Fuzzy Model Reference Learning Controller For Pitch Control System of an Aircraft

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Abstract

In this paper Fuzzy Model Reference Learning Controller (FMRLC) is designed for a pitch controller of an aircraft to obtain the desired pitch angel as required by the pilot. This controller utilizes a learning mechanism, which observes the plant outputs and adjusts the rules in a direct fuzzy controller, so that the overall system behaves like a "reference model" which characterizes the desired behavior. The performance of the FMRLC is demonstrated by simulation for various conditions with change in the aircraft dynamics caused due to change in speed of the aircraft and sensor noise.

Keywords: *FMRLC, Reference Model, Fuzzy Inverse Model, Learning Mechanism ,Pitch Control System*

1. Introduction

The large scale process control problems have been largely dealt with conventional proportional-integral-derivative (PID) controllers. Although these conventional techniques provide a minimum performance requirement, they fall short of the increasing control performance demand of robustness, optimality and adaptation to external disturbances. In the last decade new emerging intelligent control techniques have been gaining acceptance for their attractive design and implementation advantages.[1]These new control methods provides solutions for problems where no mathematical model of the system is possible and uncertainties in the operating environment are significant .Over recent years the fuzzy control has emerged as a practical alternative to classical control schemes when one is interested in controlling certain time-varying, non-linear, and ill-defined processes. While fuzzy control can provide effective practical solutions to complex aircraft problems [2-3] as an alternative to conventional control methods there are several drawbacks that may limit its use for some problems. The fuzzy controller sometimes performs inadequately if significant and unpredictable plant variations occur. With introduction of a progressively learning mechanism[4] into the system along with basic ideas of fuzzy sets and control theory improves the performance of the overall controlled system when interacting with the environment. The FMRLC[5] utilizes a learning mechanism $\theta_{ref}(kT)$ which observes the plant outputs and adjusts accordingly the rules in a direct fuzzy controller such that the overall system performs satisfactorily.

2. FMRLC

The functional block diagram for the FMRLC is shown in Figure-.1. It has four main parts i.e. the plant, the fuzzy controller to be tuned, the reference model, and the learning mechanism (an adaptation mechanism). The FMRLC uses the learning mechanism to observe numerical data from a fuzzy control system (i.e., $\theta ref(kT)$ and $\theta(kT)$ where T is the sampling period). Using this numerical data, it characterizes the fuzzy control system’s current performance and automatically synthesizes or adjusts the fuzzy controller so that some given performance objectives are met. These performance objectives (closed-loop specifications) are characterized via the reference model shown in Figure 1. The learning mechanism seeks to adjust the fuzzy controller so that the closed-loop system (the map from $\theta ref(kT)$ to $\theta(kT)$) acts like the given reference model (the map from $\theta ref(kT)$ to $\theta m(kT)$).The fuzzy control system loop which is the lower part of Figure-1 operates to make $\theta(kT)$ to track $\theta ref(kT)$ by manipulating $\delta(kT)$.The upper-level adaptation control loop which is the upper part of Figure 1 seeks to make the output of the plant $\theta(kT)$ to track the output of the reference model $\theta ref(kT)$ by manipulating the fuzzy controller parameters.

