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PERFORMANCE EVALUATION OF A GENERAL CLASS OF MULTI-LEVEL QUEUE SCHEDULING SCHEME

[1]Diwakar Shukla, [2]Shweta Ojha, [3]Saurabh Jain

[1]Department of Mathematics and Statistics, Sagar University, Sagar M.P. 470003, India.
e-mail: diwakarshukla@rediffmail.com
[2]Department of Computer Science & Applications, Sagar University, Sagar M.P. 470003, India.
e-mail: meshwetaojha@gmail.com
[3]Department of Computer Science & Applications, Sagar University, Sagar M.P. 470003, India.
e-mail: iamsaurabh_4@yahoo.co.in

Abstract:

There are many CPU scheduling algorithms in the literature like FIFO, Round Robin, Shortest Job First and so on. The Multilevel Queue Scheduling is superior to these due to its better management of a variety of processes. In this paper, a general class of multi-level queue scheduling schemes is designed and studied under a Markov Chain model. The scheduler is assumed to perform random movement on queues over predefined quantum of time. Three different scheduling schemes, as members of the class, are examined and compared under this model with the special consideration of a rest (waiting) state. It is found that the scheme III of the class is more appropriate than scheme I and scheme II in reference to the minimum rest state probability criteria. All the conclusions are well supported through a simulation study based on three different data sets. The Markov Chain model advocates for noble properties in scheduling scheme III of the class.

***Keywords**: Process scheduling, Markov chain model, State of system, Rest State, Process queue, Multi-level queue scheduling, Transition probability matrix, Central Processing Unit (CPU).*

SUBJECT CLASSIFICATION

D.4	(Operating System)
SDD.4.1	(Scheduling)
SDD.4.8	(Modeling and Predictions, Performance, Operating System)
***I.6.3	(Modeling and Simulation applications)
**C.4	(Performance of System)

1.0 INTRODUCTION

The operating system plays a major role in managing processes reaching to CPU, specially in the form of multiple queues. The arrival of a process is random along with its different categories and types in terms of size, memory requirement, time etc. All these require scheduling algorithms to work over real time environment with special reference to task, control and efficiency (see Stankovic (1984), Liu and Layland (1973), Garey and Johnson (1977) etc.). The randomization involved in scheduling procedure leads to perform a probabilistic study over the movement phenomenon. The jump of scheduler over multiple queues of processes is a line of thought and a source of motivation to think over for a stochastic study of system.

Demer et al. (1989) have presented an analysis of Fair Queuing algorithm whereas Cobb et al. (1998) picked up fair scheduling of flaros with the consideration of time shifting approach in the area of high-speed networks. Goyal et al. (1996) derieved the Hierarchical CPU scheduler in the environment where the multimedia operating system is used. In the similar line, Hieh and Lam (2003) discussed smart schedulers for multimedia users. A time driven scheduling model is proposed by Janson et al. (1985) attracted attention of researchers for the model formation over functioning and procedure on operating systems. Katcher (1993) proposed an analysis of fixed priority schedulers and Horn (1974) generated some new scheduling algorithms useful for managing queues in operating system. David (1994) presented a successful contribution over the study of real time and conventional scheduling with a comparative analysis.

Barthomew (1973), Medhi (1991) and Parzern (1962) given an elaborate study of a variety of stochastic processes and their applications in various fields. Medhi (1976) developed a Markov chain model for the study of uncertain rainfall phenominon. Naldi (2002) presented a Markov chain model for understanding the internet traffic sharing among various operators in a competitive market. Shukla et al. (2007) used a Markov chain model for the study of transition probabilities in space division switches in computer networks. This paper takes into account a class of multilevel queue scheduling schemes with the assumption of random jumps of scheduler and a rest state along with keen interest on comparative study of different schemes.

1.1 A GENERAL CLASS OF MULTI-LEVEL QUEUE SCHEDULING

Consider a scheduling with three queues Q_1, Q_2, Q_3, each having large number of processes $P_j, P_j^{'}, P_j^{''}\left(j = 1,2,3...\right)$ respectively, waiting for processing. Let these queues Q_i $\left(i = 1,2,3\right)$ are states of a scheduling system and there is a specific fourth state $Q_4 = W$ treated as a rest state. A quantum is a small pre-defined slot of time given for processing to the waiting processes in queues. Symbol n denotes the n^{th} quantum allotted by scheduler to a process for execution $\left(n = 1,2,3,4....\right)$.

A proposed structure of the general class of multi-level scheduling is laid down below considering fig 1.1 and fig 1.2.

(1) A new process enters through any of the three queues Q_i only, with initial probabilities $pr_1, pr_2, pr_3 \left(\sum_{i=1}^{3} pr_i = 1\right)$.

(2) The movement of the scheduler is random over the three different states Q_1, Q_2, Q_3 and to the rest state $Q_4 = W$. Scheduler continues within the s^{th} state $Q_s\left(s = 1,2,3,4\right)$ for a large number of quantum until Q_i is empty.

(3) In beginning, the scheduler picks up any Q_i with a pre-defined priority, then picks up first process of that queue and allot a quantum of time for processing.

(4) The process remains with the processor until the quantum is over. If it completes within that, it gets out of Q_i.

(5) While a process is incomplete, within a quantum, scheduler assigns next quantum to the next process of the same queue and so on. The earlier incomplete process moves to the next queue $Q_{i+1}\left[\left(i+1\right) \le 3\right]$ and waits there for the next quantum to allot for its processing.

(6) The quantum allotment procedure, within Q_i, by scheduler, continues until Q_i is empty. When all Q_1, Q_2, Q_3 are empty, the scheduler jumps to the rest state $Q_4 = W$. Therefore, W be an idle state but also accepts random transition of scheduler over itself from any empty Q_i.

ISSN 1512-1232

(7) The scheduler attempts processing in queue Q_3 on "first come first serve" basis. Any incomplete process or new process, if appears in Q_3, remains with Q_3 only until processed completely.

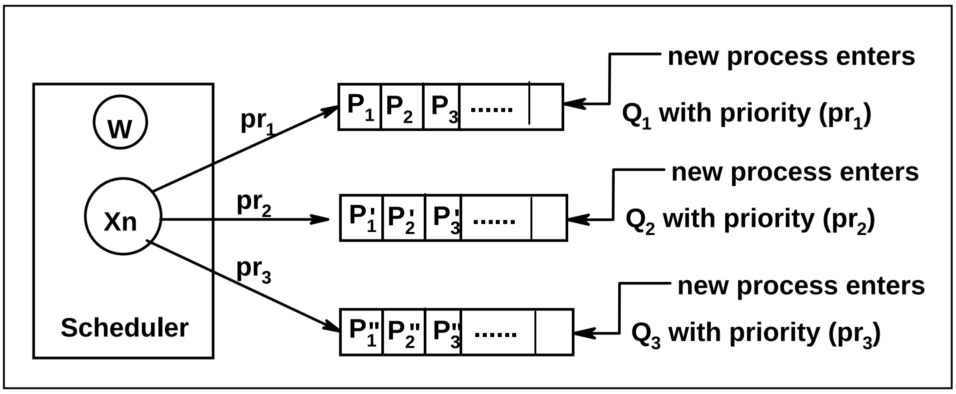

Fig. 1.1 **(General Multi-level Queue System Diagram)**

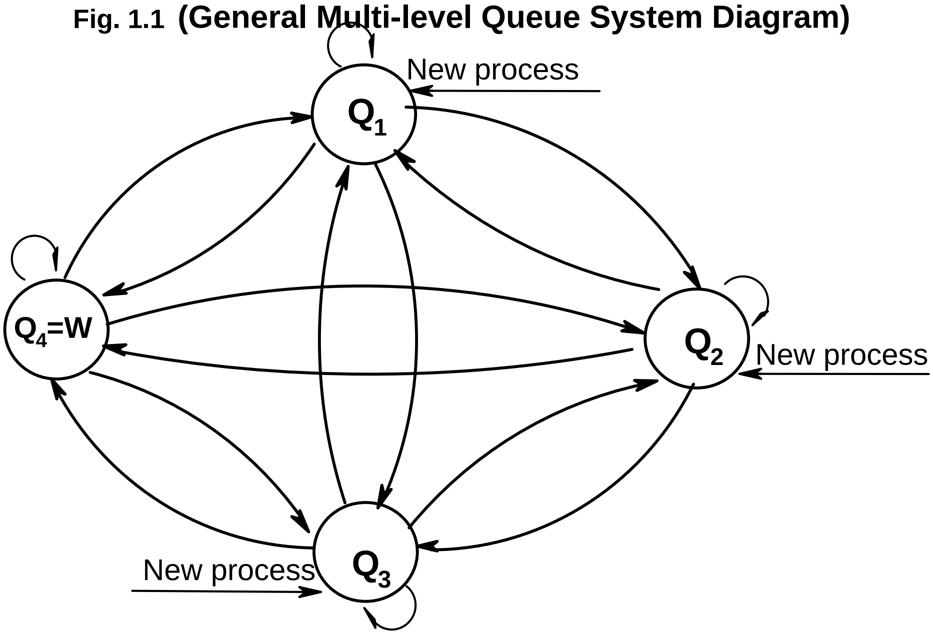

Fig 1.2 (System Diagram)

