Generalized Gaussian Model for Multi Channel Image Deconvolution

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Abstract:

In Multichannel blind deconvolution (MBD) the goal is to calculate possibly scaled and delayed estimates of the source signals from their convolutive mixtures, using approximate knowledge of the source characteristics only. Nearly all of the solutions to MBD proposed so far require from the source signals to be pair-wise statistically independent and to be timely correlated. In practice, this can only be satisfied by specific, synthetic signals.

In this paper we propose (MDB) algorithm which can recover convolution of sub- and super- Gaussian source image signals with self-adaptive nonlinearity. The MBD algorithm in the framework of natural Riemannian gradient is derived using the parameterized Generalized Gamma density model. The nonlinear function in MBD algorithm is self-adaptive and is controlled by the shape parameters of generalized gamma density model. Computer simulation results confirm the validity and high performance of the proposed algorithm. Implementation tests on real 2-D images are documented.

Keywords*: Multichannel Blind source deconvolution, image processing, Generalized gaussian.*

1. Introduction:

In speech and image recognition systems (which are based on the pattern recognition theory) the first processing step is to process the sensor data in such a way, that the useful original source patterns are extracted without noise and disruption. The applied methods give satisfactory results if the characteristics of the disruption or source characteristics are properly predicted. But methods are needed, that can automatically adjust to the sensor signal. One possible way of seeking general solutions to source extraction lies in the increase of sensors (e.g. microphones or camera views) and the processing of a combined sensor signals (containing given pattern) at the same time. Typical blind source separation (BSS) methods seek separation when the mixing process is unknown. However, loose prior knowledge regarding the mixing process often exists, due to its physical origin. In particular, this process can be represented by a parametric form, rather than a trivial representation of raw numbers. For example, consider convolutive image mixtures caused by defocus blur. This blur can be parameterized, yet the parameters' values are unknown. Such mixtures occur in tomography and microscopy [11, 7]. They also occur in semi-reflections [11], e.g., from a glass window: a scene imaged behind the semireflector is superimposed on a reflected scene [12, 3]. Each scene is at a different distance from the camera, thus differently defocus blurred in the mixtures. We claim that BSS can benefit from such a parameterization, as it makes the estimation more efficient while helping to alleviate ambiguities. In the case of semireflections, our goal is to decompose the mixed and blurred images into the separate scene layers, by minimizing the mutual information (MI) of the estimated objects. An attempt by Ref. [11] used exhaustive search, hence being computationally prohibitive. Ref. [4] attempted convolutive image separation by minimization of higher order cumulate. That method suffers from a scale ambiguity: the sources are reconstructed up to an unknown filter. Moreover, the method's complexity increases fast with the support of the separation kernel. The complexity of convolutive source separation has been reduced in the domain of acoustic signals, by using frequency methods [10, 13]. There, BSS is decomposed into several small point-wise problems by applying a short-time-Fourier transform (STFT). Then, standard BSS tools are applied to each of the STFT channels. However, these tools suffer from fundamental ambiguities, which may reduce the overall separation quality. Ref. [6] suggested that these ambiguities could be overcome by nonlinear operations in the image domain. However, this method encountered performance problems when simulated over natural images. We show that these problems can be efficiently solved by exploiting a parametric model for the unknown blur. Moreover, we use Z-transform at unit circle (sparsity of STFT) coefficients to yield a practically unique solution, which is derived fast. The algorithm is demonstrated in simulations of semi-reflected natural scenes.

2. Fundamental Models and Assumptions

Let us consider a multichannel, linear time invariant (LTI), discrete-time dynamical systems described in the most general form as

$$x(k) = \sum_{p=-\infty}^{p=\infty} H_p s(k-p), \tag{1}$$

where H_p is an m x n n-dimensional matrix of mixing coefficient at time-lag p (called the impulse response at time-lag p) and s(k) is an n-dimensional vector of source signals with mutually independent components. It should be noted that the casuality in time domain is satisfied only when $H_p = 0$ for all p<0. In most general case, we attempt to estimate the sources by employing an other multichannel, LTI, discrete-time, stable dynamical system described as

$$y(k) = \sum_{p=-\infty}^{p=\infty} W_p x(k-p) \tag{2}$$

where $y(k) = [y_1(k), y_2(k), ..., y_n(k)]^T$ is an n-dimensional vector of the outputs and W_p is an n x m-dimensional coefficient matrix at time lag p. We use the operator form notation