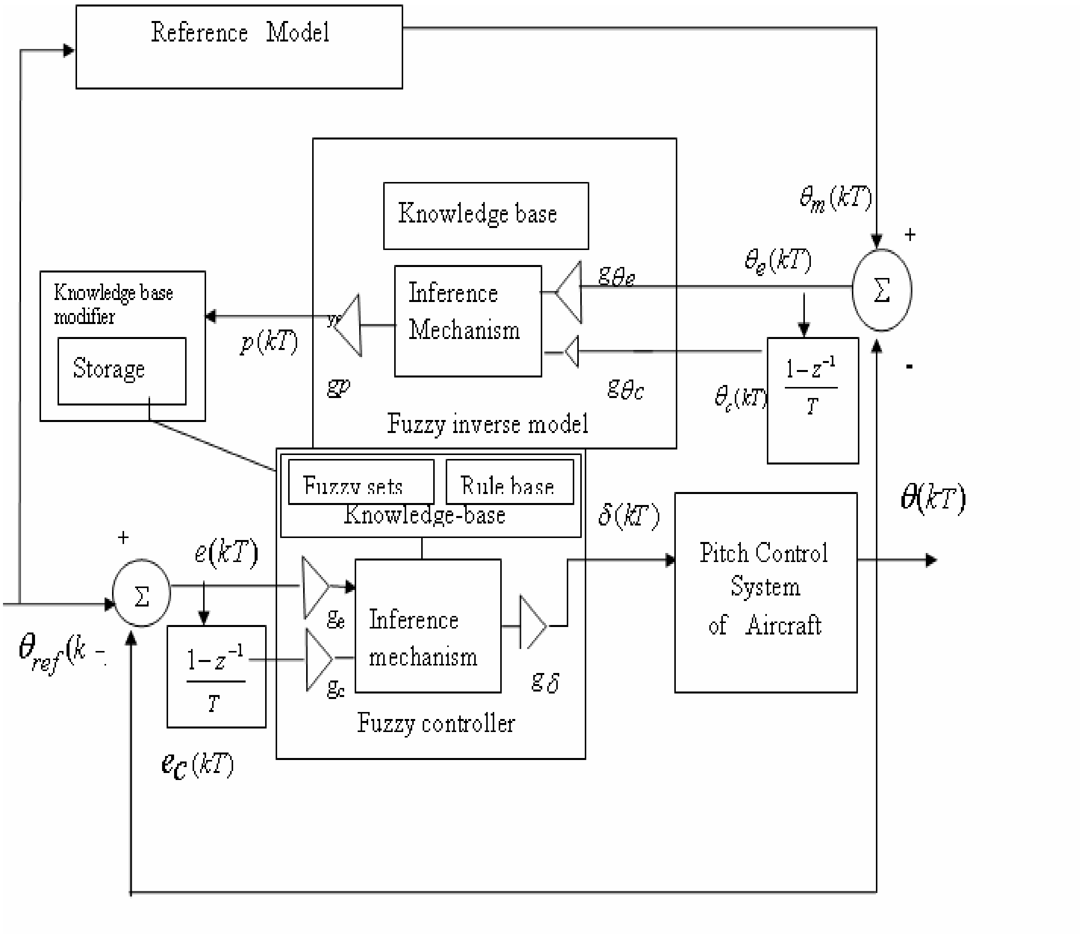


Figure 1. FMRLC for Aircraft Pitch Control

Next each component of the FMRLC is discussed in detail for the pitch control system of the aircraft which is taken here as the plant .

2.1. Plant

Here plant is taken as the pitch control system of the Bravo fighter aircraft. The input to the plant is the elevator deflection (δ) and the output is the pitch angle (θ) .The longitudinal dynamics[8] of a aircraft can be epresented with following set of equations.

$$\dot{u} = X_u u + X_w w - g\cos\gamma_0\theta$$

$$\dot{w} = Z_u u + Z_w w + U_o q - g\sin\gamma_0\theta + Z_{\delta E}\ \delta_E \quad (1)$$

$$\dot{q} = M_u u + M_w w + M_{\dot{w}}\dot{w} + M_q q + M_{\delta E}\ \delta_E$$

$$\dot{\theta} = q$$

Substituting the values of stability derivatives(Z_W , $M_{\dot{w}}$, M_q , M_W , U_o , $Z_{\delta E}$, $M_{\delta E}$) of aircraft[8] for flight condition-3 and 4 given the following transfer functios are obtained as follows

Flight Condition-3

$$\frac{\delta(s)}{\theta(s)} = \frac{-0.4500(1+1.6094s)}{[1+(0.0319 + 0.1844i)s]\ [1+(0.0319 -0.1844i)s]} \tag{2}$$

Flight Condition-4

$$\frac{\delta(s)}{\theta(s)} = \frac{-0.1350(1+2.6045\ s)}{[1+(0.0170 + 0.1469)s][1+(0.0170 - 0.1469)s]} \tag{3}$$

Reference Model

A first order system as shown in equation 3 is taken here as reference model for output $\theta(kT)$ to track a smooth, stable first order response of $\theta m(kT)$ .The performance of the overall system is computed with respect to the reference model by generating the error signal using learning mechanism.

$$\frac{\theta_m(s)}{\theta_{ref}(s)} = \frac{K}{s+a} \tag{4}$$

Bilinear Transform is used to find the discrete equivalent of the continuous time transfer function substituting $s = \frac{2}{T}\left(\frac{z-1}{z+1}\right)$ in (4)

$$\frac{\theta_m(z)}{\theta_{ref}(z)} = \frac{kT/2}{\left(\frac{z-1}{z+1}\right) + \frac{aT}{2}} = \frac{(kT/2)(z+1)}{(z-1)+(z+1)\frac{aT}{2}} \tag{5}$$

$$[z(1+\frac{aT}{2}) - (1-\frac{aT}{2})]\theta_m(z) = \frac{kT}{2}(z+1)\theta_{ref}(z) \tag{6}$$

Taking the inverse *Z* transform and rearranging the equation 6

$$\theta_m(kT+T) = \frac{(1-\frac{aT}{2})}{(1+\frac{aT}{2})}\theta_m(KT) + \frac{(kT/2)}{(1+\frac{aT}{2})}[\theta_{ref}(kT+T)+\theta_{ref}(kT)] \tag{7}$$