2.0 MARKOV CHAIN MODEL

Let $\{X^{(n)}, n \geq 1\}$ be a Markov chain where $X^{(n)}$ denotes the state of scheduler at the n^{th} quantum of time. The state space for random variable X is $\{Q_1, Q_2, Q_3, Q_4\}$ and scheduler $X^{(n)}$ jumps over these states in different predefined quantum of time. The initial selection probabilities of states are:

$$\left.\begin{aligned} P\left[X^{(0)} = Q_1\right] &= pr_1 \\ P\left[X^{(0)} = Q_2\right] &= pr_2 \\ P\left[X^{(0)} = Q_3\right] &= pr_3 \\ P\left[X^{(0)} = Q_4\right] &= 0 \end{aligned}\right] \tag{2.1}$$

with $pr_1 + pr_2 + pr_3 = \sum_{i=1}^{3} pr_i = 1$

ISSN 1512-1232

Suppose $s_{ij}(i, j = 1,2,3,4)$ be transition probabilities of $X^{(n)}$ over states. The unit-step transition probability matrix is:

$$
\begin{array}{c|cccc}
 & \multicolumn{4}{c}{\longleftarrow X^{(n)} \longrightarrow} \\
 & Q_1 & Q_2 & Q_3 & Q_4 \\
\hline
Q_1 & s_{11} & s_{12} & s_{13} & s_{14} \\
X^{(n-1)}\; Q_2 & s_{21} & s_{22} & s_{23} & s_{24} \\
Q_3 & s_{31} & s_{32} & s_{33} & s_{34} \\
Q_4 & s_{41} & s_{42} & s_{43} & s_{44} \\
\hline
\end{array}
\tag{2.2}
$$

subject to condition $s_{14} = \left(1 - \sum_{i=1}^{3} s_{1i}\right)$, $s_{24} = \left(1 - \sum_{i=1}^{3} s_{2i}\right)$, $s_{34} = \left(1 - \sum_{i=1}^{3} s_{3i}\right)$,

$s_{44} = \left(1 - \sum_{i=1}^{3} s_{4i}\right)$ and $0 \le s_{ij} \le 1$.

The state probabilities, after the first quantum, can be obtained by a simple relationship:

$$
\begin{aligned}
P\left[X^{(1)} = Q_1\right] = {} & p\left[X^{(0)} = Q_1\right] p\left[X^{(1)} = Q_1 / X^{(0)} = Q_1\right] + \\
& p\left[X^{(0)} = Q_2\right] p\left[X^{(1)} = Q_1 / X^{(0)} = Q_2\right] + \\
& p\left[X^{(0)} = Q_3\right] p\left[X^{(1)} = Q_1 / X^{(0)} = Q_3\right]
\end{aligned}
$$

$$
\left.
\begin{aligned}
&= \sum_{i=1}^{3} pr_i s_{i1} \\
P\left[X^{(1)} = Q_2\right] &= \sum_{i=1}^{3} pr_i s_{i2} \\
P\left[X^{(1)} = Q_3\right] &= \sum_{i=1}^{3} pr_i s_{i3} \\
P\left[X^{(1)} = Q_4\right] &= \sum_{i=1}^{3} pr_i s_{i4}
\end{aligned}
\right] \quad .(2.3)
$$

Similarly, state probabilities after second quantum can be obtained by simple relationship:

$$
\begin{aligned}
P\left[X^{(2)} = Q_1\right] = {} & p\left[X^{(1)} = Q_1\right] p\left[X^{(2)} = Q_1 / X^{(1)} = Q_1\right] + \\
& p\left[X^{(1)} = Q_2\right] p\left[X^{(2)} = Q_1 / X^{(1)} = Q_2\right] + \\
& p\left[X^{(1)} = Q_3\right] p\left[X^{(2)} = Q_1 / X^{(1)} = Q_3\right] + \\
& p\left[X^{(1)} = Q_4\right] p\left[X^{(2)} = Q_1 / X^{(1)} = Q_4\right]
\end{aligned}
$$

ISSN 1512-1232

$$
\left.
\begin{aligned}
&= \sum_{i=1}^{4} \Big(\sum_{j=1}^{3} pr_j s_{ji}\Big) s_{i1} \\
P\left[X^{(2)} = Q_2\right] &= \sum_{i=1}^{4} \Big(\sum_{j=1}^{3} pr_j s_{ji}\Big) s_{i2} \\
P\left[X^{(2)} = Q_3\right] &= \sum_{i=1}^{4} \Big(\sum_{j=1}^{3} pr_j s_{ji}\Big) s_{i3} \\
P\left[X^{(2)} = Q_4\right] &= \sum_{i=1}^{4} \Big(\sum_{j=1}^{3} pr_j s_{ji}\Big) s_{i4}
\end{aligned}
\right] \qquad (2.4)
$$

Remark 2.1 The generalized expressions for n quantum are:

$$
\left.
\begin{aligned}
P\left[X^{(n)} = Q_1\right] &= \sum_{m=1}^{4} \ldots\ldots \sum_{l=1}^{4} \sum_{k=1}^{4} \sum_{i=1}^{4} \sum_{j=1}^{3} p_{rj} s_{ji} s_{ik} s_{kl} \ldots\ldots s_{m1} \\
P\left[X^{(n)} = Q_2\right] &= \sum_{m=1}^{4} \ldots\ldots \sum_{l=1}^{4} \sum_{k=1}^{4} \sum_{i=1}^{4} \sum_{j=1}^{3} p_{rj} s_{ji} s_{ik} s_{kl} \ldots\ldots s_{m2} \\
P\left[X^{(n)} = Q_3\right] &= \sum_{m=1}^{4} \ldots\ldots \sum_{l=1}^{4} \sum_{k=1}^{4} \sum_{i=1}^{4} \sum_{j=1}^{3} p_{rj} s_{ji} s_{ik} s_{kl} \ldots\ldots s_{m3} \\
P\left[X^{(n)} = Q_4\right] &= \sum_{m=1}^{4} \ldots\ldots \sum_{l=1}^{4} \sum_{k=1}^{4} \sum_{i=1}^{4} \sum_{j=1}^{3} p_{rj} s_{ji} s_{ik} s_{kl} \ldots\ldots s_{m4}
\end{aligned}
\right] \qquad .(2.5)
$$

3.0 SOME SPECIAL MULTI-LEVEL SCHEDULING SCHEMES

By imposing restrictions and conditions over the ways and procedures, one can generate various scheduling schemes from the generalized class in section 1.1. Three schemes are discussed in sub-section 3.1 to 3.3.

3.1 SCHEME-I WHEN PROCESS ENTERS TO FIRST QUEUE ONLY

Under process entry restriction, the scheme-I is described in fig 3.1

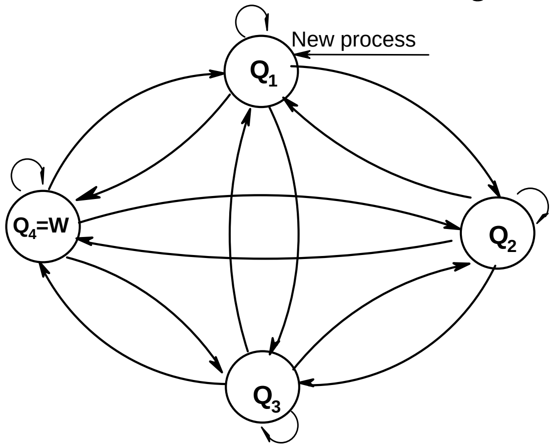

fig 3.1(Transition Diagram of Scheme-I)

ISSN 1512-1232

Unit step transition probability matrix for $X^{(n)}$ under scheme-I is

$P[X^{(0)} = Q_1] = 1$;
$P[X^{(0)} = Q_2] = 0$;
$P[X^{(0)} = Q_3] = 0$;
$P[X^{(0)} = Q_4] = 0$;

$X^{(n-1)}$ \ $X^{(n)}$	Q_1	Q_2	Q_3	Q_4
Q_1	s_{11}	s_{12}	s_{13}	s_{14}
Q_2	s_{21}	s_{22}	s_{23}	s_{24}
Q_3	s_{31}	s_{32}	s_{33}	s_{34}
Q_4	s_{41}	s_{42}	s_{43}	s_{44}

