensors.

Throughout our analysis, we suppose that we have one source emitting the wave $s(t)$ supposed not to be dispersed, and respectively noting by $X(t)$ and $Y(t)$ the signal received on the first sensor and the same signal received on the second sensor,and which is delayed and defased of the first one:

$$X(t) = s(t) + b_x(t) \tag{3}$$

$$Y(t) = s(t - \tau)e^{i\varphi} + b_y(t) \tag{4}$$

wherein,

- $S(t)$ represents the unknown reference signal.
- $s(t - \tau)e^{i\varphi}$ is the same delayed signal of τ and the defased one of φ.
- $b_x(t)$ and $b_y(t)$ are two noise sources supposed to be of high frequencies (see [7]).

The problem lies in estimating the delay τ and the defasing φ.

3 A Recall about Wavelets

For the rest of this work, we will confine ourselves to the unidimensional case, because we are interested in a particular seismic trace.

Set $f : R \longrightarrow C$ an integrable function ($f \in L^1(R)$),the Fourier transform of the function f , noted by $\widehat{f}$, is defined by:

$$\widehat{f}(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-i\xi t} f(t)dt \tag{5}$$

The Fourier Transform can be generalized to temperate distributions, particularly to $L^2(R)$ functions (signals of finished energies).

Set $\psi \in L^2(R)$ of integrable square, ψ is admissible or checks the admissibility condition if:

$$C_\psi = \int_0^{+\infty} \left|\widehat{\psi}(\xi)\right|^2 \xi d\xi < +\infty \tag{6}$$

We label such a function the analyzing wavelet ([9]).

The admissibility condition (inequality 6) is relatively flexible in certain cases, as it is showed by the following property:

Proprety 1 *Set* $\psi \in L^1(R)$

1) *If* ψ *is admissible, then* $\widehat{\psi}(0) = \int_R \psi(t)dt = 0$.
2) *reversely, if* $\widehat{\psi}(0) = 0$ *with* $\widehat{\psi} \in L^2(R)$*, and if the derivative of* $\widehat{\psi}$ *is bound, then* ψ *is admissible.*

In the actual practice, most of the time the wavelet is taken regular, and the conditions of the property 1 are therefore verified, the admissibility condition is usually written as $\widehat{\psi}(0) = 0$.

The wavelets family $\psi_{a,b}$, $a \in R^{+*}$, $b \in R$ is defined for an analyzing wavelet ψ as:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi(\frac{t-b}{a})$$

The operator W is the continuous wavelet transform defined by:

$$\begin{array}{c} W: \; L^2(R, dt) \rightarrow L^2(R^{+*} \times R, \frac{dbda}{a^2}) \\ f \rightarrow Wf \end{array}$$

wherein:

$$Wf(a,b) = \frac{1}{\sqrt{C_\psi}} \int_{-\infty}^{+\infty} \overline{\psi}_{a,b}(t) f(t)dt \tag{7}$$

- $\overline{\psi}_{a,b}$ is complex conjugate of $\psi_{a,b}$.
- $(Wf(a,b))_{a,b}$ are the coefficients of wavelets associated with f.

A formulation of a wavelets transform in the Fourier domain (see [10]) leads to a calculating algorithm which is less costly (in terms of the number of operations), and which is given by the following formula:

$$Wf(a,b) = \frac{\sqrt{a}}{\sqrt{2\pi C_\psi}} \int_{-\infty}^{+\infty} e^{i\xi b} \widehat{\overline{\psi}}(a\xi)\widehat{f}(\xi)d\xi \tag{8}$$

The continuous wavelet transform is an isometry of $L^2(R, dt)$ in $L^2(R^{+*} \times R, dbdaa^2)$, which signifies that there is no information loss between the function and its wavelet transform (stability property [9]).