Substituting the value of K = 0.1, a = 0.1 and sampling time T = 1 mille second in equation 7

$$\theta_m(kT+T) = 0.9999\ \theta_m(kT) + \ 4.9995(10^{-5})\ [\theta_{ref}(kT+T)+\theta_{ref}(kT)] \tag{8}$$

2.2. The Fuzzy Controller

The inputs to the PD(Propertional-Derivative) fuzzy Controller are generally generated from the plant output and reference input. Here inputs to the fuzzy PD controller are the pitch angle error, $e(kT)$ and change in pitch angle error, $e_c(kT)$ expressed as

$$e(kT) = \theta_{ref}(kT) - \theta(kT)\ , \qquad e_c(kT) = \frac{[\theta_{ref}(kT) - \theta(kT)]}{T}$$

respectively, where $\theta_{ref}(kT)$ is the desired pitch angle and $\theta(kT)$ is the actual pitch angle . The controller output is the elevator angle $\delta(kT)$ of the aircraft. The fuzzy controller used here has 11 uniformly spaced triangular membership functions (MF) for each of the controller input $e(kT)$ and $e_c(kT)$) as shown in Figure-2 and for the output $\delta(kT)$ as shown in Figure-3 below.

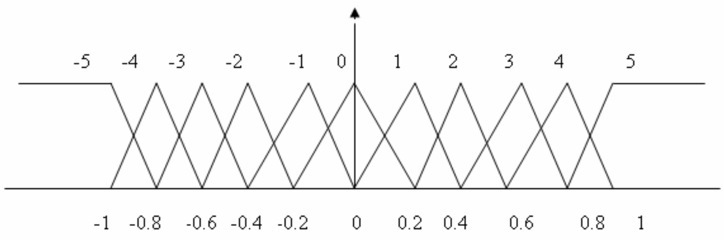


Fig 2. MF for Fuzzy Controller input,e(kT) and e_c(kT)

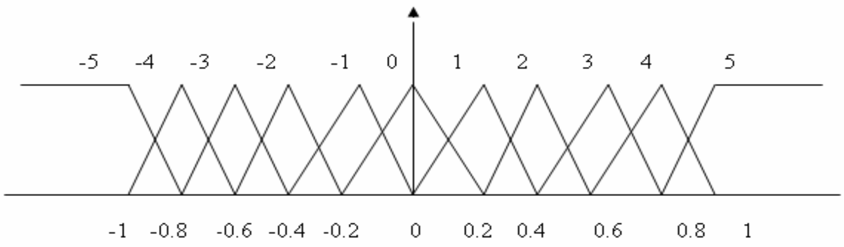


Fig 3. MF for Fuzzy Controller Out put $\delta(kT)$

Linguistic variables	Linguistic representation	Numerical representation	Center of the triangular MF
NVL	Negative Very Large	-5	-1
NL	Negative Large	-4	-0.8
NM	Negative Medium	-3	-0.6
NS	Negative Small	-2	-0.4
-Z	Negative Zero	-1	-0.2
Z	Zero	0	0
+Z	Positive Zero	1	0.2
PS	Positive Small	2	0.4
PM	Positive Medium	3	0.6
PL	Positive Large	4	0.8
PVL	Positive Very Large	5	1

Table-1 Linguistic and Numerical Representation of MF

The MF for the fuzzy controller output are assumed to be symmetric and triangular-shaped with a base width of 0.4.The input and output universes of discourse of the fuzzy controller are normalized on the range $[-1, 1]$.The gains g_e, g_c maps the actual inputs of the of fuzzy controller to the normalized universe of discourse [−1, 1] are called normalizing gains.. Similarly gain g_δ is used to map the out put of the fuzzy controller.

2.3 Rule Base

The rule base for the fuzzy controller is expressed as '*if* $\bar{e}$ *is* $\overset{-j}{E}$ *and* $\bar{e}_c$ *then* $\bar{\delta}$ *is* $\overset{-m}{U}$'. Where $\bar{e}$ and $\bar{e}_c$ denote the linguistic variables associated with controller inputs $e(kT)$ and $e_c(kT)$ respectively, $\bar{\delta}$ denotes the linguistic variables associated with the controller output δ, $\overset{-j}{E}$ and $\overset{-l}{C}$ denote the $j^{th}(l^{th})$ linguistic value associated with $\bar{e}(\bar{c})$ respectively and $\overset{-m}{U}$ denotes the consequent linguistic value associated with $\bar{u}$ .The rule base matrix for the fuzzy controller is shown in Table-2.