Remark 3.1.1 Using (2.3), the state probabilities of scheme-I, after the first quantum are:

$$P[X^{(1)} = Q_1] = s_{11}$$
$$P[X^{(1)} = Q_2] = s_{12}$$
$$P[X^{(1)} = Q_3] = s_{13}$$
$$P[X^{(1)} = Q_4] = s_{14}$$

Remark 3.1.2 Using (2.4), the state probabilities after the second quantum are:

$$P[X^{(2)} = Q_1] = \sum_{j=1}^{4} s_{1j} s_{j1}$$
$$P[X^{(2)} = Q_2] = \sum_{j=1}^{4} s_{1j} s_{j2}$$
$$P[X^{(2)} = Q_3] = \sum_{j=1}^{4} s_{1j} s_{j3}$$
$$P[X^{(2)} = Q_4] = \sum_{j=1}^{4} s_{1j} s_{j4}$$

Remark 3.1.3 The generalized expressions of scheme-I for state probabilities are:

$$P[X^{(n)} = Q_1] = \sum_{m=1} \cdots \sum_{l=1}^{4} \sum_{i=1}^{4} \sum_{j=1}^{4} s_{1j} s_{ji} s_{il} \cdots s_{m1}$$
$$P[X^{(n)} = Q_2] = \sum_{m=1} \cdots \sum_{l=1}^{4} \sum_{i=1}^{4} \sum_{j=1}^{4} s_{1j} s_{ji} s_{il} \cdots s_{m2}$$
$$P[X^{(n)} = Q_3] = \sum_{m=1} \cdots \sum_{l=1}^{4} \sum_{i=1}^{4} \sum_{j=1}^{4} s_{1j} s_{ji} s_{il} \cdots s_{m3}$$
$$P[X^{(n)} = Q_4] = \sum_{m=1} \cdots \sum_{l=1}^{4} \sum_{i=1}^{4} \sum_{j=1}^{4} s_{1j} s_{ji} s_{il} \cdots s_{m4}$$

3.2 SCHEME-II WHEN SOME TRANSITIONS ARE RESTRICTED

In the class of section 1.1, following are restricted and shown in fig 3.2.1:

(a) a new process enters to Q_1 only;

(b) scheduler cannot jump to Q_3 from Q_1 without passing Q_2;

(c) scheduler comes to Q_3 only if Q_1 and Q_2 are empty; it restricts the transition from Q_3 to Q_2; however, the transition from Q_3 to Q_1 is allowed only if a new process enters to Q_1;

(d) resting of scheduler on state W ends up only if a new process enters in Q_1, otherwise resting continues.

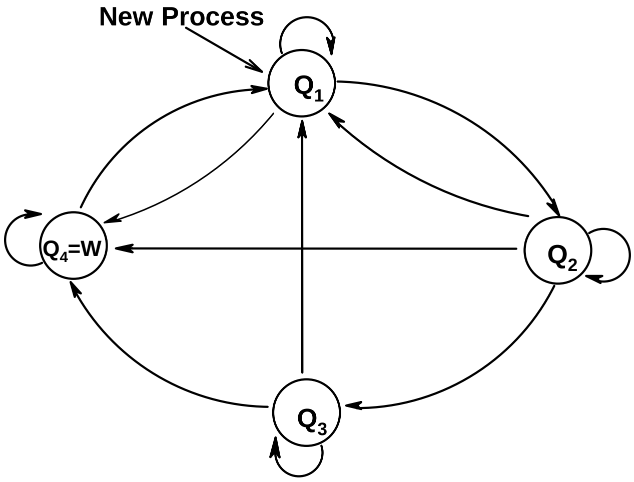

Fig. 3.2.1 (Transition Diagram of Scheme-II)

Remark 3.2.1 The scheme-II is same as the multi-level feedback scheduling discussed in literature [See Stallings (2005), Silberschatz and Galvin (1999), Tannenbaum (2000)].

Remark 3.2.2 The initial probabilities and transition probability matrix under scheme-II are:

$P\left[X^{(0)} = Q_1\right] = 1$;
$P\left[X^{(0)} = Q_2\right] = 0$;
$P\left[X^{(0)} = Q_3\right] = 0$;
$P\left[X^{(0)} = Q_4\right] = 0$;

$$X^{(n-1)} \quad \overset{\longleftarrow X^{(n)} \longrightarrow}{\begin{array}{c|cccc} & Q_1 & Q_2 & Q_3 & Q_4 \\ \hline Q_1 & s_{11} & s_{12} & 0 & s_{14} \\ Q_2 & s_{21} & s_{22} & s_{23} & s_{24} \\ Q_3 & s_{31} & 0 & s_{33} & s_{34} \\ Q_4 & s_{41} & 0 & 0 & s_{44} \end{array}}$$

Remark 3.2.3 Using (2.3), state probabilities after the first quantum for scheme-II are:

$$P\left[X^{(1)} = Q_1\right] = s_{11}$$
$$P\left[X^{(1)} = Q_2\right] = s_{12}$$
$$P\left[X^{(1)} = Q_3\right] = 0$$
$$P\left[X^{(1)} = Q_4\right] = s_{14}$$

Define indicator function $l_{(ij)}, i, j = 1,2,3,4.$ such that

ISSN 1512-1232

$l_{(ij)} = 0$ when $(i=1, j=3), (i=3, j=2), (i=4, j=2), (i=4, j=3)$

$l_{(ij)} = 1$ otherwise.

Then, using (2.4) state probabilities of scheme-II after second quantum:

$$P\left[X^{(2)} = Q_1\right] = \sum_{j=1}^{4} (l_{1j} s_{1j})(l_{j1} s_{j1})$$

$$P\left[X^{(2)} = Q_2\right] = \sum_{j=1}^{4} (l_{1j} s_{1j})(l_{j2} s_{j2})$$

$$P\left[X^{(2)} = Q_3\right] = \sum_{j=1}^{4} (l_{1j} s_{1j})(l_{j3} s_{j3})$$

$$P\left[X^{(2)} = Q_4\right] = \sum_{j=1}^{4} (l_{1j} s_{14})(l_{j1} s_{j4})$$

Remark 3.2.4 For n quantum, the generalized expressions are:

$$P\left[X^{(n)} = Q_1\right] = \sum_{m=1} \ldots\ldots \sum_{k=1}^{4} \sum_{i=1}^{4} \sum_{j=1}^{4} \left(l_{1j} s_{1j}\right)\left(l_{ji} s_{ji}\right)\left(l_{ik} s_{ik}\right) \ldots\ldots \left(l_{m1} s_{m1}\right)$$

$$P\left[X^{(n)} = Q_2\right] = \sum_{m=1} \ldots\ldots \sum_{k=1}^{4} \sum_{i=1}^{4} \sum_{j=1}^{4} \left(l_{1j} s_{1j}\right)\left(l_{ji} s_{ji}\right)\left(l_{ik} s_{ik}\right) \ldots\ldots \left(l_{m2} s_{m2}\right)$$

$$P\left[X^{(n)} = Q_3\right] = \sum_{m=1} \ldots\ldots \sum_{k=1}^{4} \sum_{i=1}^{4} \sum_{j=1}^{4} \left(l_{1j} s_{1j}\right)\left(l_{ji} s_{ji}\right)\left(l_{ik} s_{ik}\right) \ldots\ldots \left(l_{m3} s_{m3}\right)$$

$$P\left[X^{(n)} = Q_4\right] = \sum_{m=1} \ldots\ldots \sum_{k=1}^{4} \sum_{i=1}^{4} \sum_{j=1}^{4} \left(l_{1j} s_{1j}\right)\left(l_{ji} s_{ji}\right)\left(l_{ik} s_{ik}\right) \ldots\ldots \left(l_{m4} s_{m4}\right)$$

3.3 SCHEME-III WHEN SOME TRANSITIONS ARE RESTRICTED WITH SECURITY MEASURES

Transition from Q_1 to W is possible in scheme II when Q_1 empty, but in scheme-III, a set of new imposed conditions are:

(a) transition from Q_1 to W is restricted;

(b) transitions must occur in sequence, that is, from Q_1 to Q_2, Q_2 to Q_3, and then Q_3 to W as shown in fig 3.3.1.

This provides a security measure for the scheduler because it cannot be on resting state unless all the queues are empty.