$$H(z) = \sum_{p=-\infty}^{p=\infty} H_p z^{-p}, \tag{3}$$

$$W(z) = \sum_{p=-\infty}^{p=\infty} W_p z^{-p}, \tag{4}$$

In practical applications, we need to implement the blind deconvolution problem with a finite impulse response (FIR) multichannel filter with matrix transfer function

$$W(z) = \sum_{p=0}^{p=L} W_p z^{-p}, \tag{5}$$

or apply a non-causal (double-finite) feed forward multichannel filter

$$W(z) = \sum_{p=-K}^{p=L} W_p z^{-p}, \tag{6}$$

where K and L are two given positive integers. The global transfer function is defined by

$$G(z) = W(z)H(z) \tag{7}$$

In order to insure that the mixing system is recoverable, we put the following constraints on the convolutive/mixing systems.

1. the filter H(z) is stable, i.e., its impulse response satisfies the absolute summability condition

$$\sum_{p=-\infty}^{p=\infty} \|H_p\|_2 < \infty \tag{8}$$

where $\|\cdot\|_2$ denotes the Euclidean norm.

2. The filter matrix transfer function H (z) is full rank on the unit circle ($|z| = 1$), that is , it has no zero on the unit circle.

3. Separation Deconvolution Criteria

The blind deconvolution task is to find a matrix transfer function W(z) such that

$$G(z) = W(z)H(z) = P\Lambda D(z) \tag{9}$$

where $P \in R^{nxn}$ is a permutation matrix $\Lambda \in R^{nxn}$ is a nonsingular diagonal-scaling matrix, and the diagonal matrix $D(z) = diag\{D_1(z),..., D_n(z)\}$ represent a bank of arbitrary stable filters with transfer functions $D_i(z) = \sum_p d_{ip} z^{-p}$. In other words, the objective of multichannel blind deconvolution, in most general case, is to recover the source vector s(k) from the observation vector x(k), up to possible scaled, reordered, and filtered estimates. However if we assume that sources are i.i.d, then we can relax the conditions to the form:

$$G(z) = W(z)H(z) = P\Lambda D_0(z) \tag{10}$$

where $D_0(z) = diag\{z^{-\Delta_1},...., z^{-\Delta_n}\}$. In such case, the original source signals can be reconstructed up to arbitrary scaled, reordered, and delayed estimates. In other words, we can preserve their waveforms exactly. For some models it is difficult or even impossible to find an exact inverse of the channels in the scenes described above, since no knowledge of the channel and the source signals is available in advance. Hence, instead of finding an inverse decomposition (9) or (10) in one step, we often attempt to find a matrix W(z) that satisfies the generalized zero-forcing (ZF) condition given by

$$G(z) = W(z)H(z) = \Gamma D_0(z), \tag{11}$$

where Γ is an n x n nonsingular memoryless (constant) mixtures matrix and $D_0(z) = diag\{z^{-\Delta_1},...., z^{-\Delta_n}\}$.

3.1 Multichannel blind deconvolution in the frequency domain

The simplest idea of extending the blind source separation and ICA algorithms to multichannel blind deconvolution is to use the frequency domain techniques. A convolutive mixture in the time domain corresponds to an instantaneous mixture of complex-valued signals and parameters in the frequency domain [2, 9, and 8]. An n-point windowed DFT (Discrete Fourier Transform) is used to convert time domain signals $x_i(k)$ into frequency domain complex-valued-time-series signals:

$$X_i(\omega, b) = \sum_{k=0}^{N-1} e^{-j\omega k} x_i(k) win(k - b\Delta)$$
$$for \omega = 0, \frac{2\pi}{N},...., \frac{N-1}{N} 2\pi, \tag{12}$$

where *win* denotes a window function and Δ is the shift interval of the window. The number of frequency bins is equal to the frame length N and it is correspond to the length of FIR filters of the deconvolutive system. By using the fourier transform the convolutive and deconvolutive models are represented by

$$X(\omega, b) = H(\omega)S(\omega, b), \tag{13}$$

$$Y(\omega, b) = W(\omega)X(\omega, b), \tag{14}$$

and $S(\omega, b) = [S_1(\omega, b),..., S_n(\omega, b)]^T$, $X(\omega, b) = [X_1(\omega, b),..., X_m(\omega, b)]^T$ *and* $Y(\omega, b) = [Y_1(\omega, b),..., Y_n(\omega, b)]^T$

