# The mathematical modeling of astrophysics problems

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***Abstract***

*This work proposes an asymptotic method of solution for a system of nonlinear nonhomogeneous equations of one class of initial-boundary problems with an unknown external boundary in the domain. The system of equations describes an adiabatic spherical and symmetrical motion of a gravitating gas , while a moving detonating wave (a spherical surface where the solution undergoes the first kind of discontinuity) is the external boundary of the domain.*

*As the first test problem in this work considered nonavtomodel problem of a central explosion followed by a thermonuclear detonation of a nonhomogeneous bounded with vacuum, gas sphere that is balanced in its own gravitating field. The asymptotic method of a thin shock layer is used for the motion law and the thermodynamic characteristics of the medium are calculated. For the zero approximation of the detonating wave motion layer of Couch's problem in particular case are solved exactly and in general case – with numerical methods. Interpolation formulas and asymptotics are founded. As the second test problem in this work considered nonavtomodel problem of a central explosion followed by a thermonuclear detonation of a nonhomogeneous bounded with interstellar space (vacuum), gas sphere (nonhomogeneous star) that is balanced in its own gravitating field. The initial-boundary problem for a system of nonlinear nonhomogeneous equations are solved with asymptotic method of a thin shock layer. The first two approximations for the motion law and the thermodynamic characteristics of the medium are calculated. For the zero approximation of the detonating wave motion layer of Couch's problem are solved with numerical methods. Numerical results are founded.*

The mathematical modeling of astrophysics processes are one of the most actual problem of modern applied mathematics [1,2].

To resolve a number of astrophysical problems one has to investigate the dynamics of the gas bodies that interact with a gravitating field. It is clear that the conceptions of astrophysical problems investigation can be based on the statement and solution of a number of gas motion dynamic problems. These problems are regarded as theoretic models that include important peculiarities of the motion and evolution of stars.

The methods, devices and considerations of modern theoretical gas dynamics and aerodynamics must be used for the construction and investigation of such models and the statement and solution of corresponding mechanic problems related to astrophysics ones.

Numerical modeling of problems of processes that take place on the nuclei of the stars has been widely used for establishing of the phenomena of supernovae flashes [3-5]. Main attention is paid to physical processes related to thermonuclear reactions and spreading of neutrinos radiation. Less attention is paid to the gas dynamics as a whole. It was considered for a long time that neutrinos, formed in electric seizure and radiated by the central nucleus of a star must transfer a radial component of its impulse to the external layers of a star must transfer a radial component of its impulse to the external layers of the star, thus causing an supernovae explosion. However, we had to reject such mechanism of explosion after the discovery (theoretically and experimentally) in weak interaction of neutral currents that lead to keeping of neutrinos in the star nucleus [6].

This work proposes an asymptotic method of solution for a system of nonlinear no homogeneous equations of one class of initial-boundary problems with an unknown external boundary in the domain. The system of equations describes an adiabatic spherical and symmetrical motion of a gravitating gas , while a moving detonating wave (a spherical surface where the solution undergoes the first kind of discontinuity) is the external boundary of the domain.

As the first test problem in this work considered nonautomodel problem of a central explosion followed by a thermonuclear detonation of a no homogeneous bounded with vacuum, gas sphere that is balanced in its own gravitating field. The asymptotic method of a thin shock layer is used for the motion law and the thermodynamic characteristics of the medium are calculated. For the zero approximation of the detonating wave motion layer of Couch's problem in particular case are solved exactly and in general case – with numerical methods. Interpolation formulas and asymptotics are founded.

As the second test problem in this work considered nonautomodel problem of a central explosion followed by a thermonuclear detonation of a no homogeneous bounded with interstellar space (vacuum) , gas sphere (no homogeneous star) that is balanced in its own gravitating field. The initial-boundary problem for a system of nonlinear no homogeneous equations are solved with asymptotic method of a thin shock layer. The first two approximations for the motion law and the thermodynamic characteristics of the medium are calculated. For the zero approximation of the detonating wave motion layer of Couch's problem are solved with numerical methods. Numerical results are founded.