e \ ec	-5	-4	-3	-2	-1	0	1	2	3	4	5
-5	1	1	1	1	1	1	.8	.6	.4	.2	0
-4	1	1	1	1	1	.8	.6	.4	.2	0	-.2
-3	1	1	1	1	.8	.6	.4	.2	0	-.2	-.4
-2	1	1	1	.8	.6	.4	.2	0	-.2	-.4	-.6
-1	1	1	.8	.6	.4	.2	0	-.2	-.4	-.6	-.8
0	1	.8	.6	.4	.2	0	-.2	-.4	-.6	-.8	-1
1	.8	.6	.4	.2	0	-.1	-.3	-.6	-.8	-1	-1
2	.6	.4	.2	0	-.2	-.4	-.6	-.8	-1	-1	-1
3	.4	.2	0	-.2	-.4	-.6	-.8	-1	-1	-1	-1
4	.2	0	-.2	-.4	-.6	-.8	-1	-1	-1	-1	-1
5	0	-.2	-.4	-.6	-.8	-1	-1	-1	-1	-1	-1

Table-2 Rule Base Matrix for Fuzzy PD Controller

The scaling gains for the error (g_e), change in error (g_c) and the controller output (g_u) are chosen via the design procedure. The error $e(kT)$ is restricted not exceed more than 90 degree and the rate of pitch angle is not allowed to change more than 0.01 rad/sec. The elevator also will not have a deflection more than ±45 degree to control the pitch angle. So these designing constraints decides the following values

$ge = (2/\pi)$, $gc = 0.01$ and $g_u = 0.45(\pi)$

2.4. Fuzzy Inverse Model

The inputs to the fuzzy inverse model are the error $\theta_e(kT)$ and change in error $\theta_c(kT)$ shown in figure 1.

$$\theta_e(kT) = \theta_m(kT) - \theta(kT)$$

$$\theta_C(kT) = (\theta_e(kT) - \theta_e(kT - T))/T$$

The input MFs and rule base for fuzzy inverse model is same as fuzzy controller. With similar logic the values of scaling gains are decided to be $g_{\theta e} = 2/\pi$, $g_{\theta c}$ =500 and g_p=0.4 .

2.5. Knowledge-Base Modifier

Let b_m denotes the center of the MF associated with m^{th} MF. Knowledge-base modification is performed by shifting centers[9] b_m MFs of the output linguistic value U_m that are associated with the fuzzy controller rules. The shifting of centers is contributed to the previous control action $\delta(kT-T)$. Let $b_m(kT)$ denotes the center of the *mth* output MF at time instant *kT*. For all rules

$$b_m(kT) = b_m(kT-T) + p(kT)$$

3. Simulation of Nonlinear System

The aircraft model to be simulated is represented by ordinary differential equations.

$$\dot{x}(t) = f(x(t), r(t), t)$$
$$y = g(x(t), r(t), t) \qquad (9)$$

Where $x = [x_1, x_2, x_3, \ldots x_n]^T$ is a state vector and $f = [f_1, f_2, f_3, \ldots f_n]^T$ is another state vector of non linear function. *g* is also a non linear function that maps the state and reference input to the output of the system with *x(0)* being the initial state. *f, g* are both time varying function because explicitly they depend upon time. To simulate the non linear aircraft model in digital computer Runge-Kutta method[10] is discussed below. A numerical approximation to the above differential equation 9 may be expressed using the 4th order Runge-Kutta method with zero initial condition are as follows.

$$x(kh+h) = x(kh) + 1/6(k_1 + 2k_2 + 2k_3 + k_4) \qquad (10)$$

Where

$$k_1 = hf(x(kh), r(kh), kh)$$
$$k_2 = hf(x(kh) + \frac{k_1}{2}, r(kh + \frac{h}{2}), kh + \frac{h}{2})$$
$$k_3 = hf(x(kh) + \frac{k_2}{2}, r(kh + \frac{h}{2}), kh + \frac{h}{2})$$
$$k_4 = hf(x(kh) + k_3, r(kh+h), kh+h)$$

Runge-Kutta method is very easy to use as it only involves to calculate the values of k_1, k_2, k_3 and k_4 which are plugged into equation 10 to obtain next state of *x*.

4. Simulation of Pitch Control System

The general form of pitch control system of the aircraft is given below.

$$\frac{\theta(s)}{\delta_E(s)} = \frac{K(1+\tau_3 s)}{(1+\tau_1 s)(1+\tau_2 s)} \qquad (11)$$

Let $a = (\frac{1}{\tau_1} + \frac{1}{\tau_2}), b = \frac{1}{\tau_1\tau_2}, c = \frac{k\tau_3}{\tau_1\tau_2}, d = \frac{k}{\tau_1\tau_2}$. Then equation 11 can be represented as

$$\frac{\theta(s)}{\delta_E(s)} = \frac{(cs+d)}{(s^3 + as^2 + bs)} \qquad (12)$$

Inverse Laplace Transform of Equation 12 results

$$\dddot{\theta}(t) = -a\ddot{\theta}(t) - b\dot{\theta}(t) + c\dot{\delta}(t) + d\delta(t) \qquad (13)$$