4 The Estimation of the time delay and defasing

4.1 The Case of a Single Wave

We have presented the context in which we will estimate the delay and defasing of the seismic profile. For this fact, we consider two traces defined by the equations (3) and (4), and we suppose that the seismic impulsion is real, symmetrical, and of a compact support; in other words, $s(t) = 0$, $\forall t \notin [-T, T]$, and no energy has been stocked in the ground [6]:

$$\int_R s(t)dt = 0 \tag{9}$$

We also suppose that the noises $b_x(t)$ and $b_y(t)$ are of high frequencies, which is rendered by the existence of $\xi_0 > 0$, and for all ξ , $|\xi| < \xi_0$

$$\widehat{b_x}(\xi) = \widehat{b_y}(\xi) = 0 \tag{10}$$

Set, then, a supposed analyzing wavelet ψ of a compact support in the Fourier domain, which means:

$$\exists \xi_{\psi,1} > \xi_{\psi,2} > 0 \quad / \ \widehat{\psi}(\xi) = 0 \ for \ \ |\xi| \notin [\xi_{\psi,1}, \xi_{\psi,2}] \tag{11}$$

If we suppose that ψ is of a rapid decrease:

$$\forall \varepsilon > 0, \exists C > 0 \ \ /\psi(t) < \frac{\varepsilon}{t^2} < , \forall t \notin [-C, C]$$

So, we will have the following property:

Proprety 2 *Considering the hypotheses 9, 10 and 11 and for a scale factor a, such as*

$$\frac{\xi_{\psi,2}}{\xi_0} \leq a \leq \frac{T}{C}$$

we have,

$$WY(a,b) = e^{i\varphi} Ws(a, b - \tau)$$

Proof.

The wavelet transform of the trace $Y(t)$ defined in the Fourier domain gives:

$$WY(a,b) = \frac{\sqrt{a}}{\sqrt{2\pi C_\psi}} \left[e^{i\varphi} \int_{-\infty}^{+\infty} e^{i\xi(b-\tau)} \widehat{\psi}(a\xi)\widehat{s}(\xi)d\xi + \int_{-\infty}^{+\infty} e^{i\xi b} \widehat{\psi}(a\xi)\widehat{b_y}(\xi)d\xi \right]$$

The noise is supposed to be of high frequencies, thus $\widehat{\psi}(a\xi)\widehat{b_y}(\xi) = 0$ for $|\xi| < \xi_0$.

For choosing the scale factor a, such as $a \geq \frac{\xi_{\psi,2}}{\xi_0}$, and since $\widehat{\psi}$ is of a compact support, we can write:

$$\widehat{\psi}(a\xi)\widehat{b_y}(\xi) = 0 \ , \forall\xi \in C$$

Consequently, the noise contributes no longer in the wavelet transform of the seismic trace $Y(t)$, and we obtain:

$$WY(a,b) = e^{i\varphi}Ws(a, b - \tau)$$

We have noticed that the defasing of the signal $Y(t)$ presents the $WY(a,b)$ phase; therefore, the defasing can be calculated from the coefficients of wavelets.

The wavelet ψ is a rapid decrease, so for $|b - \tau| < 2T$ and for a scale factor a such as $a \leq \frac{T}{C}$, we have:

$$\begin{aligned} Ws(a, b-\tau) &= \frac{1}{\sqrt{aC_\psi}}\int_{-T}^{T}\psi(\frac{t-b-\tau}{a})s(t)dt \\ &\leq \frac{a^{\frac{3}{2}}s(o)}{\sqrt{C_\psi}}C^{ste}(T). \end{aligned}$$

Besides, if the wavelet ψ is chosen from the impulsion s' "close" form, and for choosing a scale factor a such as $1 \leq a \ \leq \frac{T}{C}$ (particulary for $a = 1$ in practice), $WY(a,b)$ reaches its maximum for the value $b = \tau$.

In fact, for $a = 1$ and $\psi \simeq s$ (since the seismic impulsion $s(t)$ is a wavelet (property 1) which checks the admissibility condition). The wavelet transform $WY(a,b)$ becomes:

$$WY(1,b) = \frac{1}{\sqrt{C_\psi}}\int_R e^{i\xi(b-\tau)}\widehat{s}^2(\xi)d\xi$$

Consequently, calculating the delay is calculating the maximum of the continuous wavelet transform of the signal $Y(t)$.