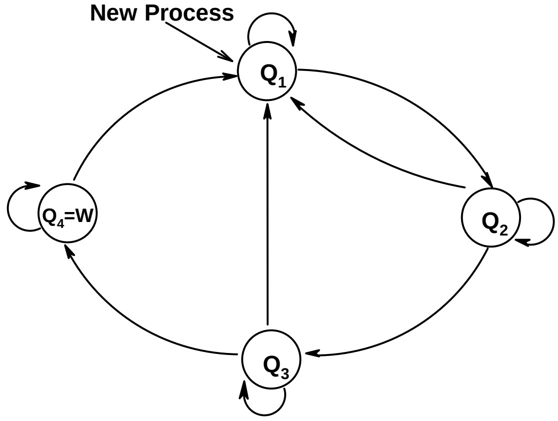

Fig. 3.3.1 (Transition Diagram of Scheme-III)

ISSN 1512-1232

For scheme-III, initial probabilities and the transition probability matrix are:

$$P\left[X^{(0)} = Q_1\right] = 1$$
$$P\left[X^{(0)} = Q_2\right] = 0$$
$$P\left[X^{(0)} = Q_3\right] = 0$$
$$P\left[X^{(0)} = Q_4\right] = 0$$

$X^{(n)}$ (columns) / $X^{(n-1)}$ (rows)

	Q_1	Q_2	Q_3	Q_4
Q_1	s_{11}	s_{12}	0	0
Q_2	s_{21}	s_{22}	s_{23}	0
Q_3	s_{31}	0	s_{33}	s_{34}
Q_4	s_{41}	0	0	s_{44}

Remark 3.3.1 Using (2.3), (2.4) and (2.5) the state probabilities after the first, second and third quantum are:

$$P\left[X^{(1)} = Q_1\right] = s_{11}\ ; \qquad P\left[X^{(2)} = Q_1\right] = s_{11}s_{11} + s_{12}s_{21}$$
$$P\left[X^{(1)} = Q_2\right] = s_{12}\ ; \qquad P\left[X^{(2)} = Q_2\right] = s_{11}s_{12} + s_{12}s_{22}$$
$$P\left[X^{(1)} = Q_3\right] = 0\ ; \qquad P\left[X^{(2)} = Q_3\right] = s_{12}s_{23}$$
$$P\left[X^{(1)} = Q_4\right] = 0\ ; \qquad P\left[X^{(2)} = Q_4\right] = 0$$

$$P\left[X^{(3)} = Q_1\right] = \left(s_{11}s_{11} + s_{12}s_{21}\right)s_{11} + \left(s_{11}s_{12} + s_{12}s_{22}\right)s_{21} + \left(s_{12}s_{23}\right)s_{31}$$
$$P\left[X^{(3)} = Q_2\right] = \left(s_{11}s_{11} + s_{12}s_{21}\right)s_{12} + \left(s_{11}s_{12} + s_{12}s_{22}\right)s_{22}$$
$$P\left[X^{(3)} = Q_3\right] = \left(s_{11}s_{12} + s_{12}s_{22}\right)s_{23} + \left(s_{12}s_{23}\right)s_{33}$$
$$P\left[X^{(3)} = Q_4\right] = \left(s_{12}s_{23}\right)s_{34}$$

Remark 3.3.2 Using similar pattern, for n quantum the generalized expression is:

$$P\left[X^{(n)} = Q_1\right] = \sum_{i=1}^{n} P\left[X^{(n-1)} = Q_i\right]s_{i1}$$
$$P\left[X^{(n)} = Q_2\right] = \sum_{i=1}^{n} P\left[X^{(n-1)} = Q_i\right]s_{i2}$$
$$P\left[X^{(n)} = Q_3\right] = \sum_{i=1}^{n} P\left[X^{(n-1)} = Q_i\right]s_{i3}$$
$$P\left[X^{(n)} = Q_4\right] = \sum_{i=1}^{n} P\left[X^{(n-1)} = Q_i\right]s_{i4}$$

4.0 CASE OF EQUAL VALUE TRANSITION PROBABILITIES

Consider below in [A] equal transition probability matrix for a constant number 'a', $0 \le a \le 1$ and $3a < 1$. The n^{th} step transition for scheme-I is expressed in [B]

ISSN 1512-1232

[A] [B]

$\longleftarrow X^{(n)} \longrightarrow$

$X^{(n-1)}$ (rows)

	Q_1	Q_2	Q_3	Q_4
Q_1	a	a	a	$1-3a$
Q_2	a	a	a	$1-3a$
Q_3	a	a	a	$1-3a$
Q_4	a	a	a	$1-3a$

$$P\left[X^{(n)} = Q_1\right] = a$$
$$P\left[X^{(n)} = Q_2\right] = a$$
$$P\left[X^{(n)} = Q_3\right] = a$$
$$P\left[X^{(n)} = Q_4\right] = 1-3a$$

While scheme-II, which is multi-level feedback queue scheduling, is taken into account, the equal transition matrix is:

$\longleftarrow X^{(n)} \longrightarrow$

$X^{(n-1)}$ (rows)

	Q_1	Q_2	Q_3	Q_4
Q_1	a	a	0	$1-2a$
Q_2	a	a	a	$1-3a$
Q_3	a	0	a	$1-2a$
Q_4	a	0	0	$1-3a$

Table 4.1 (Seven Quantum Transition Probabilities Under scheme-II)

	STATES			
Quantum number	$P\left[X^{(n)} = Q_1\right]$	$P\left[X^{(n)} = Q_2\right]$	$P\left[X^{(n)} = Q_3\right]$	$P\left[X^{(n)} = Q_4\right]$
$n=1$	a	a	0	$1-2a$
$n=2$	a	$2a^2$	a^2	$1-a-3a^2$
$n=3$	a	a^2+2a^3	$3a^3$	$1-a-a^2-5a^3$
$n=4$	a	$a^2+a^3+2a^4$	a^3+5a^4	$1-a-a^2-2a^3$ $-7a^4$
$n=5$	a	$a^2+a^3+a^4$ $+2a^5$	$a^3+2a^4+7a^5$	$1-a-a^2-2a^3$ $-3a^4-9a^5$
$n=6$	a	$a^2+a^3+a^4$ $+a^5+2a^6$	$a^3+2a^4+3a^5$ $+9a^6$	$1-a-a^2-2a^3$ $-3a^4-4a^5-11a^6$

ISSN 1512-1232

$n=7$	a	$a^2+a^3+a^4$ $+a^5+a^6+2a^7$	$a^3+2a^4+3a^5$ $+4a^6+11a^7$	$1-a-a^2-2a^3$ $-3a^4-4a^5-5a^6-13$

Under scheme-III, the transition matrix is:

$$
\begin{array}{c|cccc}
X^{(n-1)} \backslash X^{(n)} & Q_1 & Q_2 & Q_3 & Q_4 \\
\hline
Q_1 & a & 1-a & 0 & 0 \\
Q_2 & a & a & 1-2a & 0 \\
Q_3 & a & 0 & a & 1-2a \\
Q_4 & a & 0 & 0 & 1-a
\end{array}
$$

Table 4.2 (Seven Quantum Transition Probabilities Under Scheme-III)

	STATES			
Quantum number	$P[X^{(n)} = Q$	$P[X^{(n)} = Q$	$P[X^{(n)} = Q$	$P[X^{(n)} = Q_4]$
$n=$	a	a	**0**	**0**
$n=$	a	$2a-2a^2$	$1-3a-2a^2$	**0**
$n=$	a	$a+a^2-2a$	$3a-9a^2+$	$1-5a+8a^2-4$
$n=$	a	$a+a^3-2a$	$a+2a^2-1$ $+10a^4$	$1-3a-2a^2$ $+12a^3-8a^4$
$n=$	a	$a+a^4-2a$	$a-a^2+3a$ $17a^4+14a$	$1-3a+a^2-3a$ $+16a^4-12a^5$
$n=$	a	$a-24a^4+$ $25a^5-2a^6$	$a-a^2-a^3$ $4a^4+21a^5$	$1-3a+a^2+a^3$ $-4a^4-14a^5+$
$n=$	$a-24a^5+$	$a-24a^5+$ $25a^6-2a^7$	$a-a^2-a^3$ $-25a^4+77$ $-31a^6+22$	$1-3a+a^2$ $+a^3+a^4+$ $3a^5+6a^6+52$

5.0 NUMERICAL ANALYSIS

The basic and scientific plan for data analysis related to state transition probabilities for three scheduling schemes I, II, III of the proposed class are in fig 5.1.