$\dot{x}_i$ so chosen that $\dot{f}_i$ depends only on x_i and
δ for $i = 1,2,3$ so that

$$\dot{x}_3(t) = \dddot{\theta}(t) - c\dot{\delta}(t)$$
$$\dot{x}_2(t) = \ddot{\theta}(t) \qquad (14)$$

$$\dot{x}_1(t) = x_2(t)$$

$$\dot{x}_1(t) = x_2(t) = f_1(x(t), \delta(t)) \tag{15}$$

$$\dot{x}_2(t) = x_3(t) + c\delta(t) \tag{16}$$

$$\dot{x}_3(t) = -a\ddot{\theta}(t) - b\dot{\theta}(t) + d\delta(t)) \tag{17}$$

Now substituting the value of $\ddot{\theta}(t)$ from equation 14 in equation 17

$$\dot{x}_3(t) = -ax_3(t) - bx_2(t) - (ac - d)\delta(t) \tag{18}$$

The equations 15,16 and 18 with initial conditions $\theta(0) = \dot{\theta}(0) = \ddot{\theta}(0) = 0$ can be represented in matrix form as

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -b & a \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ c \\ d - ac \end{bmatrix} [\delta(t)] \tag{19}$$

Now the aircraft pitch control model expressed in equation 19 is ready for simulation. It is observed that FMRLC modifies the rule base by changing the centers from their initial value .At the end part of simulation the values of MF which are in the center of the rule base keeps on changing. It happens so because the error and change of error becomes close to zero

5. Simulation Results

Case -1

The reference signal here is a pulse of duration 35 seconds .From t=0 to 15 seconds the flight travels with flight condition-3 and after 15 seconds the flight travels with flight condition-4. The speed at flight condition-3 and flight condition-4 are 350 m/sec and 650 m/sec respectively.It is noted in figure-4.(a) at the initial stage of time the oscillation presents in θ because the FMRLC has no idea of adoption to control the pitch angle . But as the time proceeds the controller gets adopted with suitably changing the center of the MF mentioned as mentioned above. The out put of the fuzzy controller $\delta(kT)$ is shown in figure-4(b) indicates the controller out is significant when the pitch angle is not equal to the reference value and at the transition of reference signal. The output of the fuzzy inverse model θ_m is shown in figure-4(c).