1. Let us discuss the equations of the adiabatic spherical and symmetrical motion of a gas that are written in Lagrange's form [ 7 ]:

$$\frac{\partial^2 r}{\partial t^2}+4\pi r^2\frac{\partial P}{\partial m}+\frac{km}{r^2}=0, \quad P=(\gamma-1)f(m)\rho^{\gamma}, \quad \rho=[4\pi r^2\frac{\partial r}{\partial m}]^{-1} \tag{1.1}$$

Here m is the r(m,t) radius sphere mass, k is the gravitation constant, γ is the adiabatic indicator , f(m) is the function connected with the distribution of entropy by Lagrange's m coordinate. r = r(m,t) is medium motion law, p(m,t) is medium pressure, ρ(m,t) is medium density.

The first equation of system (1.1) is the motion equation, the second equation is the adiabation equation, the third equation is the mass continuity equation. r(m,t), p(m,t), ρ(m,t) functions are unknown.

The integral equation of the energy of the gas layer situated between the m=0 and m=M(t) surfaces is as follows:

$$T+U+V=E+\int_{t_0}^{t}[\dot{M}(\frac{1}{2}(\frac{\partial r}{\partial t})^2+\frac{P}{(\gamma-1)\rho}-\frac{kM}{R}+Q)-4\pi R^2\frac{\partial r}{\partial t}P]_1 d\tau \tag{1.2}$$

$$T=\frac{1}{2}\int_0^M(\frac{\partial r}{\partial t})^2 dm, \quad U=\frac{1}{(\gamma_2-1)}\int_0^M\frac{P}{\rho}dm, \quad V=-k\int_0^M\frac{mdm}{r}, \quad \dot{M}\equiv\frac{dM(t)}{dt}$$

Here T,U,V are the kinetic, inner and potential (gravitation) energies of the gas, Q is the energy excreted during the burning of a gas mass unit of the m = M(t) surface, E is the explosion energy, m = M(t) is the law of motion shock ($Q=0$) or detonation ($Q\neq 0$) wave with gas mass, R= r ( M(t), t) is the radius of a shock or detonation wave. 1, 2 indices denote correspondingly the gas position in front of and behind the wave.

Boundary conditions on the m = M(t) discontinuity of Euler's variables are as follows:

$$[\rho(\dot{R}-\frac{\partial r}{\partial t})]_1^2=0\,,\qquad [P+\rho(\frac{\partial r}{\partial t}-\dot{R})^2]_1^2=0\,,$$
$$[\frac{1}{2}(\frac{\partial r}{\partial t}-\dot{R})^2+\frac{\gamma}{\gamma-1}\frac{P}{\rho}]_1^2=Q\,,\qquad [\varphi]_1^2\equiv\varphi_2-\varphi_1 \tag{1.3}$$

Boundary conditions on the $m=M(t)$ discontinuity of Lagrange's variables are as follows:

$$[r]_1^2=0,\quad \left[\frac{\partial r}{\partial t}\dot{M}-4\pi r^2 p\right]_1^2=0$$
$$\left[\dot{M}\left(\frac{1}{2}\left(\frac{\partial r}{\partial t}\right)^2+\frac{p}{(\gamma-1)\rho}-\frac{kM}{R}\right)-4\pi r^2 p\frac{\partial r}{\partial t}\right]_1^2=Q\dot{M} \tag{1.4}$$