4.2 The Case of Several Waves

For simplifying the calculations, we have considered the case of a single wave detected on two sensors. However, we proceed in the same way in the case of several waves, and we consider the case of the one source emitting the impulsion $s(t)$.

The approximation of the seismic trace modelled by the following equation (see [1] and [10]) :

$$Y(t) = \sum_{k=0}^{N} a_k e^{i\varphi_k} s(t - \tau_k) + b_k(t) \tag{12}$$

In this case, the seismic trace can be seen as a sum of signals typical of the delayed and defased seismic impulsion for different amplitudes.

The calculation of the wavelets transform will be the same, and we obtain an equation of the type:

$$WY(a,b) = \sum_{k=0}^{N} a_k e^{i\varphi_k} Ws(a, b - \tau_k)$$

In this case, we suppose that the waves are two by two distant enough in time; therefore, the calculation of the delay and defasing depends on calculating the local maximum of the wavelet transform of the signal $Y(t)$.

5 An Adapted Wavelet

We present a method of calculating a wavelet adapted for our problem from the self-correlation function of the seismic trace $Y(t)$ (equation 12).

We suppose that the waves are two by two sufficiently distant, which is rendered by the following condition:

$$\forall k \neq r \ , \ Supp(s(\cdot - \tau_k)) \cap Supp(s(\cdot - \tau_r)) = \emptyset \tag{13}$$

We note by A_Y the self-correlation function associated with Y :

$$A_Y(\omega) = \int_R Y(t)Y(t+\omega)dt$$

we have

$$\begin{aligned} A_Y(\omega) &= \int_R \left[\sum_k a_k s(t-\tau_k)e^{i\varphi_k} + b_y(t)\right]\left[\sum_r a_r s(t+\omega-\tau_r)e^{i\varphi_r} + b_y(t+\omega)\right] dt \\ &= \sum_{k,r} a_k a_r e^{i(\varphi_k+\varphi_r)} \int_R s(t-\tau_k)s(t+\omega-\tau_r)dt \\ &+ \sum_k a_k e^{i\varphi_k} \int_R s(t-\tau_k)b_y(t+\omega)dt \\ &+ \sum_r a_r e^{i\varphi_r} \int_R s(t+\omega-\tau_r)b_y(t)dt + \int_R b_y(t)b_y(t+\omega)dt. \end{aligned}$$

taking into consideration the hypothesis (13) of $s(\cdot - \tau_k)$ and of $s(\cdot - \tau_r)$ we can then write:

$$\begin{aligned} A_Y(\omega) &= \sum_k a_k^2 e^{2i\varphi_k} A_s(\omega) + A_{b_y}(\omega) + \sum_k a_k e^{i\varphi_k} \int_R s(t-\tau_k)b_y(t+\omega)dt \\ &+ \sum_r a_r e^{i\varphi_r} \int_R s(t+\omega-\tau_r)b_y(t)dt \end{aligned}$$

We suppose that the noise b_y and the seismic impulsion s do not take part in the same frequency bands, this is expressed by:

$$\exists \ \xi_{s,b} \ , \ \xi_{s,h} / \ \ 0 < \xi_{s,b} \ < \ \xi_{s,h} \ and \widehat{b_y}(\xi) = 0 \ , \forall \ \ |\xi| < \xi_{s,h}$$

and that

$$|\widehat{s}(\xi)| < \epsilon \ , \quad \forall \, |\xi| \notin [\xi_{s,b}, \xi_{s,h}]$$

$$\begin{aligned} \widehat{A_Y}(\xi) &= \sum_k a_k^2 e^{2i\varphi_k} \widehat{A_s}(\xi) + \widehat{A_{b_y}}(\xi) + \sqrt{2\pi}[\sum_k a_k e^{i\varphi_k} e^{i\xi\tau_k} \widehat{s}(-\xi)\widehat{b_y}(\xi) \\ &+ \sum_r a_r e^{-i\xi\tau_r} \widehat{s}(\xi)\widehat{b_y}(-\xi)] \end{aligned}$$