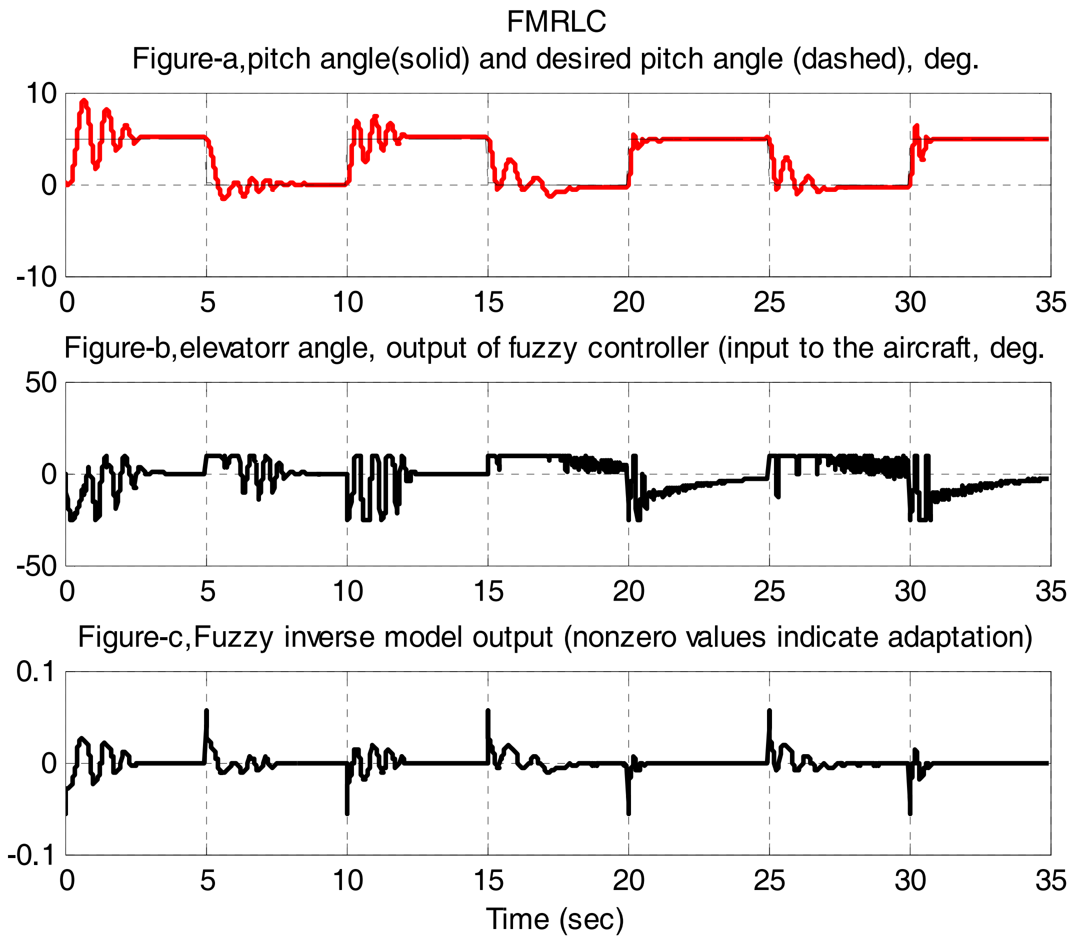
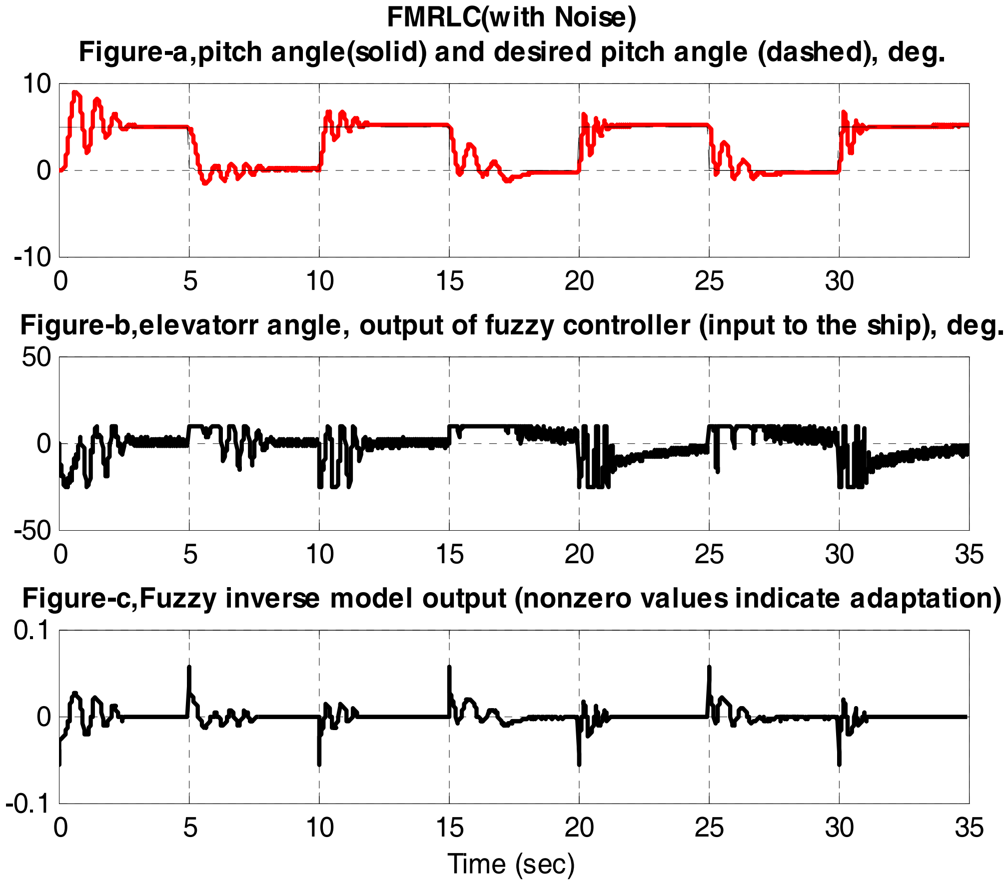


Figure 4. Result of Simulation Case-1

Case-2 With Sensor Noise

The reference signal is a pulse signal of duration 35 seconds and amplitude of 5 degree. .From t=0 to 15 seconds the flight travels with flight condition-3 and after t=15seconds with flight condition-4. It may so happens the sensor measuring the pitch angle may be added with noise. So a random noise is added uniformly with pitch angle θ with help of a random function i.e. $0.01\frac{\pi}{180}(2\,rand\text{-}1)$ .The effect of noise is shown in the following figure-15. The figure-5(a) explains the trajectory of the actual pitch angle $\theta(kT)$ and the reference mode output $\theta_m(kT)$. Figure-5(b) shows the fuzzy controller output $\delta(kT)$ which is the input to the plant is continuous because the adoption takes place continuously due to continuous presence of random noise in sensor. The continuous output $\delta(kT)$ tries the $\theta(kT)$ to follow the reference pitch angle $\theta_{ref}(kT)$ .Thus the controller is noise adoptive The error $e(kT)$ and change of error $e_c(kT)$ are shown in figure-5(d) and figure-5(e) respectively. The value of $\theta_e(kT)$ and $\theta_c(kT)$ are shown in figure-5(f) and figure-5(g)