If boundary conditions (1.3) or (1.4) are solved with respect to parameters of the gas behind the wave we get the following:

$$\rho_2=\frac{\gamma_2+1}{\gamma_2-1}\rho_1\left[1+\frac{1}{\gamma_2-1}\left(\frac{\gamma_2}{\gamma_1}\frac{a_1^2}{\left(\dot{R}-\left(\frac{\partial r}{\partial t}\right)_1\right)^2}+1-g\right)\right]^{-1},$$
$$a_1^2=\frac{\gamma_1 p_1}{\rho_1}$$
$$p_2=\frac{1}{\gamma_2+1}\left[p_1+\rho_1\left(\dot{R}-\left(\frac{\partial r}{\partial t}\right)_1\right)^2+\rho_1\left(\dot{R}-\left(\frac{\partial r}{\partial t}\right)_1\right)^2 g\right]$$
$$\dot{R}-\left(\frac{\partial r}{\partial t}\right)_2=\frac{1}{\gamma_2+1}\left(\dot{R}-\left(\frac{\partial r}{\partial t}\right)_1\right)\left[\gamma_2+\frac{\gamma_2}{\gamma_1}\frac{a_1^2}{\left(\dot{R}-\left(\frac{\partial r}{\partial t}\right)_1\right)^2}-g\right]$$
$$g=\left[\left(1-\frac{\gamma_2}{\gamma_1}\frac{a_1^2}{\left(\dot{R}-\left(\frac{\partial r}{\partial t}\right)_1\right)^2}\right)^2+\frac{2(\gamma_2+1)(\gamma_1-\gamma_2)a_1^2}{\gamma_1(\gamma_1-1)\left(\dot{R}-\left(\frac{\partial r}{\partial t}\right)_1\right)^2}-\frac{2(\gamma_2^2-1)Q}{\left(\dot{R}-\left(\frac{\partial r}{\partial t}\right)_1\right)^2}\right]^{1/2} \tag{1.5}$$

Besides, the continuity of Euler's and Lagrange's variables ought to be taken into account

$$[r]_1^2=0\,,\ [m]_1^2=0 \tag{1.6}$$

In fact, we get a initial-boundary problem for the system (1.1) of nonlinear, no homogeneous equations, where the $r(m,t)$, $p(m,t)$, $\rho(m,t)$ functions are unknown.

Initial conditions ($t = t_0$, phone) determine the initial state of a gas sphere and are the exact $r_1(m,t)$, $P_1(m,t)$, $\rho_1(m,t)$ solutions of the (1.1) system.

Thus, the initial-boundary problem is considered in the domain $\Omega$ :

$$\Omega = \{t \in (t_0, t_*), \qquad m \in (0, M(t))\},$$

where $t_0$ is the moment of explosion, $t_*$ is the moment of time when the wave comes out on the surface of the body (when $t_0 \geq 0, t_* > 0$) or the moment of collapse (when $t_0 < 0, t_* = 0$).

Boundary conditions on the external unknown boundary $m = M(t)$ are like (1.4), (1.5) and in centre

$$r(m,t) = 0, \quad \text{when} \quad m = 0.$$

2. For the most of the gases $\varepsilon = \dfrac{\gamma_2 - 1}{\gamma_2 + 1}$ is a small parameter [7 - 10].

Besides, it is included in (1.1) as a system of equations, in the boundary conditions (1.5) and in the integral equation (1.2), whence the $R(t)$ law of wave motion is established.

Thus, the analysis of the system of equations and boundary conditions makes it clear that the solution can be sought for behind the wave with respect to the small parameter $\varepsilon$ as a kind of several decompositions.

But the decompositions becomes irregular near the symmetry centre ($m = 0$) [7 - 10]. For the solution regularization in this domain we use the method of consecutive approximation the essence of which is that the members of the series area $\varepsilon$ maintained in the zero approximation $\rho_0(m,t)$ of the expression $\rho(m,t)$. Then the first approximation for the medium motion and wave laws is found from the continuity equation by means of the boundary condition $r(0,t) = 0$ and the zero approximation.

The first approximations of the $p(m,t)$ and $\rho(m,t)$ functions will be found in the rest of the system (1.1) of equations.