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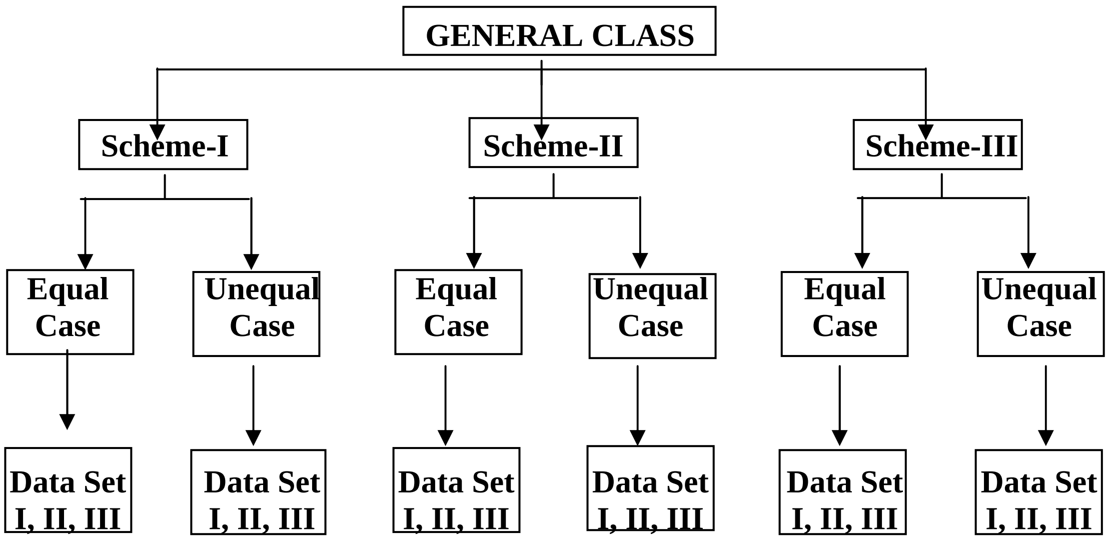

Fig: 5.1

5.1 USING DATA SET I

Under scheme-I: Consider data set of "equal and unequal transition elements" matrices as per (2.2) and (4.1) with initial probabilities:

$pr_1 = 0.5,\ pr_2 = 0.3,\ pr_3 = 0.2$

Unequal **Equal**

$$X^{(n-1)} \quad \begin{array}{c|cccc} & Q_1 & Q_2 & Q_3 & Q_4 \\ \hline Q_1 & 0.2 & 0.4 & 0.05 & 0.25 \\ Q_2 & 0.25 & 0.15 & 0.4 & 0.2 \\ Q_3 & 0.35 & 0.45 & 0.05 & 0.15 \\ Q_4 & 0.05 & 0.05 & 0.4 & 0.5 \\ \hline \end{array} \qquad \overleftrightarrow{X^{(n)}}$$

$$X^{(n-1)} \quad \begin{array}{c|cccc} & Q_1 & Q_2 & Q_3 & Q_4 \\ \hline Q_1 & 0.3 & 0.3 & 0.3 & 0.1 \\ Q_2 & 0.3 & 0.3 & 0.3 & 0.1 \\ Q_3 & 0.3 & 0.3 & 0.3 & 0.1 \\ Q_4 & 0.3 & 0.3 & 0.3 & 0.1 \\ \hline \end{array} \qquad \overleftrightarrow{X^{(n)}}$$

Table 5.0.1 $\left[P\left[X^{(n)} = Q_i\right]\right]$ **(The transition probabilities for equal and unequal cases)**

Quantums	Unequal				Equal			
	Q_1	Q_2	Q_3	Q_4	Q_1	Q_2	Q_3	Q_4
$n=1$	0.245	0.335	0.155	0.215	0.3	0.3	0.3	0.1
$n=2$	0.19775	0.22875	0.24	0.259	0.3	0.3	0.3	0.1
$n=3$	0.193687	0.234363	0.216988	0.260688	0.3	0.3	0.3	0.1
$n=4$	0.186308	0.223308	0.218554	0.260688	0.3	0.3	0.3	0.1
$n=5$	0.182492	.02192780	0.2128410	0.25315	0.3	0.3	0.3	0.1
$n=6$	0.178468	0.214323	0.208724	0.247962	0.3	0.3	0.3	0.1
$n=7$	0.174726	0.209890	0.204273	0.242771	0.3	0.3	0.3	0.1

ISSN 1512-1232

Under Scheme-II: Consider the following probability matrices with initial transition probabilities $pr_1 = 1$, $pr_2 = 0$, $pr_3 = 0$;

Unequal

$$\begin{array}{c|cccc} X^{(n-1)} \downarrow \backslash X^{(n)} \rightarrow & Q_1 & Q_2 & Q_3 & Q_4 \\ \hline Q_1 & 0.2 & 0.45 & 0 & 0.35 \\ Q_2 & 0.17 & 0.33 & 0.41 & 0.09 \\ Q_3 & 0.1 & 0 & 0.39 & 0.51 \\ Q_4 & 0.15 & 0 & 0 & 0.85 \end{array}$$

Equal

$$\begin{array}{c|cccc} X^{(n-1)} \downarrow \backslash X^{(n)} \rightarrow & Q_1 & Q_2 & Q_3 & Q_4 \\ \hline Q_1 & 0.3 & 0.3 & 0 & 0.4 \\ Q_2 & 0.3 & 0.3 & 0.3 & 0.1 \\ Q_3 & 0.3 & 0 & 0.3 & 0.4 \\ Q_4 & 0.3 & 0 & 0 & 0.7 \end{array}$$

Table 5.0.2 $\left[P\left[X^{(n)} = Q_i\right]\right]$ **(Transition probabilities for equal and unequal cases)**

Quantums	Unequal				Equal			
	Q_1	Q_2	Q_3	Q_4	Q_1	Q_2	Q_3	Q_4
$n=1$	0.2	0.45	0	0.35	0.3	0.3	0	0.4
$n=2$	0.169	0.2385	0.1845	0.408	0.3	0.18	0.09	0.43
$n=3$	0.153995	0.154755	0.16974	0.52151	0.3	0.144	0.081	0.475
$n=4$	0.152308	0.120367	0.129648	0.597677	0.3	0.1332	0.0675	0.4993
$n=5$	0.15354	0.10826	0.099913	0.638287	0.3	0.12996	0.06021	0.50983
$n=6$	0.154847	0.104819	0.083353	0.656982	0.3	0.128998	0.057051	0.513961
$n=7$	0.155671	0.104271	0.075483	0.664575	0.3	0.128696	0.055812	0.515492

Under Scheme-III: Take the following probability matrices

Unequal

$$\begin{array}{c|cccc} X^{(n-1)} \downarrow \backslash X^{(n)} \rightarrow & Q_1 & Q_2 & Q_3 & Q_4 \\ \hline Q_1 & 0.2 & 0.8 & 0 & 0 \\ Q_2 & 0.4 & 0.35 & 0.25 & 0 \\ Q_3 & 0.13 & 0 & 0.68 & 0.19 \\ Q_4 & 0.45 & 0 & 0 & 0.55 \end{array}$$

Equal

$$\begin{array}{c|cccc} X^{(n-1)} \downarrow \backslash X^{(n)} \rightarrow & Q_1 & Q_2 & Q_3 & Q_4 \\ \hline Q_1 & 0.3 & 0.7 & 0 & 0 \\ Q_2 & 0.3 & 0.3 & 0.4 & 0 \\ Q_3 & 0.3 & 0 & 0.3 & 0.4 \\ Q_4 & 0.3 & 0 & 0 & 0.7 \end{array}$$

ISSN 1512-1232

Table 5.0.3 $\left[P\left[X^{(n)} = Q_i\right]\right]$ **(Transition probabilities for equal and unequal cases)**

Quantums	Unequal				Equal			
	Q_1	Q_2	Q_3	Q_4	Q_1	Q_2	Q_3	Q_4
$n=1$	0.2	0.8	0	0	0.3	0.7	0	0
$n=2$	0.36	0.44	0.2	0	0.3	0.42	0.28	0
$n=3$	0.274	0.442	0.246	0.038	0.3	0.336	0.252	0.112
$n=4$	0.28068	0.3739	0.27778	0.067640	0.3	0.3108	0.21	0.1792
$n=5$	0.272245	0.355409	0.282365	0.08998	0.3	0.30324	0.18732	0.20944
$n=6$	0.273811	0.342189	0.280861	0.103139	0.3	0.300972	0.0177492	0.221536
$n=7$	0.274562	0.338815	0.276533	0.110090	0.3	0.300292	0.173636	0.226072

5.2 USING DATA SET II

Under Scheme-I: Consider the probability matrix with initial probabilities.