taking into consideration the hypotheses 9, 10 and 11, we have

$$\widehat{b_y}(\xi) = 0 \ then \widehat{A_Y}(\xi) = \sum_k a_k^2 e^{2i\varphi_k} \widehat{A_S}(\xi), for \ \ |\xi| < \xi_{s,h}$$

whereas

$$\widehat{A_Y}(\xi) = \sqrt{2\pi}\left|\widehat{Y}(\xi)\right|^2$$

consequently

$$\left|\sum_k a_k^2 e^{2i\varphi_k}\right| |\widehat{s}(\xi)|^2 = \left|\widehat{Y}(\xi)\right| \quad for \ \ |\xi| < \xi_{s,h}$$

and for $|\xi| \notin [\xi_{s,b}, \xi_{s,h}]$, we have:

$$\widehat{A_Y}(\xi) = \sqrt{2\pi}(\left|\widehat{b_y}(\xi)\right|^2 + \mu(\xi))$$

with

$$|\mu(\xi)| < \epsilon\left[\epsilon + \left|\widehat{b_y}(\xi)\right| + \left|\widehat{b_y}(-\xi)\right|\right]$$

whereas, the seismic impulsion s (equation 9) is a wavelet. Besides, it is supposed to be a real symmetry: $\widehat{s}$ is also a real symmetrical, we choose then the wavelet adapted to the problem in question defined in the Fourier domain by:

$$\widehat{s_{ad}}(\xi) = \begin{cases} \frac{1}{\left|\sqrt{\sum_k a_k^2 e^{2i\varphi_k}}\right|}\widehat{Y}(\xi) & for \quad |\xi| < \xi_{s,h}. \\ 0 & otherwise \end{cases} \tag{14}$$

and by the inverse transform, we obtain the adapted wavelet s_{ad}.

6 Numerical Tests

We have seen that the wavelet adapted to the estimation of the time delay and defasing of the seismic signals must be a "close form" to that of the seismic impulsion $s(t)$, of a compact support, symmetrical, and without oscillations.

To justify our choice of this adapted wavelet (the seismic wavelet), we have tested our method on synthetic signals, and by diverse classical wavelets, which verify the seismic impulsion properties (biorthogonal wavelet and I. Daubechies wavelet,...). The obtained results are numerically satisfactory.

On the Matlab, we have tested this method on a synthetic seismic profile composed of one wave.

The profile form is presented in figure1, and we have compared the obtained results with those given by the gradient method [5] and [11]. To simplify, we consider the case of two traces

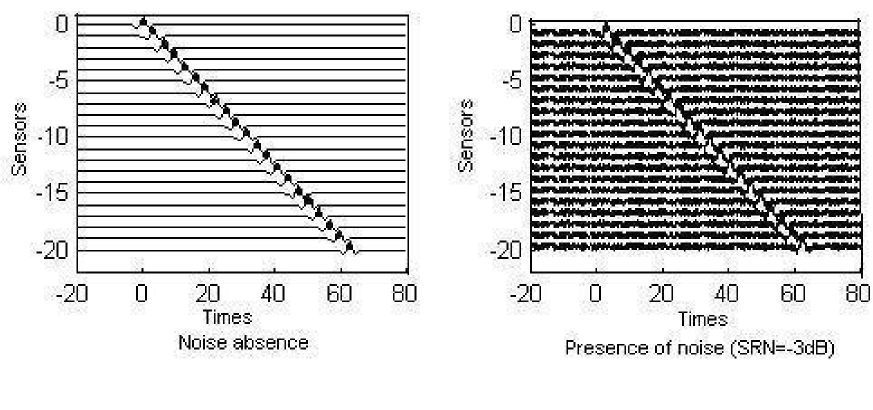

Figure 1: Synthetic profile (20 sensors)

(simulation by a function of the Mexican hat type), the second being delayed of 3.14 and defased of $0.41 rad$ in regard to the first trace, and we suppose that shannon theorem is well verified.