The described our method makes it possible to solve quate a wide class of initial-boundary problems of a system of equations (1.1). It is natural that the choice of decomposition depends on the initial state of the gas sphere (the exact solution of (1.1) system before the wave) and on the energy of the explosion.

3. Let us discuss the problem of the central explosion at the $t = 0$ moment of a no homogeneous gas sphere ( star ) balanced in its own gravitation field as the first test problem.

Thus, the exact solution of the system of equations (1.1) that corresponds the no homogeneous gas sphere balanced in its own gravitation field ( $\left(\dfrac{\partial r}{\partial t}\right)_1 = 0$ ) is taken as an initial condition (phone). The gravitation constant, the sphere centre density and the sphere radius are taken as main units of dimension

$$\rho = 1 - r^{\alpha}, \quad m = 4\pi r^3 \left(\frac{1}{3} - \frac{r^{\alpha}}{3+\alpha}\right), \tag{3.1}$$

$$P = 4\pi\left[\frac{1}{6}(1 - r^2) - \frac{6+\alpha}{3(3+\alpha)(\alpha+2)}(1 - r^{\alpha+2}) + \frac{1 - r^{2\alpha+2}}{2(3+\alpha)(1+\alpha)}\right], \quad \alpha \in (0,1)$$

It arises from (3.1) that the pressure $p$ and density $\rho$ is equal to zero on the sphere surface ($r=1$), i.e. the $r=1$ sphere is a boundary between a star and the interstellar space, as the density of the interstellar gas $\rho \sim 10^{-24}$ gr/sm$^3$.

Let us introduce a small parameter [ 7 – 10]

$$\varepsilon = \frac{\gamma_2 - 1}{\gamma_2 + 1}$$

The analysis of the energy integral equation (1.2) and the condition of the existence of a detonating wave (1.5) before the moment of the body coming out on the surface leads us to the condition

$$E = E_0 / \varepsilon^2, \quad E_0 = \underline{\underline{O}}(1)$$

Besides, the time of detonating wave motion before the sphere comes out on the surface will be of $\sqrt{\varepsilon}$ series.

That's why for the sake of simplicity we can additionally protract the time

$$\tau = t/\sqrt{\varepsilon}$$

The analysis of the system of equations (1.1) and the boundary conditions (1.5) makes it clear that the solution can be sought behind the detonating wave as the following decomposition:

$$r = R_0(\tau) + \varepsilon H(m,\tau) + \dots, \quad R(\tau) = R_0(\tau) + \varepsilon R_1(\tau) + \dots,$$

$$P = P_0(m,\tau) + \varepsilon P_1(m,\tau) + \dots, \quad \rho = \frac{\rho_0(m,\tau)}{\varepsilon} + \rho_1(m,\tau) + \dots, \quad \tau = t/\sqrt{\varepsilon} \tag{3.2}$$

Including the (3.2) decomposition into the (1.1) system of equations, in the (1.2) integral equation and in the (1.5) boundary conditions, we shall obtain the zero approximation of the problem solution using the regularization method [7 - 10].

$$P_0 = R_0^{'^2}(1 - R_0^{\alpha}) + \frac{R_o^{''}(M_0 - m)}{4\pi R_0^2}, \qquad M_0 = 4\pi R_0^3\left(\frac{1}{3} - \frac{R_0^{\alpha}}{3+\alpha}\right) \tag{3.3}$$

$$\rho_0 = P_0^{1/\gamma_2}[R_0^{'^2}(T_0)]^{-1/\gamma_2}\left[1 + \frac{(1/\gamma_2 - \gamma_*)a_1^2(m) + 2Q_0}{R_0^{'^2}(T_0)}\right]^{-1}$$

$$a_1^2(m) = \frac{4\pi\gamma_1}{1 - r^{\alpha}}\left[\frac{1}{6}(1 - r^2) - \frac{6+\alpha}{3(3+\alpha)(\alpha+2)}(1 - r^{\alpha+2}) + \frac{1 - r^{2\alpha+2}}{2(3+\alpha)(\alpha+1)}\right]$$