$pr_1 = 0.5,\ pr_2 = 0.3,\ pr_3 = 0.2$

Unequal

$X^{(n)}$ (columns), $X^{(n-1)}$ (rows)

	Q_1	Q_2	Q_3	Q_4
Q_1	0.25	0.15	0.17	0.43
Q_2	0.11	0.32	0.07	0.50
Q_3	0.22	0.38	0.13	0.27
Q_4	0.14	0.29	0.37	0.20

Equal

$X^{(n)}$ (columns), $X^{(n-1)}$ (rows)

	Q_1	Q_2	Q_3	Q_4
Q_1	0.2	0.2	0.2	0.4
Q_2	0.2	0.2	0.2	0.4
Q_3	0.2	0.2	0.2	0.4
Q_4	0.2	0.2	0.2	0.4

Table 5.1.1 $\left[P\left[X^{(n)} = Q_i\right]\right]$ **(Transition probabilities for equal and unequal cases)**

Quantums	Unequal				Equal			
	Q_1	Q_2	Q_3	Q_4	Q_1	Q_2	Q_3	Q_4
$n=1$	0.202	0.247	0.132	0.419	0.2	0.2	0.2	0.4
$n=2$	0.16537	0.28101	0.22382	0.3298	0.2	0.2	0.2	0.4
$n=3$	0.167666	0.295422	0.198906	0.338006	0.2	0.2	0.2	0.4
$n=4$	0.165493	0.293291	0.200103	0.341113	0.2	0.2	0.2	0.4
$n=5$	0.165414	0.293639	0.200889	0.340058	0.2	0.2	0.2	0.4
$n=6$	0.165457	0.293731	0.200612	0.340199	0.2	0.2	0.2	0.4
$n=7$	0.165437	0.293703	0.200642	0.340217	0.2	0.2	0.2	0.4

ISSN 1512-1232

Under Scheme-II: Consider the following data of matrices

Unequal

$X^{(n-1)}$ (rows) → $X^{(n)}$ (columns)

	Q_1	Q_2	Q_3	Q_4
Q_1	0.25	0.15	0	0.6
Q_2	0.11	0.32	0.07	0.50
Q_3	0.22	0	0.13	0.65
Q_4	0.14	0	0	0.86

Equal

$X^{(n-1)}$ (rows) → $X^{(n)}$ (columns)

	Q_1	Q_2	Q_3	Q_4
Q_1	0.2	0.2	0	0.6
Q_2	0.2	0.2	0.2	0.4
Q_3	0.2	0	0.2	0.6
Q_4	0.2	0	0	0.8

Table 5.1.2 $\left[P\left[X^{(n)} = Q_i\right]\right]$ **(Transition probabilities for equal and unequal cases)**

Quantums	Unequal				Equal			
	Q_1	Q_2	Q_3	Q_4	Q_1	Q_2	Q_3	Q_4
$n=1$	0.25	0.15	0	0.6	0.2	0.2	0	0.6
$n=2$	0.163	0.0855	0.0105	0.741	0.2	0.08	0.04	0.68
$n=3$	0.156205	0.05181	0.00735	0.784635	0.2	0.056	0.024	0.72
$n=4$	0.156216	0.04001	0.004582	0.799192	0.2	0.0152	0.016	0.7328
$n=5$	0.15635	0.036236	0.003396	0.804018	0.2	0.05024	0.01344	0.73632
$n=6$	0.156383	0.035048	0.002978	0.805591	0.2	0.050048	0.012736	0.737216
$n=7$	0.1563890	0.034673	0.00284	0.806098	0.2	0.05001	0.012557	0.737434

Under Scheme-III: The data is given below:

Unequal

$X^{(n-1)}$ (rows) → $X^{(n)}$ (columns)

	Q_1	Q_2	Q_3	Q_4
Q_1	0.25	0.75	0	0
Q_2	0.11	0.32	0.57	0
Q_3	0.22	0	0.13	0.65
Q_4	0.14	0	0	0.86

Equal

$X^{(n-1)}$ (rows) → $X^{(n)}$ (columns)

	Q_1	Q_2	Q_3	Q_4
Q_1	0.2	0.8	0	0
Q_2	0.2	0.2	0.6	0
Q_3	0.2	0	0.2	0.6
Q_4	0.2	0	0	0.8

ISSN 1512-1232

Table 5.1.3 $\left[P\left[X^{(n)} = Q_i\right]\right]$ **(Transition probabilities for equal and unequal cases)**

Quantums	Unequal				Equal			
	Q_1	Q_2	Q_3	Q_4	Q_1	Q_2	Q_3	Q_4
$n=1$	0.25	0.75	0	0	0.2	0.8	0	0
$n=2$	0.145	0.4275	0.4275	0	0.2	0.32	0.48	0
$n=3$	0.177325	0.24555	0.29925	0.277875	0.2	0.224	0.288	0.288
$n=4$	0.176079	0.21157	0.178866	0.433485	0.2	0.2048	0.192	0.4032
$n=5$	0.167331	0.199762	0.143847	0.48906	0.2	0.20096	0.16128	0.43776
$n=6$	0.163921	0.189222	0.132564	0.514092	0.2	0.200192	0.152832	0.446976
$n=7$	0.162954	0.183556	0.1252024	0.528286	0.2	0.200038	0.150682	0.44928

5.3 USING DATA SET III

Under Scheme-I: $pr_1 = 0.5$, $pr_2 = 0.3$, $pr_3 = 0.2$

Unequal

$X^{(n)}$ (columns), $X^{(n-1)}$ (rows)

	Q_1	Q_2	Q_3	Q_4
Q_1	0.32	0.12	0.26	0.30
Q_2	0.21	0.43	0.17	0.19
Q_3	0.06	0.54	0.12	0.28
Q_4	0.42	0.02	0.31	0.25

Equal

$X^{(n)}$ (columns), $X^{(n-1)}$ (rows)

	Q_1	Q_2	Q_3	Q_4
Q_1	0.25	0.25	0.25	0.25
Q_2	0.25	0.25	0.25	0.25
Q_3	0.25	0.25	0.25	0.25
Q_4	0.25	0.25	0.25	0.25

Table 5.2.1 $\left[P\left[X^{(n)} = Q_i\right]\right]$ **(Transition probabilities for equal and unequal cases)**

Quantums	Unequal				Equal			
	Q_1	Q_2	Q_3	Q_4	Q_1	Q_2	Q_3	Q_4
$n=1$	0.235	0.297	0.205	0.263	0.25	0.25	0.25	0.25
$n=2$	0.26033	0.27187	0.21772	0.25008	0.25	0.25	0.25	0.25
$n=3$	0.258495	0.270714	0.217555	0.253236	0.25	0.25	0.25	0.25
$n=4$	0.258981	0.269971	0.21784	0.253209	0.25	0.25	0.25	0.25
$n=5$	0.258986	0.269863	0.217865	0.253286	0.25	0.25	0.25	0.25
$n=6$	0.258999	0.269832	0.217875	0.253293	0.25	0.25	0.25	0.25
$n=7$	0.259	0.269826	0.217877	0.253296	0.25	0.25	0.25	0.25

ISSN 1512-1232

Under Scheme-II: The data is below:

Unequal

$X^{(n-1)}$ \ $X^{(n)}$	Q_1	Q_2	Q_3	Q_4
Q_1	0.32	0.12	0	0.56
Q_2	0.21	0.43	0.17	0.19
Q_3	0.06	0	0.12	0.82
Q_4	0.42	0	0	0.58

Equal

$X^{(n-1)}$ \ $X^{(n)}$	Q_1	Q_2	Q_3	Q_4
Q_1	0.25	0.25	0	0.50
Q_2	0.25	0.25	0.25	0.25
Q_3	0.25	0	0.25	0.50
Q_4	0.25	0	0	0.75

Table 5.2.2 $\left[P\left[X^{(n)} = Q_i\right]\right]$ **(Transition probabilities for equal and unequal cases)**

Quantums	Unequal				Equal			
	Q_1	Q_2	Q_3	Q_4	Q_1	Q_2	Q_3	Q_4
$n=1$	0.32	0.12	0	0.56	0.25	0.25	0	0.5
$n=2$	0.3628	0.09	0.0204	0.5268	0.25	0.125	0.0625	0.5625
$n=3$	0.357476	0.082236	0.017748	0.54254	0.25	0.09375	0.046875	0.609375
$n=4$	0.360594	0.078259	0.01611	0.245038	0.25	0.085938	0.035156	0.628906
$n=5$	0.361707	0.076922	0.015237	0.546134	0.25	0.083984	0.030273	0.635742
$n=6$	0.36219	0.076481	0.014905	0.546423	0.25	0.083496	0.028564	0.637939
$n=7$	0.362354	0.07635	0.01479	0.546506	0.25	0.083374	0.028015	0.638611

Under Scheme-III: The data is as under:

Unequal

$X^{(n-1)}$ \ $X^{(n)}$	Q_1	Q_2	Q_3	Q_4
Q_1	0.32	0.68	0	0
Q_2	0.21	0.43	0.36	0
Q_3	0.06	0	0.12	0.82
Q_4	0.42	0	0	0.58

Equal

$X^{(n-1)}$ \ $X^{(n)}$	Q_1	Q_2	Q_3	Q_4
Q_1	0.25	0.75	0	0
Q_2	0.25	0.25	0.50	0
Q_3	0.25	0	0.25	0.50
Q_4	0.25	0	0	0.75

Table 5.2.3 $\left[P\left[X^{(n)} = Q_i\right]\right]$ **(Transition probabilities for equal and unequal cases)**

Quantums	Unequal				Equal			
	Q_1	Q_2	Q_3	Q_4	Q_1	Q_2	Q_3	Q_4
$n=1$	0.32	0.68	0	0	0.25	0.75	0	0
$n=2$	0.2452	0.51	0.2448	0	0.25	0.375	0.375	0
$n=3$	0.200252	0.386036	0.212976	0.200736	0.25	0.28125	0.28125	0.1875
$n=4$	0.242236	0.302167	0.16453	0.291067	0.25	0.257812	0.210938	0.28125
$n=5$	0.273091	0.294652	0.128524	0.303734	0.25	0.251957	0.181641	0.316406
$n=6$	0.284545	0.312402	0.121498	0.281555	0.25	0.250488	0.171387	0.328125

ISSN 1512-1232

$n = 7$	0.282202	0.327824	0.127044	0.26293	0.25	0.250122	0.168091	0.331787

6.0 GRAPHICAL STUDY AND DISCUSSION

The analytical discussion is performed with three scientific approaches:
(a) Comparison of unequal and equal probability of transitions within a scheme;
(b) Comparison of scheduling schemes under equal transition probability;
(c) Comparison of scheduling schemes under unequal transition probability.

For Scheme-I: When the system is completely unrestricted and the process is allowed to enter into from any of the queue with unequal initial probabilities, the variation $P\left[X^{(n)} = Q_i\right]$ over three different data sets is shown in fig. 6.1 to fig. 6.6.

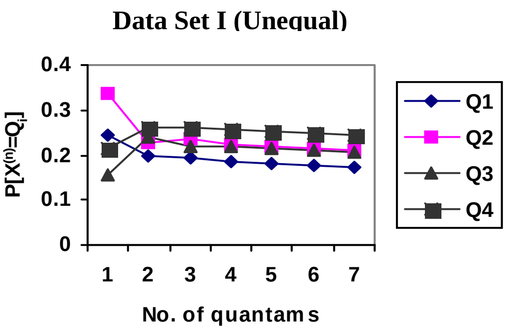

Fig. 6.1 (Scheme-I)

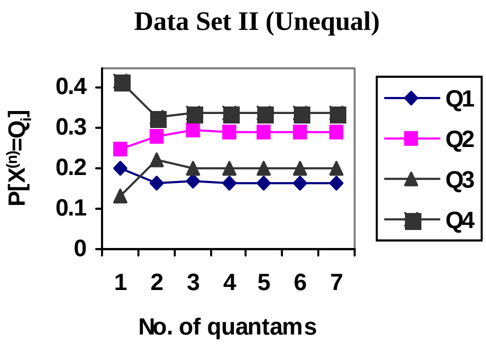

Fig. 6.2 (Scheme-I)

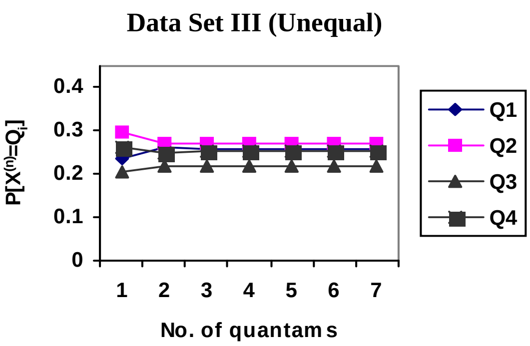

Fig. 6.3 (Scheme-I)

Data Set I (equal)

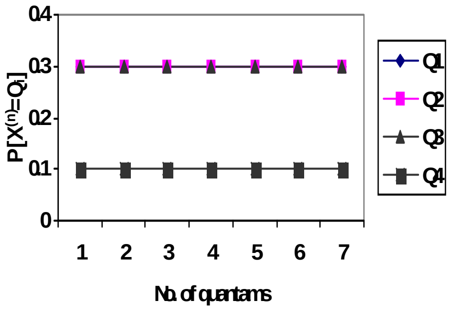

Fig. 6.4 (Scheme -I)

Data Set II (equal)

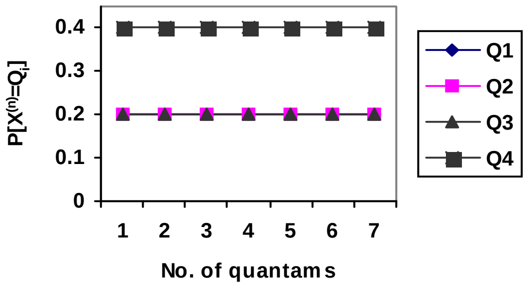

Fig. 6.5 (Scheme -I)

Data Set III (equal)

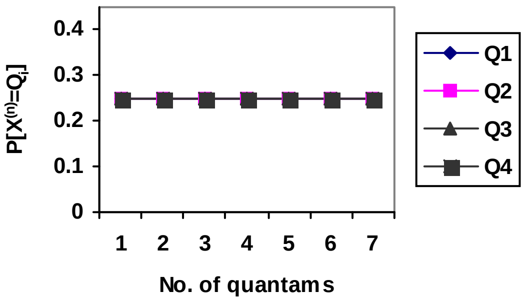

Fig. 6.6 (Scheme -I)

The pattern of transition over states Q_1, Q_2, Q_3 and Q_4 of the scheduler reflects a stability over a number of quantum for $n \geq 2$ (when unequal transitions). The remarkable point is that the probability of state Q_4 (rest state) is higher in all data sets than for other states. This shows a loss of efficiency of scheme-I under model criteria. The higher chance that scheduler spends more and more over the rest state than on working states Q_1, Q_2 and Q_3. Therefore, completely unrestricted scheduling scheme-I leads to a loss of CPU time. While considering the same with equal transition matrix, the state probabilities are found independent of the quantum variation, but it also supports the above fact.

For Scheme-II: The scheme is similar to usual multi-level queue scheduling where the system starts from Q_1, moves towards Q_2 and Q_3, but can shift over to Q_4 from any of the $Q_i\,(i = 1,2,3,)$. The fig. 6.7 to fig. 6.12 has the variability pattern of chances for $P\left[X^{(n)} = Q_i\right]$ over different quantum.

ISSN 1512-1232

Data Set I (unequal)

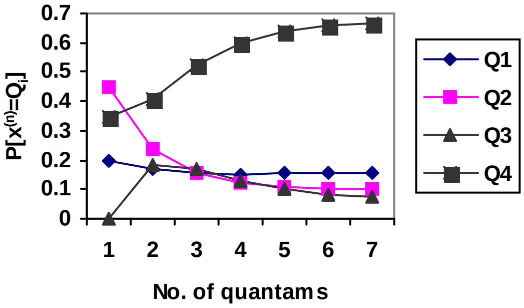

Fig. 6.7 (Scheme-II)

Data Set II (unequal)

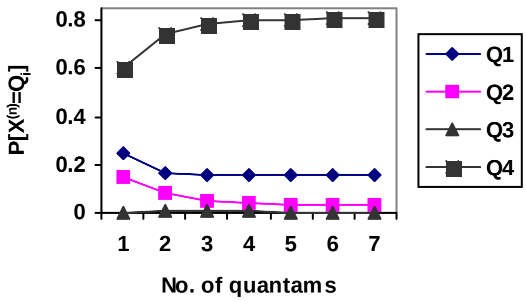

Fig. 6.8 (Scheme-II)

Data Set III (unequal)

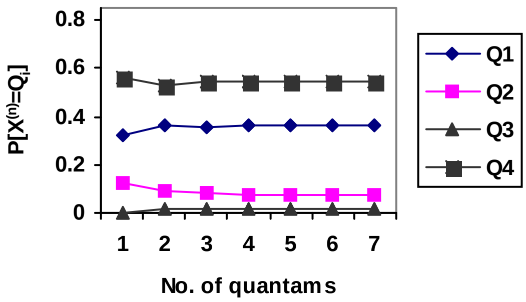

Fig.6.9 (Scheme-II)

Data Set I (equal)

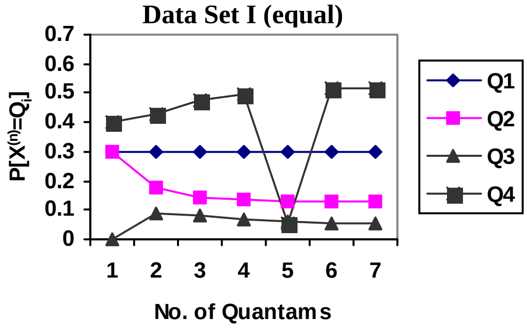

Fig. 6.10 (Scheme-II)

