A Time-scale Representation for the Estimation of the Parameters of Dispersion of the Surface Seismic Waves

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Abstract
In this paper we present a new approach for the estimate of the parameters of dispersion of the surface seismic waves often present in a seismic document and sometimes disturb the interpretation of data. Contrary to the traditional methods which estimate two parameters, we propose a representation time -scale (wavelets) adapted to the estimation of third parameters (of dispersion) modelling the seismic wave propagation in a complex geological medium.
To validate our method, some numerical tests and comparisons in Matlab with synthetic seismic profile will be presented.

Keywords*: Surface seismic waves, wavelet, Time-scale, delay, phase shift, coefficient of dispersion.*

1. Introduction

The study of the surface seismic wave propagation has a fundamental interest for several reasons:

In geology, particularly in civil engineering (construction of the bridge, agriculture, buildings…) the estimate of the parameters of surface seismic waves propagation enables to characterize the properties physics of the ground.

However, in geophysics, the surface waves are regarded as noise in the seismic image which hides useful information and their extraction (wave separation) which enable to increase the signal report on noise (SRN), consequently to facilitate the interpretation of the seismic document (see [1], [2]).

In image processing seismic, several method of filtering are "sensitive" to the "good" correction of the dispersion, in particular, for example the wave separation by singular value decomposition (SVD, see [3]).

The dispersion phenomenon of the seismic surface waves is related to the properties of homogeneity and elasticity of the field: if the propagation medium is made up of several formation, the various harmonics of the wave are propagated at different velocity, thus dispersion is accompanied by a separation between the group velocity and the phase velocity and sometimes a widening of the wave train (deformation of its form) modelling by the introduction of a term of two order (see [2]).

Historically, several methods allowing to estimate the delay and the phase shift, among these methods, those based on the statistics of the Higher order, the time-frequency representation, the frequency-velocity representation or time-frequency-velocity representation and none of these approaches estimate the three parameters of the dispersion (delay, phase shift and coefficient of dispersion, see [4], [5]).

We propose another way by the wavelet transform applied to a seismic profile, indeed we have improve and adapt our method proposed in [8] in this paper we proposed to estimate the dispersion in the simple case modelled by a time delay and a defasing (phase-shift).

The fundamental key of our method is to find an adapted wavelet (must be a "close form" to that of the seismic impulsion) and a scale factor and by the search of the maximal modulus of the wavelet transform we estimate the three parameters of the dispersion.

In order to show the performance of our approach and to confront the theoretical results with the applications we tested our methods on a synthetic profile and with different wavelets adapted and the results are numerically satisfactory.

2. Presentation of the Problem

In a seismic profile (section or image seismic), we can detect the presence of several types of waves: the volume waves (compression P, shearing S) and the surface waves (waves of Rayleigh, Love, Stonely and the tube waves) (see [2]).

When the geological medium is elastic, homogeneous, isotropic and infinite, the waves which are propagated are the volume waves which are not dispersive and make it possible to have information on the physical properties (structural) of the rocks.

If the medium is not infinite, but limited by a free face (ground into seismic terrestrial), in fact, the surface waves are propagated along surface and have a penetration depth in the ground which depends on the frequency this implies a dispersion.

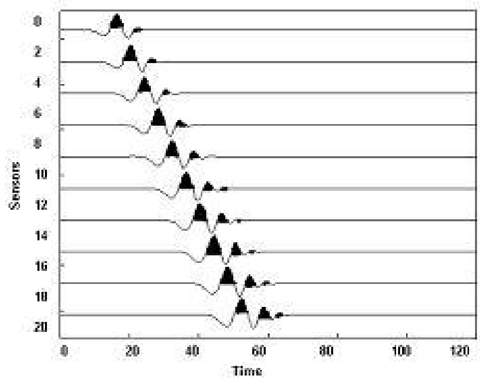

FIG.1: Profile synthetic containing of a dispersive wave

Our objective, thus is the analysis by representation in the time-scale domain (wavelets) of the dispersive wave propagation (surface waves see FIG.1).