$$\gamma_* \equiv \frac{1}{\gamma_1}, \;\; \gamma_1 - 1 = \underline{\underline{0}}(1); \;\; \gamma_* \equiv \frac{\gamma_1 - \gamma_2}{\gamma_1 - 1} = \underline{\underline{0}}(1), \;\; \gamma_1 - 1 = \underline{\underline{0}}(\varepsilon)$$

$$\gamma_* \equiv 0, \qquad \gamma_1 = \gamma_2, \qquad Q = Q_0/\varepsilon, \qquad Q_0 = \underline{\underline{0}}(1),$$

where $r = r(m)$ are determinate from equation

$$m = 4\pi r^3\left(\frac{1}{3} - \frac{r^{\alpha}}{3+\alpha}\right),$$

and $T_0 = T_0(m)$ is the moment of time when the detonating wave passes the particle with Lagrange's m coordinate.

The function $R_0(\tau)$ in the (3.2) is the Cauchy's following problem solution:

$$\pi(1-R_0^{\alpha})R_0^{'^2} R_0^3(\frac{1}{3}-\frac{R_0^{\alpha}}{3+\alpha}) = E_0 , \quad R_0(0)=0 , \qquad E = E_0 / \varepsilon^2 , \quad E_0 = \underline{\underline{O}}(1) \qquad (3.4)$$

In particular, when $\alpha = 5/6$ (3.4) problem are solved exactly

$$F(R_0) - F(0) = [\frac{23E_0}{6\pi}]^{1/2}\tau$$

$$F(R_0) = (R_0^{5/6} - \frac{41}{36})(R_0^{5/3} + \frac{23-41R_0^{5/6}}{18})[\frac{1}{4}(R_0^{5/6} - \frac{41}{36})^2 + \frac{2233}{3456}] -$$
$$-\frac{130225}{13436928}\ln|R_0^{5/6} - \frac{41}{36} + (R_0^{5/3} + \frac{23-41R_0^{5/6}}{18})^{1/2}| + \frac{41}{56}(R_0^{5/3} + \frac{23-41R_0^{5/6}}{18})^{3/2} \qquad (3.5)$$

In general case the (3.4) problem are solved with numerical methods (Euler's and Range - Cute methods).

The Cauchy's problem (3.4) asymptotic of solution are calculated, when $\tau \to 0_+$ and from (3.5) when $\tau \to \tau_-^*$

$$(\tau^* : R_0(\tau^*) = 1)$$

$$R_0(\tau) \cong (\frac{75E_0}{4\pi})^{1/5}\tau^{2/5} \quad , \quad \tau \to 0_+ ,$$

$$R_0(\tau) \cong 1 + \frac{1}{F^{'}(1)}(\frac{23E_0}{6\pi})^{1/2}(\tau - \tau^*) \quad , \quad \tau \to \tau_-^* \quad ,$$

$$\tau^* = [F(1) - F(0)](\frac{6\pi}{23E_0})^{1/2} \quad , \qquad F'(1) = 0.058$$

With numerical results and asymptotic are founded the following interpolation formulas, when $\tau \in [0, \tau^*]$

$$R_0(\tau) \cong (\frac{75E_0}{4\pi})^{1/5}\tau^{2/5} + 1961.26\tau^2 - 27.05\tau , \ \alpha = 0.1 , \ \tau^* = 0.0283$$

$$R_0(\tau) \cong (\frac{75E_0}{4\pi})^{1/5}\tau^{2/5} + 3.15\tau^2 + 17.05\tau , \ \alpha = 0.25 , \ \tau^* = 0.0567$$

$$R_0(\tau) \cong (\frac{75E_0}{4\pi})^{1/5}\tau^{2/5} - 28.67\tau^2 + 13.12\tau , \ \alpha = 0.5 , \ \tau^* = 0.0955$$