6.1 The Case of a Wave in Noise Absence

We introduce the average quadratic errors of the delay and defasing respectively noted by Err_τ and Err_φ, and defined in the general case by:

$$Err\_\tau = \frac{1}{N_c - 1} \sum_{i=1}^{N_c-1} [\overline{\tau_i} - \tau_i]^2$$
$$Err\_\varphi = \frac{1}{N_c - 1} \sum_{i=1}^{N_c-1} [\overline{\varphi_i} - \varphi_i]^2$$

With
τ_i :the real delay value
$\overline{\tau_i}$:the approximate delay value
φ_i :the real defasing value
$\overline{\varphi_i}$:the approximate defasing value
N_c :number of sensors
This quality criterion will be used to judge the quality and performance of the suggested method.

6.1.1 A Seismic Wavelet

The tabular (TAB.1) presents the numerical results obtained when using an adapted wavelet (a seismic wavelet). The estimation in this case is perfect: the modulus reaches its maximum in the exact value of the delay 3.14.

Wavelet	τ_a	Err_τ	φ_a	Err_φ
Seismic	3.14	0	0.41	$0.30815.10^{-34}$

TAB.1 The obtained results by an adapted wavelet

We note (TAB.1) that we obtain an estimation of the exact defasing and the error is almost null.

6.1.2 Classical Wavelets

To justify our choice, we have applied this method on wavelets which have a form close to the seismic wavelet: a biorthogonal wavelet, and the Daubechies's wavelet in two null moments (TAB.2).

Wavelet	τ_a	Err_τ	φ_a	Err_φ
Daubechies ($N = 2$)	3.14	0	0.41	$0.20831.10^{-31}$
Biorthogonal	3.14	0	0.41	$0.11093.10^{-30}$

TAB.2 The obtained results by classical wavelets

The defasing estimation in this case is exact, and the Daubechies and biorthognal wavelets are well adapted in this case for the delay estimation.

6.2 The Case of a Wave in the Presence of Noise

We apply the same tests in the presence of a white gaussien noise, with diverse powers, the figure 2 demonstrates the same signals in the presence of an additive noise.

6.2.1 A Seismic Wavelet

To study the evolution of the average quadratic errors of the delay and defasing, we apply noises of more or less great amplitudes (10dB,-3dB,-15dB,-17dB,-19dB,-20dB,-22dB,-25dB,-27dB and -30dB) : different signal report on noise SRN.