FMRLC(with Noise)
Figure-a,pitch angle(solid) and desired pitch angle (dashed), deg.
Figure-b,elevatorr angle, output of fuzzy controller (input to the ship), deg.
Figure-c,Fuzzy inverse model output (nonzero values indicate adaptation)
Time (sec)

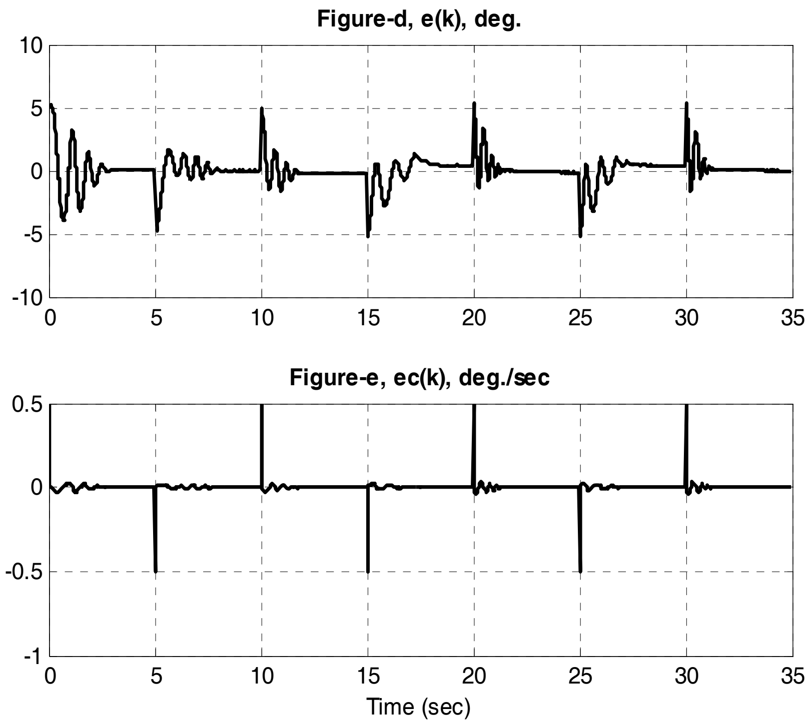


Figure-d, e(k), deg.
Figure-e, ec(k), deg./sec
Time (sec)

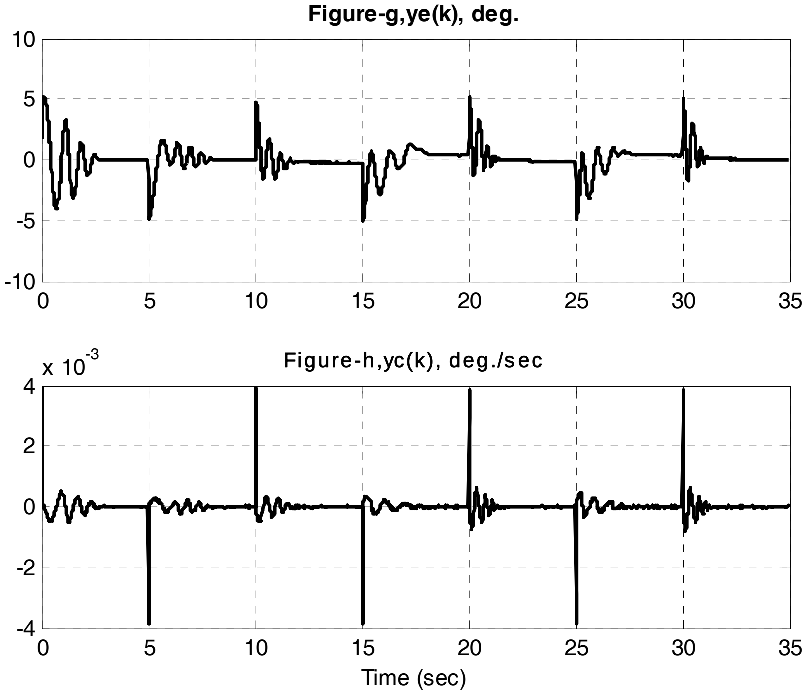


Figure -5: Result of Simulation Case-2

6. Control surface of FMRLC

The non linear control surface of FMRLC for case-2 is shown in the following figure-6.The non linearity in control surface depends upon so many factors like reference input , method of adoption of fuzzy controller and the dynamics of the system. If the parameters of the plant changes due to sensor noise or disturbances the control surface's nonlinearity also changes. Adjustment continues till end of the simulation. The control surface also establishes the error and change of error is reduced to zero which causes the peaks in control surface.