We propose an analysis one-dimensional 1D (mono-trace) of a seismic trace delayed, with an phase shift and presenting an dispersion (coefficient of dispersion).

3. Mathematical Modeling of Dispersion

The objective of this part is to present a simplified mathematical model of dispersive waves and not to characterize the different types of waves present in a seismic acquisition.

In geophysics, the emitted signal (seismic impulse) $s(t)$ is returned after reflexion, refraction or diffraction by the interfaces between the various geological layers of the basement.

If we note by $s_p(t), p = 1,...,P$ the signals returned by the interfaces and N the number of sensors, then a model of the seismic trace received on the i^{rd} sensor can be written (see [6], [7]):

$$x_i(t) = \sum_{p=1}^{P} a_{ip} * s_p(t) + b_i(t) \qquad (3.1)$$

With

- a_{ip} Represents the transfer function (Green) between the source p (interface) and sensor i.
- $b_i(t)$ Is a additive noise supposed Gaussian, centered.

In more the share of the cases the propagation medium is supposed isotropic and not infinite (see [2]), then the phase of the transfer function of the filter propagation in function of the frequency noted A_{ip} and which characterizes the dispersive wave can be written:

$$A_{ip}(\xi) = \varphi_{\mathrm{ip}} + r_{ip}\xi + \alpha_{ip}\xi^2 \qquad (3.2)$$

With

- φ_{ip} is the phase shift of the p^{rd} wave in the i^{rd} sensor.
- r_{ip} is the delay.
- α_{ip} is the coefficient of dispersion .

Since we will analyze the only trace and for reasons of simplification, we consider the case of only one wave detected on a series of N sensor (see FIG.1) and thus consider a seismic trace $x(t)$:

$$x(t) = a_x * s(t) + b(t) \qquad (3.3)$$

The phase of the transfer function a_x will be noted then

$$A(\xi) = \varphi + r\,\xi + \alpha\xi^2 \qquad (3.4)$$

We propose to estimate the three parameters φ, r and α.

4. The wavelet transform

During this work, we will confine ourselves to the one dimension case, because we are interested in a particular seismic trace.

To fix the notations, the Fourier transform of a function $f \in L^1(\Re)$ is written:

$$\hat{f}(\xi) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(t) e^{-i\xi t} dt$$

The Fourier Transform can be generalized to temperate distributions, particularly to $L^2(\Re)$ functions (signals of finished energies).

Set $\psi \in L^2(\Re)$ of integrable square, ψ is admissible or checks the admissibility condition if:

$$C_\psi = \int_{-\infty}^{+\infty} \frac{|\hat{\psi}(\xi)|^2}{\xi} d\xi < +\infty \qquad (4.1)$$

We label such a function the analyzing wavelet.

Remark 1 *If $\hat{\psi}(0) = 0$ with $\hat{\psi} \in L^2(\Re)$ and if the derivative of ψ is bound, then ψ is admissible.*

In practice, the wavelet is taken enough regular and the conditions of the preceding remark are checked, consequently the admissibility condition is often written $\hat{\psi}(0) = 0$.

The wavelets family $\psi_{a,b}$, $a \in \Re^{+*}$, $b \in \Re$ is defined for an analyzing wavelet ψ as:

$$\psi_{a,b}(t) = \frac{1}{\sqrt{a}} \psi\left(\frac{t-b}{a}\right)$$

The continuous wavelet transform is the operator W:

$$W: \quad L^2(\Re, dt) \rightarrow L^2\left(\Re^{+*} \times \Re, \frac{dbda}{a^2}\right)$$
$$f \mapsto Wf$$

With
$$Wf(a,b) = \frac{1}{\sqrt{C_\psi}} \int_{-\infty}^{+\infty} \overline{\psi}_{a,b}(t) f(t) dt$$

The continuous wavelet transform is isometric thus it does not have there a loss of information between the function and its wavelet transform.