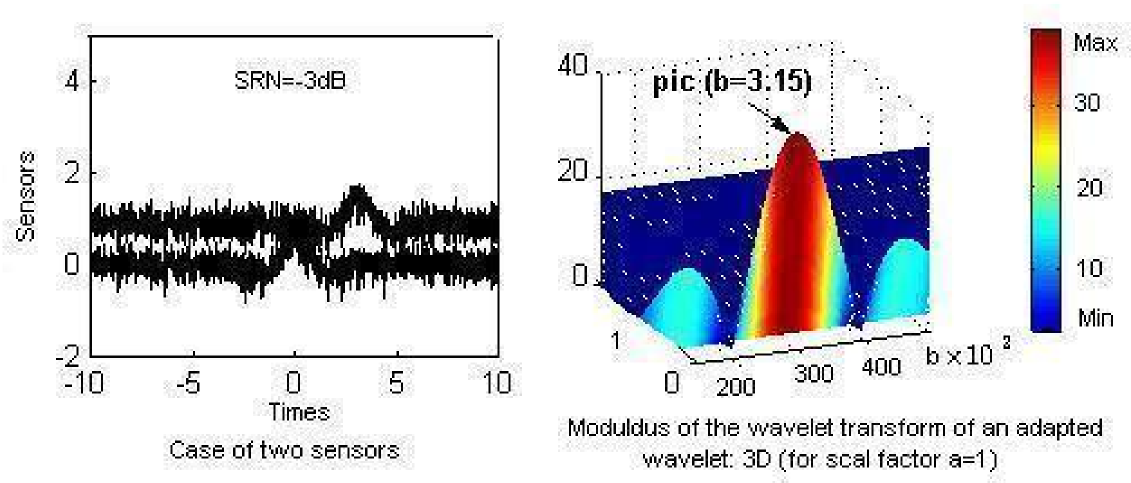

Figure 2: The Synthetic profile and the modulus of wavelet transform

The pic of the seismic wavelet transform, for example, in the case (-3dB) is attained at the 3.15 value with an average quadratic error $Err\_\tau = 1.1000.10^{-4}$, which validates the theorical obtained result.

$SRN\ (dB)$	Err_τ	Err_φ
10	10^{-6}	$0.16.10^{-7}$
-3	$0.11.10^{-5}$	$0.45.10^{-6}$
-10	$0.13.10^{-5}$	$0.47.10^{-6}$
-12	10^{-4}	$0.50.10^{-3}$
-15	$0.90.10^{-5}$	$0.15.10^{-2}$
-17	$0.21.10^{-4}$	$0.18.10^{-2}$
-19	$0.43.10^{-4}$	$0.11.10^{-1}$
-20	$0.13.10^{-3}$	$0.13.10^{-1}$
-22	$0.12.10^{-1}$	$0.18.10^{-1}$
-25	$0.2.10^{-1}$	$0.20.10^{-1}$
-27	$0.41.10^{-1}$	$0.25.10^{-1}$
-30	$0.90.10^{-1}$	$0.56.10^{-1}$

TAB.3 The obtained results by an adapted wavelet and for different noise values

We notice in the case of two waves detected on two sensors,that the wavelet transform presents two pics (two maximal locals see figure 3).

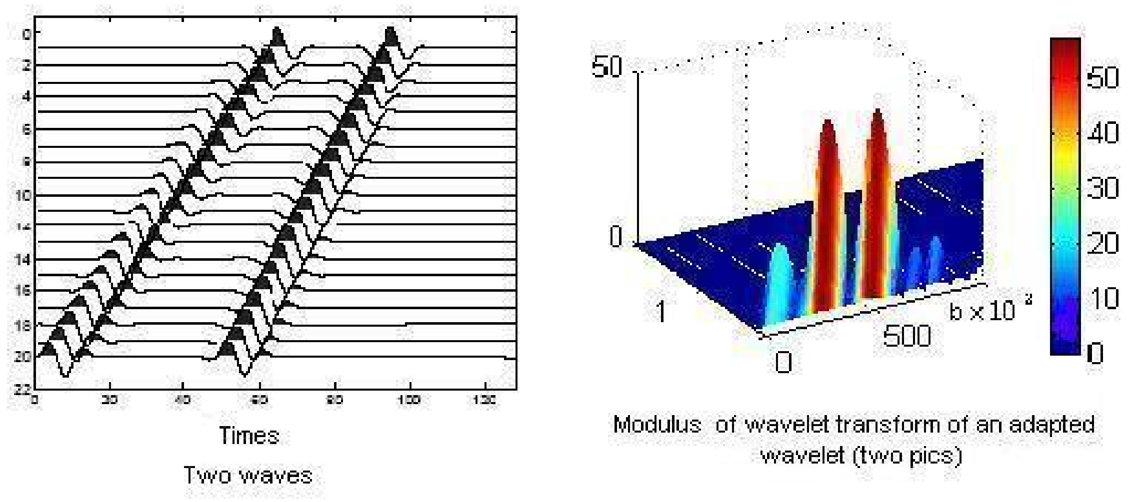

Figure 3: The Synthetic profile and the modulus of wavelet transform for two waves

6.2.2 Classical Wavelets

We have realized the same tests on Daubechies wavelets in two null moments ($N = 2$) and on biorthogonal wavelets, and with the same noise amplitude, and we have noticed that the pic of the wavelets transform is altered in relation to the delay, witch can be theoretically explained by the unsymmetry of the Daubechies wavelets in two null moments. The biorthogonal wavelets are symmetrical, and should not bring about the shifting; still, when it comes to practice, they have the same performance.