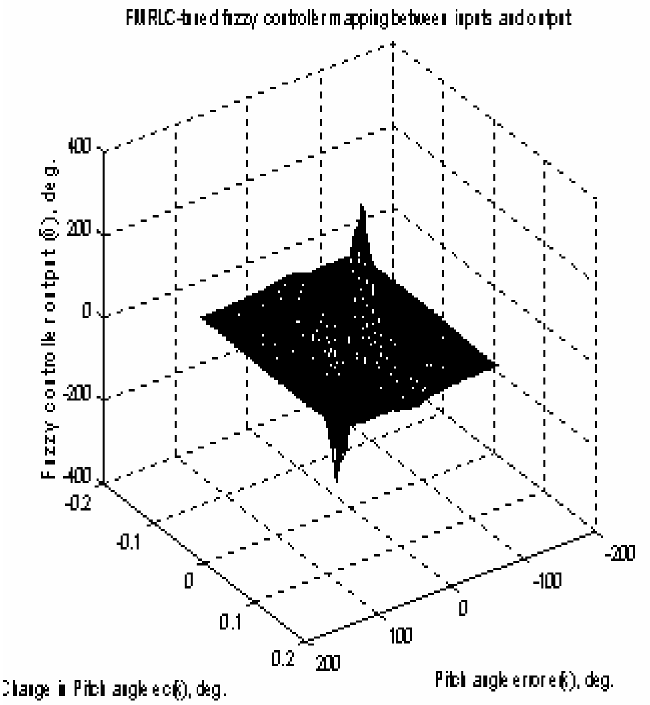


Figure-6: Control Surface of FMRLC

7. Conclusion

The principal objectives of this paper were to provide a design methodology for the FMRLC and design a FMRLC for a nonlinear time-varying pitch control problem of an aircraft. The non zero value of the fuzzy controller output exhibits it's adoptive nature when the actual pitch angle differs from it's reference value and at the time of transition of the reference signal. When the speed of the aircraft is changed the control signal to the pitch control system also changes to cope up with the change in speed. It is also shown in the simulation that the sensor noise does not affect the output of the FMRLC because the controller output continuously changes to nullify the effect of this noise. In conclusion the learning and adoptive mechanism of FMRLC dynamically tunes the rule base of the direct fuzzy controller to adopt the change of the system parameters. FMRLC designed in this case is found to be adoptive and robust with strong learning ability.

References

[1] K. Self, "Designing with Fuzzy Logic", *IEEE Spectrum*, pp. 42–105 and 105, November 1990.

[2] Kwong, W. A., Passino, K. M., Laukonen, E.G., Yurkovich, S., "Expert Supervision of Fuzzy Learning Systems with Applications to Reconfigurable Control for Aircraft", Proceedings of the 33rd IEEE Conference On Decision and Control, Vol. 4, Lake Buena Vista, FL, December 1994, pp. 4116-4121.

[3] Kwong, W. A., Passino, K. M., Laukonen, E. G., Yurkovich, S., "Expert Supervision of Fuzzy Learning Systems for Fault-Tolerant Aircraft Control", Proceedings of the IEEE, Vol. 83, No. 3, March 1995, pp. 466-483.

[4] Chun-Liang Lin And Huai-Wen Su, Intelligent Control Theory in Guidance and Control System Design: an Overview Proc. Natl. Sci, Counc. ROC(A),Vol. 24, No. 1, pp 15-30, 2000.

[5] Layne J.R., Passino K.M., "Fuzzy Model Reference Learning Control," Journal of Intelligent and Fuzzy Systems, Vol. 4, No.1, pp.33–47, 1996.

[6] Yuman Yuan, Yuguang Feng, Wenjin Gu, "Fuzzy Model Reference Learning Control for Aircraft Pitch Autopilot Design", 8th International Conference on Control, Automation. Robotics and Vision, China, 6-9th December 2004.

[7] D.S. Reay, M.W.Dunnigan, "Learning Issues in Model Reference Based Fuzzy Control", IEE Proc -Control Theory Appl., Vol 144, No 6, November 1991.

[8] Automatic Flight Control Systems, Mclean Donald, Prentice Hall International (UK)Ltd, 1990.

[9] Biomimicry for Optimization, Control and Optimization, Kelvino M Passino, Springer-verlag, London Limited-2005.

[10] Koga M, Tanimura N, Sato T, "Numerical Simulation using Runge-Kutta Method for the System with Uncertain Discontinuous Change", Proceedings of the 41st SICE Annual ConferenceVol.5, pp.2749 – 2753, Aug 2002.

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