We shall obtain the following from the continuity equation in the next ( first ) approximation

$$\frac{4\pi r^3}{3} = \frac{4\pi R^3}{3} - \int_m^M \frac{\varepsilon dm}{\rho_0(m,\tau)} \qquad (3.6)$$

We shall use the boundary condition in the centre: m = 0 when r = 0, to establish the first approximation $R_1(\tau)$ of the detonating wave motion law. We shall obtain the following

$$R(\tau) = [\frac{3}{4\pi}\int_0^M \frac{\varepsilon dm}{\rho_0(m,\tau)}]^{1/3} \qquad (3.7)$$

Using the motion law of the gas found behind the detonating wave (3.6), (3.7) we shall calculate $p_1(m,\tau)$, $\rho_1(m,\tau)$ from (1.1) system of equations.

**4**. Let us discuss the second test problem. The exact solution of the system of equations (1.1) that corresponds the no homogeneous gas sphere (star) balanced in its own gravitation field is taken as an initial condition ( phone). The gravitation constant, the sphere centre density and the sphere radius are taken as main units of dimension

$$\rho = (1-r)^n ,$$

$$m = 4\pi\left\{-\frac{1}{n+1} r^2 (1-r)^{n+1} - \frac{2r(1-r)^{n+2}}{(n+1)(n+2)} + \frac{2}{(n+1)(n+2)(n+3)}\left[1-(1-r)^{n+3}\right]\right\} \quad (4.1)$$

$$P = \frac{4\pi}{n+1}\left(-\frac{(1-r)^{2(n+1)}}{2(n+1)} - \frac{2}{n+2}\int_r^1 \frac{(1-r)^{2(n+1)}}{r} dr + \right.$$
$$\left. + \frac{2}{(n+2)(n+3)}\int_r^1 \frac{(1-r)^n}{r^2} dr + \frac{2}{(n+2)(n+3)}\int_r^1 \frac{(1-r)^{2n+3}}{r^2} dr \right)$$
$$n = \text{const} > 0$$

It arises from (4.1) that the pressure p and density $\rho$ is equal to zero on the sphere surface $(r=1)$, i.e. the $r=1$ sphere is a boundary between a gravitating gas sphere (star) and the vacuum (the interstellar space, as the density of the interstellar gas $\rho \sim 10^{-24}$ gr/sm$^3$).

The exact solution (4.1) of the system of equations (1.1) describes vary real model of the star and of the interstellar medium.

Let us introduce a small parameter

$$\varepsilon = \frac{\gamma_2 - 1}{\gamma_2 + 1}$$

The analysis of the system of equations (1.1) and the boundary conditions (1.5) makes it clear that the solution can be sought behind the detonating wave as the following asymptotic decomposition:

$$r(m,\tau) = R_0(\tau) + \varepsilon H(m,\tau) + ..., \quad R(\tau) = R_0(\tau) + \varepsilon R_1(\tau) + ... ,$$
$$P(m,\tau) = P_0(m,\tau) + \varepsilon P_1(m,\tau) + ... , \quad \rho(m,\tau) = \frac{\rho_0(m,\tau)}{\varepsilon} + \rho_1(m,\tau) + ... , \quad (4.2)$$
$$\tau = t/\sqrt{\varepsilon}$$

Including the (4.2) decomposition into the (1.1) system of equations, in the (1.2) integral equation and the (1.5) boundary conditions, we shall obtain the zero approximation of the problem solution using the regularization method [7 -10].

