One Stochastic Model For the Struggle Against Malaria

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Abstract

In this work is described one stochastic model for the struggle against Malaria which reduces Anopheles reproducing dynamics on the Markov's Chains. For illustration is considered the simplest two factored situation. On the one hand this gives us an opportunity for the foundation of multifactor models in future and on the other hand for the development of preliminary and correcting prognoses theory in parallels with the well approbated models.

Keywords: *Malaria, Mosquito, Markov's Chain, Stochastic Model, 2 Factors.*

§1 Introduction and target settings

It is known that calling reason of Malaria – plasmodium transferring mosquito Anopheles spreading dynamics depends on certain physical, biological and ecological factors [1],[10]. Disease nozoareal usually coincide with insect propagation areas and depends on several factors and most on plasmodium and its carrier's phenotipical development speed [4].

For plasmodium and anopheles spreading and development speed, as for all ectodermic organisms, general factor is, as negative as positive, environment temperature [5,6]. Of course in huge amounts of models which appreciate anopheles population critical factor is temperature. Jean-Marc O Depiney and authors in their mathematical model, which describes anopheles dynamic and malaria spreading, underscored five main factors where temperature had taken as a main factor, we can appreciate larvae development speed using this model by the moisture and temperature also using other 3 factors (see [1], formula (2), [8], [11]). Around this two main factors are classified all other factors. It was mentioned that, influence of temperature is inquired through the enzyme reaction kinetics in cell (see [4]) on the basis of [1], [7], [8], [9] we can mark out two main groups of factors, first – factors which support and which suppress mosquito population growth.

Naturally arises the following question, is it possible to distribute these factors in environment of the opposite Markov's chain after their quantitative description, similar to the distribution of discrete automats in a random environment (see [2],[3]), below is given a positive answer on the posted question an established by an example of one idealized scheme.

§2 Main notations and definitions

Let R be the set of real numbers and R^n be the n- dimensional vector space endowed with the standard inner product; for vectors x and y from R^n where the inner product is defined by $(x\,|\,y)$ and $(x\,|\,y)=\sum_{i=1}^{n} x_i y_i$ where $x=(x_1,x_2,x_3,...,x_n), y=(y_1,y_2,y_3,...,y_n)$ in the natural basis $(e_1,e_2,e_3,...,e_n)$. We say that R^n is R algebra, if a bilinear map (reflection) $[x,y]$ of R^n onto is given. It is easy to see that R algebra is defined by a set of linear operators

$$A^{(1)}, A^{(2)}, A^{(3)},..., A^{(n)} \tag{2.1}$$

Acting in R^n by the following way

$$[x, y]_k = \sum_{i,j=1}^{n} a_{i,j}^{(k)} x_i y_j \quad (k = 1,2,\dots,n) \qquad (2.2)$$

Where $\left(a_{i,j}^{(k)}\right)_{i,j=1}^{n}$ is matrix representation of the operator $A^{(k)}$ in the given basis. Conversely, every R algebra is given by (2.1) and (2.2). For the given R algebra and every $\sigma = (\sigma_1, \sigma_2, \sigma_3, \dots, \sigma_n)$ we will deal with the following linear operators acting in R^n.

$$\tilde{A}_\sigma = \sum_{k=1}^{n} A^{(k)} \sigma_k \qquad (2.3)$$

and we'll say that $\sigma = (\sigma_1, \sigma_2, \sigma_3, \dots, \sigma_n)$ vector is distributed through R algebra. Later on we suppose that

$$\sigma_i > 0 \quad (i = 1,2,3,\dots,n); \ \sum_{i=1}^{n} \sigma_k = 1 \qquad (2.4)$$

We say that R^n forms stochastic algebra, if the operators $A^{(k)}$ $(k = 1,2,3,\dots,n)$ are stochastic, it means that if

$$A^{(k)} = P^{(k)}; \ \left(a_{i,j}\right)_{i,j=1}^{n} = \left(P_{i,j}^{(k)}\right)_{i,j=1}^{n}; \ P_{i,j}^{(k)} \geq 0; \ \sum_{j=1}^{n} P_{i,j}^{(k)} = 1 \ \ \forall i = 1,2,\dots,n \qquad (2.5)$$

It is clear that in the stochastic algebra the liner operator

(2.6)

defined by (2.3) and (2.5) with from (2.4), is stochastic too thus we say, that (2.7)

a vector from (2.6) is distributed in the stochastic algebra. This last notion,

with stochastic operators and with are essential and main for us. The point, is that to any stochastic operator corresponds Markov's chain wih a constant step

and finite positions

the dynamic of which is completely defined buy the powers of stochastic operator (2.8)

:

and by so-called initial distribution

(2.9)

; ;

After the conducting reasoning we can say that to the distribution, defined by (2.5), (2.6) and (2.7) vector through algebra accompanies Markov's chain dynamic of it completely defined by the operator given by (2.8) and (2.9).

Remark (2.1): Here we are discussing about the powers of the operator , and in (2.6) denotes the stochastic operator (2.5).

Remark (2.2): It's clear that procedure described above concerning the vector is not one valued. Choice of the operators must be dictate from the contents of task.

§3 Prediction with two factor system

Above was mentioned that for clearness we consider simplest case with two factors for the struggle against Anopheles. As the first factor is taken mosquito development rate at the larval stage, precisely its maximal value and, according to [1], the value of this maximum per hour. This factor promotes malaria's dispersal and we denote this by . The second factor is taken also from [1] -<nutrient competition>. As the first factor we take here maximal value of the development rate loss per hour and denote it by . It is clear that is an abstraction factor. First step for model creating means construction of vector by the following procedures:

(3.1)

the conditions (2.4) are satisfied automatically. By this procedure the opposite factors are and . Now we have to create Markov's chain, for which, according to our agreement, the (2.7) is 1 hour.

In this work we present the simple case for the development of vector , defined by (3.1):

(3.2)

It means that we consider the case of symmetric stochastic operators. They are simultaneously reducible in the natural basis ; of the coordinate plane .

The eigenvalues of two-dimensional symmetric stochastic operators (3.3)

are

;

really

It's easy to see that the operator reduces to the coordinate axes by means of the following orthogonal operator:

(3.4)

Really

the operator could be reduced to the diagonal form for arbitrary and . Therefore

(3.5)

Hence dynamic of Markov's chain, which is defined by powers of the operator is given by means of (3.5) completely.

Now we are ready for the study of distribution of defined by (3.1) vector through defined by (3.2) .According to (3.1) and (3.2) we have

For the conformability let us denote value of operator by . By (3.3), (3.4) and (3.5) it is enough to study , which we can write in the following form:

Thus we have proved the following:

Proposition (3.1). Dynamic of Markov's chain (with step equal to 1 hour), which is defined by the vector and is distributed through the stochastic algebra , is defined by powers of the operator and is given by the following formula:

Sense of and are given in the beginning of this section. As we can see our Markov's chains dynamic depends on a phase

(3.6)

completely. e.g. for and we have that

(3.7)

Taking into consideration boundary condition we obtain at last that

(3.8)

Thus for and arbitrary initial condition the desired probabilities tends to . It was expected because symmetric stochastic operators are always ergodic. We can see from (3.6), (3.7) and (3.8) that relaxation time depends only on the phase . For example, in case of (3.7) the time of relaxation is more established as soon as and factors are near to each other.

The obtained results could be generalized for the case when instead of twice stochastic matrix we consider general stochastic matrix:

It is clear that case of 2 factors which is in addition contracted by symmetric operators, is far from the real situation. The authors hope that research in that direction will be useful.

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