$SRN(dB)$	Err_τ	Err_φ
10	$0.53.10^{-7}$	$0.35.10^{-7}$
-3	$0.36.10^{-5}$	$0.77.10^{-6}$
-10	$0.28.10^{-5}$	0.7910^{-6}
-12	$0.42.10^{-2}$	$0.50.10^{-6}$
-15	$0.51.10^{-2}$	$0.52.10^{-6}$
-17	$0.55.10^{-2}$	$0.46.10^{-4}$
-19	$0.19.10^{-1}$	$0.26.10^{-3}$
-20	$0.20.10^{-1}$	$0.28.10^{-3}$
-22	$0.30.10^{-1}$	$0.31.10^{-1}$
-25	$0.80.10^{-1}$	$0.40.10^{-1}$
-27	$0.99.10^{-1}$	$0.51.10^{-1}$
-30	0.15	0.11

Daubechieswavelet

$SRN(dB)$	Err_τ	Err_φ
10	$0.57.10^{-7}$	$0.80.10^{-7}$
-3	$0.66.10^{-5}$	$0.45.10^{-5}$
-10	$0.98.10^{-5}$	$0.47.10^{-5}$
-12	$0.43.10^{-2}$	$0.30.10^{-2}$
-15	$0.51.10^{-2}$	$0.50.10^{-2}$
-17	$0.19.10^{-1}$	$0.58.10^{-2}$
-19	$0.20.10^{-1}$	$0.32.10^{-1}$
-20	$0.23.10^{-1}$	$0.41.10^{-1}$
-22	$0.40.10^{-1}$	$0.60.10^{-1}$
-25	0.11	$0.70.10^{-1}$
-27	0.13	$0.90.10^{-1}$
-30	0.21	0.13

biorthogonalwavelet

TAB.4 The obtained results by Daubechies and biorthogonal wavelets

6.2.3 The Gradient Method

We recall that the gradient method consist of calculating the minimum of the gradient defined by:

$$grad\_E_{/\tau} = \frac{dE_{i,j}(\tau,\varphi)}{d\tau} \quad grad\_E_{/\varphi} = \frac{dE_{i,j}(\tau,\varphi)}{d\varphi}$$

With $$E_{i,j}(\tau,\varphi)=\frac{1}{l-1}\sum_{m=1}^{l}\left[X_i(m)-X_j(m).e^{-2i\pi\tau(m-f_c)}.e^{-i\varphi}\right]^2$$

where $X_k=\widehat{x_k}$, $k=1,...l$, is the Fourier transform of the trace x_k.

The same profile has been studied by the gradient method, which is based on the assumption of minimizing the difference between two consecutive traces [5].The evolution of the average quadratic errors on the delay and defasing is presented in (TAB.5)

$RSN(dB)$	Err_τ	Err_φ
10	$0.45.10^{-2}$	$0.19.10^{-2}$
-3	$0.21.10^{-1}$	$0.62.10^{-2}$
-10	$0.30.10^{-1}$	$0.62.10^{-2}$
-12	$0.44.10^{-1}$	$0.55.10^{-2}$
-15	$0.54.10^{-1}$	$0.35.10^{-2}$
-17	$0.60.10^{-1}$	$0.20.10^{-1}$
-19	$0.90.10^{-1}$	$0.80.10^{-1}$
-20	$0.11.10^{-1}$	0.11
-22	0.20	0.18
-25	0.30	0.30
-27	0.62	0.41
-30	1.057	0.62

TAB.5 The obtained results by Gradient method

The Figure 4 presents the speed of the evolution curve of the average quadratic error in the delay and defasing amplitudes of noise.