By applying the formula of Parseval the definition of the continuous wavelet transform can be written:

$$Wf(a,b) = \frac{1}{\sqrt{C_\psi}} \int_{-\infty}^{+\infty} e^{i\xi b} \overline{\hat{\psi}}_{a,b}(a\xi) \hat{f}(\xi) d\xi \qquad (4.2)$$

5. Estimation of the dispersion parameters

We seek to estimate the parameters of dispersion and not to extract the noise, therefore to simplify we suppose that the profile is without noise, consequently the seismic trace (equation (3.3)) becomes:

$$x(t) = a_x * s(t)$$

We suppose that the seismic impulsion is real, symmetrical, and of a compact support:

$$s(t) = 0, \quad \forall t \notin [-T, T]$$

and no energy has been stocked in the ground (there is not absorption see [2]):

$$\int_{-\infty}^{+\infty} s(t)dt = 0 \qquad (5.1)$$

Set, then, a supposed analyzing wavelet ψ of a rapid decrease:

$$\forall \varepsilon > 0,\ \exists C > 0 / \quad |\psi(t)| < \frac{1}{t^2}, \qquad \forall t \notin [-C, C]$$

we are

$$\hat{x}(\xi) = e^{iA(\xi)}\hat{s}(\xi)$$

(not of attenuation because we supposed there is not absorption)

By the formula (4.2), the continuous wavelet transform of the seismic trace $x(t)$ can be written:

$$Wx(a,b) = \frac{\sqrt{a}}{\sqrt{2\pi C_\psi}} \int_{-\infty}^{+\infty} e^{i\xi b}\overline{\hat{\psi}}_{a,b}(a\xi)e^{i(\varphi + r\,\xi + \alpha\xi^2)}\hat{s}(\xi)d\xi$$

$$= \frac{\sqrt{a}}{\sqrt{2\pi C_\psi}} e^{i\varphi} \int_{-\infty}^{+\infty} e^{i(b-r)\xi} e^{-i\alpha\xi^2}\overline{\hat{\psi}}(a\xi)\hat{s}(\xi)d\xi$$

One has for any b such as $|b - r| < 2T$, $a < \frac{T}{C}$ and by exploiting the fact that ψ is of a rapid decrease, we are:

$$|Wx(a,b)| < \frac{s(0)}{\sqrt{C_\psi}} C^{ste}(T)$$

The seismic impulse $s(t)$ checks the equality (5.1) thus according to the remark 1, it is an wavelet analyzing , if the wavelet ψ is chosen such as $\hat{\psi}(\xi)$ has a form "close" to that of $e^{-i\xi^2}\hat{s}(\xi)$ (in particular if we choose $\hat{\psi}(\xi) \approx e^{-i\xi^2}\hat{s}(\xi)$), then for a scale factor a in the vicinity of α , the modulus of the wavelet transform:

$$|Wx(a,b)| = \frac{\sqrt{a}}{\sqrt{2\pi C_\psi}} e^{i\varphi} \left| \int_{-\infty}^{+\infty} e^{i(b-r)\xi} e^{-i(\alpha - a^2)\xi^2}\overline{\hat{s}}(a\xi)\hat{s}(\xi)d\xi \right|$$

Present a local maximum for the value of the point $(a,b) = (\sqrt{\alpha}, r)$, which gives an estimate of the delay and coefficient of dispersion.

For the estimation of phase shift it enough to notice that φ is the phase of $Wx(\sqrt{\alpha}, r)$.

6. Tests and simulations on a synthetic profile

In Matlab, we draw tests of this method on an profiles seismic synthetic (FIG.1) containing a dispersive wave with a delay of 1.15, a phase shift of 0.34 with an coefficient of dispersion of 0.26 trace by trace.