$$P_0 = R_0^{'^2}(1-R_0)^n + \frac{R_o^{''}(M_0 - m)}{4\pi R_0^2},$$

$$M_0 = \frac{8\pi}{(n+1)(n+2)(n+3)}\left\{1-(1-R_0)^{n+3} - \frac{(n+2)(n+3)}{2} R_0^2 (1-R_0)^{n+1} - \right.$$
$$\left. -(n+3)R_0(1-R_0)^{n+2}\right\}$$

$$\rho_0 = P_0^{1/\gamma_2}\left[R_0^{'^2}(T_0)\right]^{-1/\gamma_2}\left[1 + \frac{\left(1/\gamma_2 - \gamma_*\right)a_1^2(m) + 2Q_0}{R_0^{'^2}(T_0)}\right]^{-1} \quad (4.3)$$

$$a_1^2(m)=\frac{\gamma_1 P_1}{\rho_1}=\frac{\gamma_1 4\pi}{(n+1)(1-r)^n}\left\{-\frac{(1-r)^{2(n+1)}}{2(n+1)}-\frac{2}{n+2}\int_r^1\frac{(1-r)^{2(n=1)}}{r}dr+\right.$$
$$\left.+\frac{2}{(n+2)(n+3)}\int_r^1\frac{(1-r)^n}{r^2}dr+\frac{2}{(n+2)(n+3)}\int_r^1\frac{(1-r)^{2n+3}}{r^2}dr\right\}$$

$$Q=Q_0/\varepsilon\,,\qquad Q_0=\underline{\underline{0}}(1)\,,$$

$$\gamma_*\equiv\frac{1}{\gamma_1}\,,\ \ \gamma_1-1=\underline{\underline{0}}(1)\,;\ \ \gamma_*\equiv\frac{\gamma_1-\gamma_2}{\gamma_1-1}=\underline{\underline{0}}(1)\,,\ \ \gamma_1-1=\underline{\underline{0}}(\varepsilon)$$

$$\gamma_*\equiv 0\,,\qquad \gamma_1=\gamma_2\,,$$

where $r=r(m)$ are determinate from (4.1), $T_0 = T_0(m)$ is the moment of time when the detonating wave passes the particle with Lagrange's m coordinate.

The function $R_0(\tau)$ in the (4.3) is the Cauch's following special problem solution:

$$R_0^{'2}\ \frac{2\pi(1-R_0)^n}{(n+1)(n+2)(n+3)}\left[1-(1-R_0)^{n+3}-\frac{(n+2)(n+3)}{2}R_0^2(1-R_0)^{n+1}-\right.$$
$$\left.-(n+3)R_0(1-R_0)^{n+2}\right]=E_0\,,\ \ R_0(0)=0 \tag{4.4}$$

$$E=E_0/\varepsilon^2\,,\quad E_0=\underline{\underline{O}}(1)\,.$$

The Cauch's problem (4.4) exactly not solved. For the problem solution we use the Euler's numerical method. The numerical solution considerate from the time $\tau=0$ for the time $\tau_*$, where are founded from equality

$$R_0(\tau_*)=1$$

The Cauch's problem (4.4) asymptotic of solution are calculated, when $\tau\to 0_+$

$$R_0(\tau)\approx\left(\frac{75E_0\tau^2}{4\pi}\right)^{1/5},\quad \tau\to 0_+$$

The results of the numerical solution are founded.

In the next (first) approximation we shall get the following from the continuity equation (1.1)

$$\frac{4\pi r^3}{3}=\frac{4\pi R^3}{3}-\int_m^M\frac{\varepsilon dm}{\rho_0(m,\tau)} \tag{4.5}$$

We shall use the boundary condition in the centre: m=o when r=0, to establish the first approximation $R_1(\tau)$ the decomposition (4.2) of the detonating wave motion law. We shall obtain the following formula:

$$R(\tau)=\left[\frac{3}{4\pi}\int_0^M\frac{\varepsilon dm}{\rho_0(m,\tau)}\right]^{1/3} \tag{4.6}$$

Finally, using the motion law of the gas found behind the detonating wave (4.5), (4.6) we shall calculate the first approximations $p_1(m,\tau)$ and $\rho_1(m,\tau)$ from (1.1) system of equations.

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