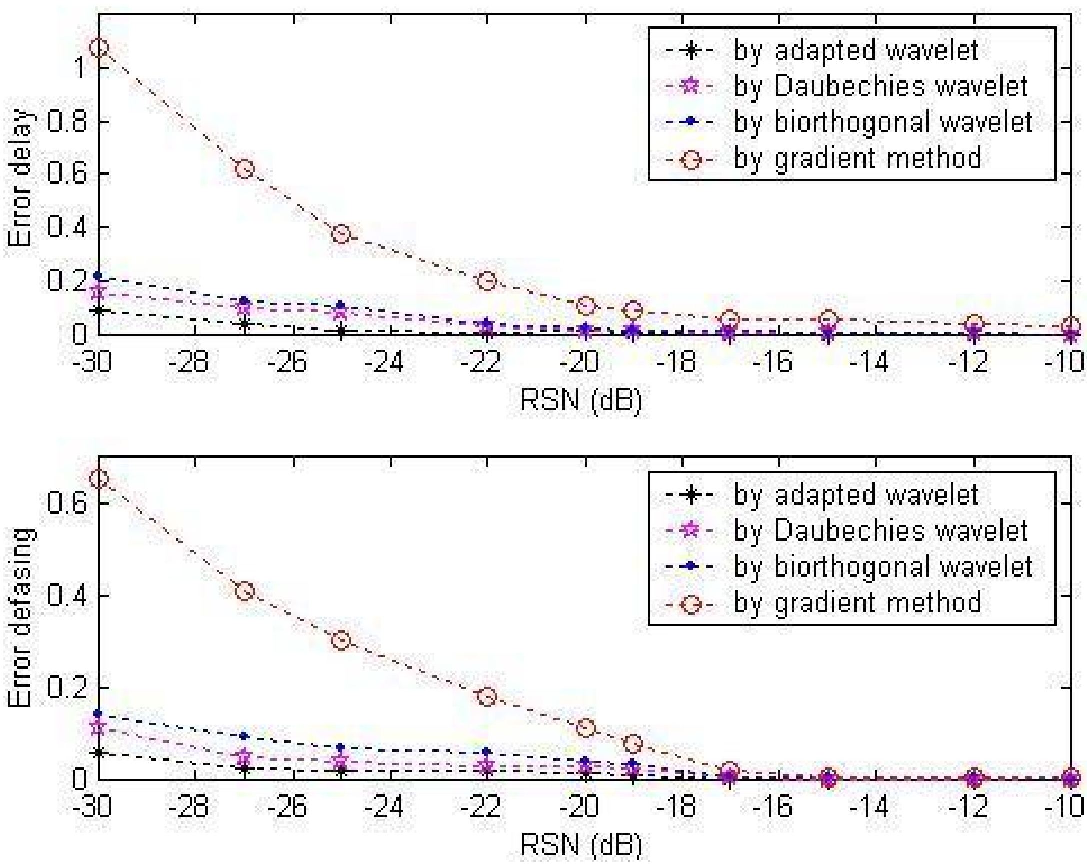

Figure 4: Evolution of average quadratic error

7 Conclusion

While realizing our tests, we have noticed that the seismic wavelets transform gives a numerically satisfactory estimation of the delay and the defasing in regard to biorthognal and Daubechies wavelets. The latter (i.e Daubechies wavelet) even if it gives estimations less good than the one given by the adapted wavelet, represents a certain shifting due to the fact that it is not symmetrical.

The delay and defasing estimations based on the gradient method are numerically satisfactory. Still, they are less good than the ones given by our method; besides, such a method seems " sensitive" to noise, which is rendered by significant errors in the case of a strong noise (weak SRN).

We have chose the formula (8) to calculate the wavelet transform, not only for its frequencial quality, but also for its calculating time which is less elevated (in terms of operations) in regard to the direct calculating algorithm formula (7) specially in the case of a seismic profile registering several waves on different sensors.

Calculating an adapted wavelet in every seismic profile permits to give " finer" estimations more adapted for every situation, which validate the importance of our approach.

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