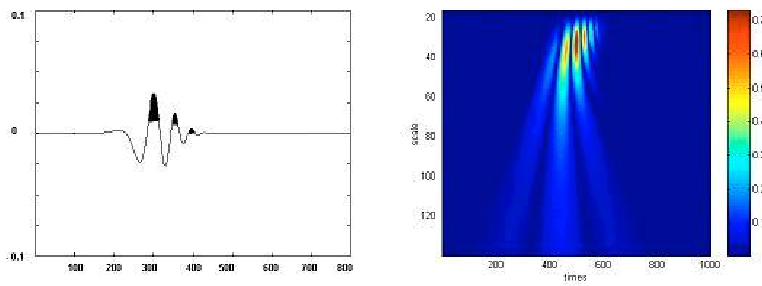

FIG.2: Seismic trace (dispersive wave) and modulus it its wavelet transform (case of the Mexican hat)

The figure (FIG.2) present the analysis of only one trace of the profile by the wavelet transform and we can already measure the delay and the coefficient of dispersion from the modulus of the wavelet transform (local maxima, see image on the right).

We saw in preceding paragraph that the choice of adapted wavelet to the estimate of dispersion rests on the form of the seismic impulse (symmetrical, with compact support and without too many oscillations: $\hat{s}(0)=0$), and the wavelet of analysis will be then in the Fourier domain $e^{-i\xi^2}\hat{s}(\xi)$.

For our case three wavelets appears adapted to our situation, the Mexican hat, the Morlet's and the Daubechies's wavelet at two null moment.

The table (TAB. 1) presents some values found and the average error quadratic on the parameter of dispersion.

Wavelet	r	$AEQ_{\_r}$	φ	$AEQ_{\_\varphi}$	α	$AEQ_{\_\alpha}$
Mexican hat	1.1552	2.7040e-005	0.3471	5.0410e-005	0.2612	1.2544e-004
Dabauchies (N=2)	1.1635	1.8225e-004	0.3224	3.0976e-004	0.2811	9.6721e-004
Morlet	1.1466	1.1560e-005	0.3110	8.4100e-004	0.2877	0.0014

TAB. 1: Numerical results

With

- $AEQ_{\_r}$: Average error quadratic of delay.
- $AEQ_{\_\varphi}$: Average error quadratic of phase shift.
- $AEQ_{\_\alpha}$: Average error quadratic of coefficient of dispersion.

It is noticed that the error with the wavelet built starting from the Mexican hat is more weak compared to the others wavelets and this finds its explanation of made that simulation is carried out on signals dispersive of Mexican hat type.

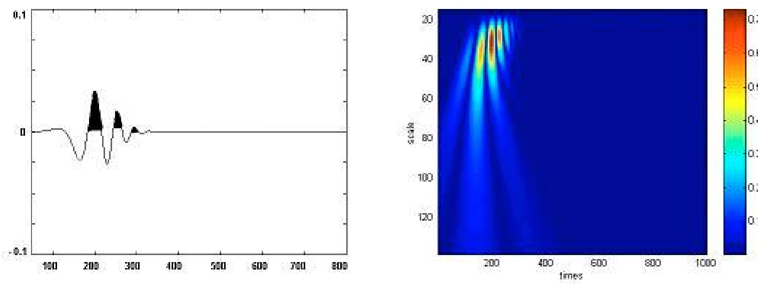

FIG.3: Seismic trace (dispersive wave) after the correction of dispersion (on the left) and its modulus of wavelet transform (on the right)

The figure (FIG.3) shows the result of the correction of dispersion (alignment of the wave).
In conclusion, the numerical results of our approach are satisfactory (within the meaning of the quadratic average errors) but in practice, on the real seismic data except quality of "a good" correction of the wave (alignment) and the comparison between an departure profiles and the same one profiles aligned can judge the quality and the performance of our approach and the choice of the adapted wavelet depends on each profiles given, only the seismic wavelet (impulse) seems to give a better correction ($\hat{\psi}(\xi)\approx e^{-i\xi^2}\hat{s}(\xi)$) however, this wavelet is generally unknown and it is a question of seeking a means to estimate it from profiles to analyze and we seek to generalize the method suggested in [8] based on the function of self-correlation.

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Article received: 2006-07-14