3.2 Cost function for MBD

To describe the ICA optimization denotes $W(\omega)$ as the separation matrix at the channel ω. In addition, denote $I^w\left(\hat{s}_1, \hat{s}_2\right)$, *and* H_{sk}^{ω} as the MI and the marginal entropies of the estimated sources at the channel ω, respectively. Then, the MI of the estimated sources at each channel is given by

$$\min_{W(z)}\left\{-\log\left|\det\left|W(z)\right|\right|+\sum_{k=1}^{K}\hat{H}_{sk}^{\omega}\right\} \tag{15}$$

where $\hat{H}_{sk}^{\omega}$ is the estimator of the channel entropy of an estimated source. Hence using the factorization MI minimization of a convolutive mixture is expected to be both more accurate and more efficient to obtain.

4. Natural Riemannian Gradient in Orthogonality Constraint

Natural Rienmannian gradient in orthogonality constraint has been recently proposed by Amari [1]. Let us assume that the observation vector X(t) has already been whitened by preprocessing and source signals are normalized i.e.,

$$E(x(t)x^T(t)) = I_m \tag{16}$$

$$E(s(t)s^T(t)) = I_n \tag{17}$$

from (16), (17), we have

$$AA^T = I_n \tag{18}$$

the m row vectors of A are orthogonal n dimensional unit vectors. The set of n dimensional subspaces in R^n is called stifle manifold. The natural Riemann gradient in the stifle manifold was calculated by Amari [1]

$$\hat{\nabla} L(W(t)) = \nabla L(W(t)) - W(t)\{\nabla L(W(t))\}^T W(t) \tag{19}$$

Using this result the natural gradient is given by

$$\hat{\nabla} L(W(t)) = f(y(t))x^T(t) - y(t) f^T(y(t))W(t) \tag{20}$$

then the learning algorithm is given by

$$W(t+1) = W(t) - \eta_t \hat{\nabla} L(W(t)) \tag{21}$$

4.1 Natural Riemannian Gradient in Orthogonality Constraint in Frequency Domain

Similar to equation (21) we can write the natural Riemannian in Orthogonality Constraint in Frequency domain by the following equation

$$W(z,k+1) = W(z,k) + \eta_t\{f(Y(z,k))X^H(z,k) - Y(z,k) f^T(Y(z,,k))W(z,k)\} \tag{22}$$

Where Y(z) and X(z), W(z) are defined by equations (1), (2) and (3) respectively

5. GENERALIZED GAUSSIAN SOURCE MODEL

A generalized exponential source model is introduced in [5]. This model encompasses both super- and sub-Gaussian sources. The generalized gaussian density is expressed as follows:

$$F(Y_q) = \frac{r_q}{2\sigma_q\Gamma(1/r_q)} e^{\left(-\frac{1}{r_q}\left|\frac{Y_q}{\sigma_q}\right|^{r_q}\right)} \tag{22}$$

where $r_q > 0$ is a variable parameter, $\Gamma(r) = \int_0^\infty Y^{r-1}\exp(-Y)dY$ is the gamma function and $\sigma_i^r = E\{|Y|^r\}$ is a generalized measure of variance known as the dispersion of the distribution.

The parameter r_q can change from zero to, through 1 (the replace distribution) and r_q =2 (standard Gaussian distribution), to r_q going to infinity (for uniform distribution).

6. Simulation algorithm

1.Start with some initial point W(z,k=0) in the multi-dimensional parameter space
2. Obtain the gradient value $\nabla J(W(z,k))$
3.Compute the value W(z,k+1) by moving from W(z,k) along the gradient descent, i.e. along -$\nabla J(W(z,k))$
$W(z,k+1) = W(z,k) - \eta(k)\nabla J(W(z,k))$
4. Test the stability of the parameters, i.e. if $|W(z,k+1) - W(z,k)| < \theta$ (threshold)

7. COMPUTER SIMULATION RESULTS

Example 1:

Consider the system involving the following three independent sources
$U_1(n)$ =a square wave of amplitude a, and fundamental frequency w0
$U_2(n)$ = a triangular wave of amplitude, and fundamental frequency w0
$U_3(n) = 0.1\sin(400n)\cos(30n)$ (23)
The mixing matrix **A** is

$$\mathbf{A} = \begin{bmatrix} 0.56 & 0.79 & -0.37 \\ -0.75 & 0.65 & 0.86 \\ 0.17 & 0.32 & -0.48 \end{bmatrix} \quad (24)$$

The algorithm was implemented using the following conditions:

- Initialization. The weights in the demixing matrix **W** were picked from a random number generator with a uniform distribution inside the range [0.0,0.5].
- The learning rate parameter was fixed at $\eta = 0.1$
- Signal duration. The time series produced at the mixer output had a sampling period 10^{-4}s and contained N =65,000 samples.

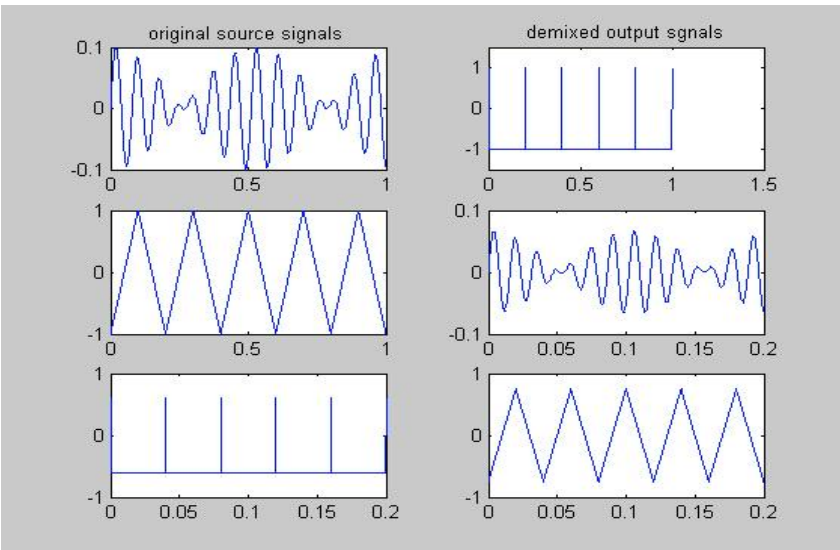

Figure (4) displays the waveforms of the source signals and the signals produced at the output of the demixer. It can be observed those after 3000 iterations; source signals are well separated

Example 2:

The approach described in our paper was implemented in matlab and it was tested on some examples of image sources (Fig. 2.). The result of the blind source deconvolution process applied to the mixtures of these images is shown in Fig. 2. as well.

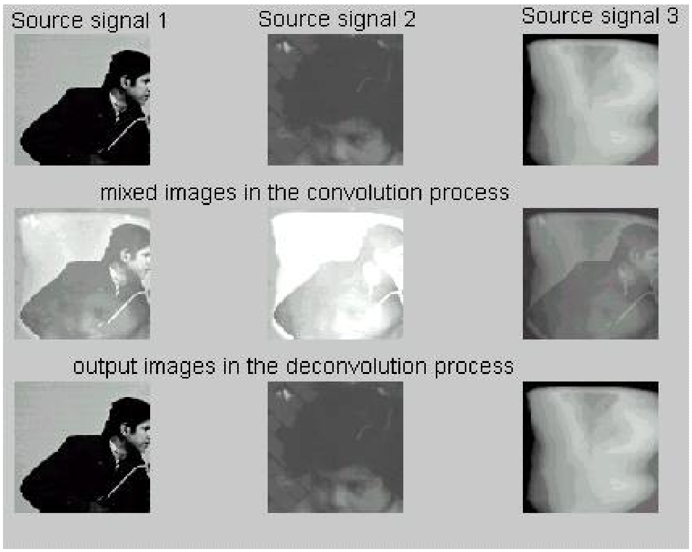

Figure 2: the source signals, the convolution signals and the deconvolution outputs

Conclusion

In this paper we propose and test a new approach to the blind deconvolution via the multiple BSS in frequency domain space we avoid the permutation and scaling problems which are common difficulties of the deconvolution approaches in the frequency domain. Our approach depends on generalized gaussian density function, which is suitable for sub-gaussian and super-gaussian signals. Finally we apply our algorithm on a mixture of natural images, which give a good results.

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