Data Set II (equal)

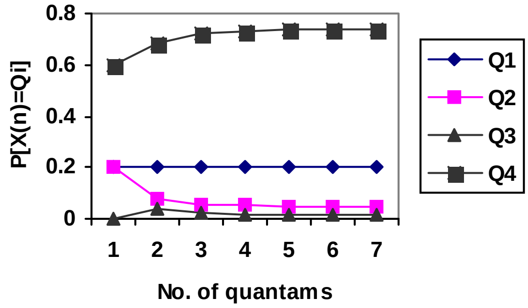

Fig. 6.11 (Scheme-II)

Data Set III (equal)

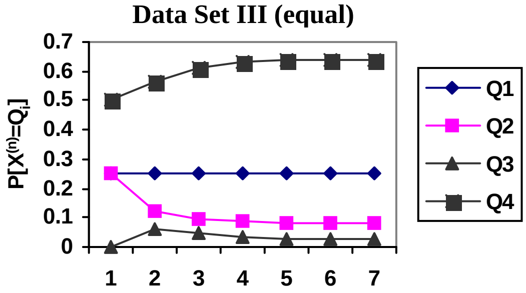

Fig. 6.12 (Scheme-II)

ISSN 1512-1232

Graphs reveal a higher probability at the rest state Q_4 than the other Q_1, Q_2, Q_3. This again leads to a loss of efficiency under model due more resting chance of the scheduler. The state probability is independent of the quantum variation for $n > 2$. But, the rest state bears an increasing variation of probability over large n. The special remark for this process-scheduling scheme-II is that probability for the state Q_3 is very low. Therefore, there are lesser chance for jobs contained in Q_3 to be processed than Q_1 and Q_2.

For Scheme-III: When hard restriction is imposed in terms of a security measure, the graphical pattern is shown in fig. 6.13 to fig. 6.18.

Data Set I (unequal)

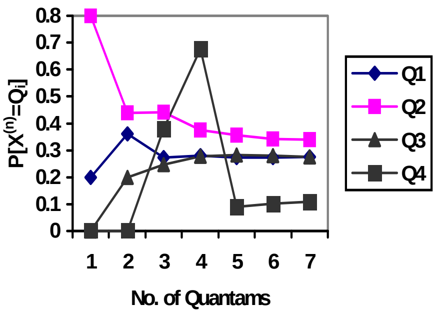

Fig.6.13 (Scheme-III)

Data Set II (unequal)

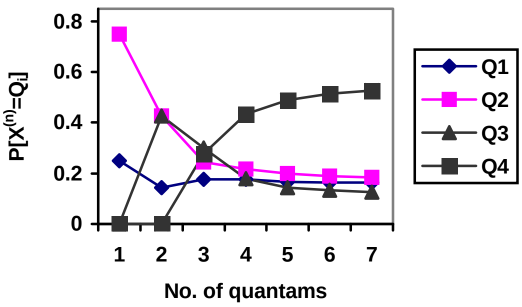

Fig. 6.14 (Scheme-III)

Data Set III (unequal)

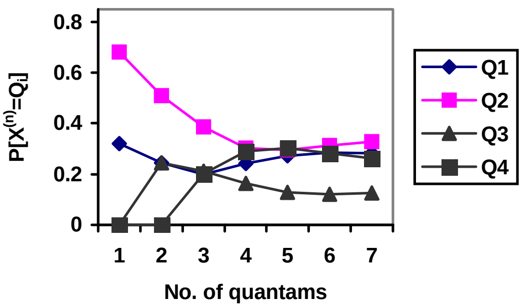

Fig. 6.15 (Scheme-III)

ISSN 1512-1232

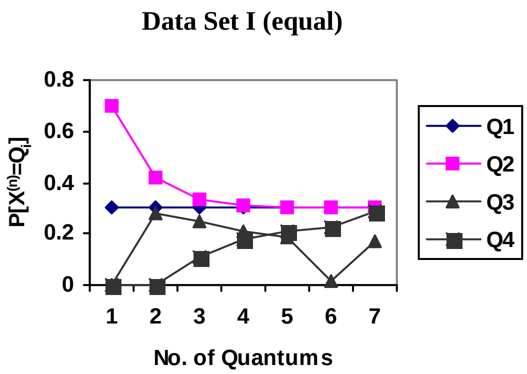

Fig. 6.16 (Scheme-III)

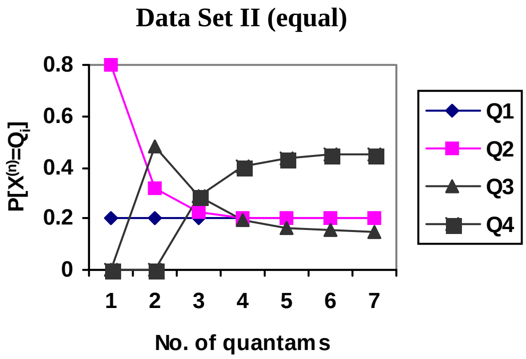

Fig. 6.17 (Scheme-III)

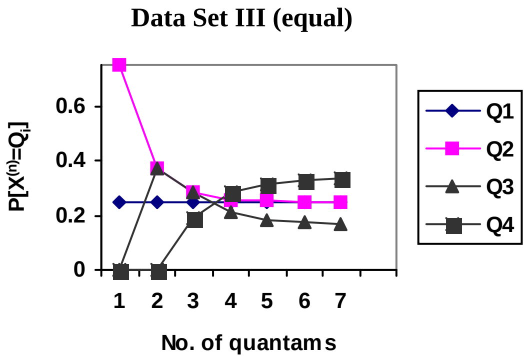

Fig. 6.18 (Scheme-III)

The probability of scheduler, for the rest state is lower than other state probabilities. This has a sign of increase in efficiency of the scheduling scheme-III under model consideration. The probability of state Q_3 is higher in the scheme than the previous schemes. Most of the transition probabilities are almost equal and well within the range of 0.15 to 0.45. Slight variation is observed for quantum $n=1,2$ and 3 but for $n \geq 4$, a stable probability pattern is found. The scheme-III provides more and more chance or importance to job processing than "resting-the-scheduler", in comparison to scheme-II. Therefore, efficiency of this scheduling procedure is higher and recommendable.

In scheme-II, the scheduler has less chance of having transition over state Q_2 and Q_3 than Q_4.

6.1 EFFECT OF EQUAL AND UNEQUALTRANSITION PROBABILITY MATRICES BETWEEN SCHEMES

A between comparison of schemes generates following remarks:

For Scheme-I: The equal transition probabilities leads to independency of quantum with the transition probability. Also the information overlaps in equal probability case. In the unequal probability matrix, elements present a better picture of transition within states.

The state Q_1 has lesser probability than Q_2 and Q_3 which is objectionable and indicate for inefficient result.

For Scheme-II: This has a little unstable pattern of variation of state probabilities for unequal transition elements. The state Q_4 has constantly higher probability (when unequal elements) in

ISSN 1512-1232

comparison to Q_1, Q_2 and Q_3. However, the state Q_3 bears the lowest state probability. The similar pattern is found with equal probability matrices.

For Scheme-III: It is observed that the pattern variation of state probabilities over quantum is almost same between equal and unequal transition elements of matrices. Further, the pattern is having not much variation over changing data. This is an interesting feature which leads to the stability of the whole system and the probability of rest state Q_4 is also not much in comparison to Q_1, Q_2 and Q_3.

7.0 CONCLUDING REMARKS

In the first type of scheduling scheme, the probability towards the rest state is very high which indicates for a loss of system efficiency. The graphical pattern does not depend much on quantum variation for $n \geq 2.$ There is a deep effect of equal and unequal probability elements set-up on this scheme. Moreover the state Q_1 has lesser probability than Q_2 and Q_3 which is not a good sign in favour.

The second scheduling scheme bears even higher probabilities for the rest state than the previous one. The third state Q_3 has a very low chance of being processed. This is a serious drawback observed in model-based study with reference to the usual multi-level queue scheduling.

The third scheme provides a stable pattern of probability variation over quantum almost in all the three data sets. For $n \geq 4$ the variation becomes independent of changes in terms of quantum. In this, most of the state probabilities lies between 0.15 to 0.45 for all sets of data.

As an overall view, the three suggested schemes of the general class are multilevel queue scheduling for an operating system but the third one is useful, efficient and recommendable over the earlier two in light of the considered Markov chain probability model.

ISSN 1512-1232

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