ISSN 1512-1461

POTENTIAL BETWEEN QUARKS AND ANTIQUARKS ACCORDING TO INFRARED ASYMPTOTICS OF THE GLUON PROPAGATOR

[1]A.Khelashvili, [2]T.Nadareishvili

[12] Iv. Javakhishvili Tbilisi State University. Institute of High Energy Physics University str. 9. 0109 Tbilisi.Georgia

[12] St.Andrew the First-called Georgian University of Patriarchy of Georgia, Chavchavadze Ave.53a, 0162, Tbilisi, Georgia.

[1]anzor.khelashvili@tsu.ge, [2]teimuraz.nadareishvili@tsu.ge

Abstract:

Exploring the infrared asymptotic behavior of the gluon propagator, one gluon exchange potential between quark and antiquark is constructed. It is very close to the Cornell potential and admits Richardson parameterization in sufficiently good precision.

***Keywords**: Dyson-Schwinger equation,Cornel and Richardson potentials, gluon propagator.*

Potential models are widely used in studying quark-antiquark bound states. Potentials motivated by QCD are based on two kinds of asymptotics: ultraviolet at short distances, i.e. Coulomb-like term and infrared – which in accordance with lattice QCD, the string model, Dyson-Schwinger equation etc., gives linear increase of confining potential at large distances. The simple extrapolation of these asymptotics is so-called Cornell potential [1]

$$V(r) = kr - \frac{4}{3}\frac{\alpha_S}{r} \qquad (1)$$

where $\alpha_S = const.$, or is considered as a running coupling constant

$$\alpha_S\left(\vec{q}^{\,2}\right) = \frac{48\pi^2}{33-2N_f}\frac{1}{\ln\left(\vec{q}^{\,2}/\Lambda^2\right)}, \qquad \vec{q}^{\,2} >> \Lambda^2 \qquad (2)$$

Here N_f is a number of “effective” fermions and Λ is QCD scale parameter.

Richardson proposed the following potential [3]

$$V\left(\vec{q}^{\,2}\right) = -\frac{4}{3}\frac{48\pi^2}{33-2N_f}\frac{1}{\vec{q}^{\,2}\ln\left(1+\vec{q}^{\,2}/\Lambda^2\right)}, \qquad (3)$$

i.e. it is normalized to have correct ultraviolet behavior of α_S for $\vec{q}^{\,2} >> \Lambda^2$ resulting from perturbation theory, and is extrapolated to infrared region by specific choice of logarithmic function.

Assuming the Richardson potential valid for infrared region as well, after expanding in powers of small $\vec{q}^{\,2}$ up to Coulomb term we have

$$V\left(\vec{q}^{\,2}\right) \approx \frac{64\pi^2}{33-2N_f}\frac{\Lambda^2}{\left(\vec{q}^{\,2}\right)^2} - \frac{32\pi^2}{33-2N_f}\frac{1}{\vec{q}^{\,2}}, \qquad \vec{q}^{\,2} << \Lambda^2 \qquad (4)$$

Therefore we can model the Cornell potential using this formula.

Further we’ll consider pure gluodynamics $\left(N_f = 0\right)$. In that case we can obtain the values of parameters by comparing Fourier transform of (4) with (1)

ISSN 1512-1461

$$k^R = \frac{8\pi\Lambda^2}{33}, \qquad \alpha_S^R = \frac{2\pi}{11} \approx 0.571 \qquad (5)$$

During past years a lot of work has been dedicated to the investigation of infrared asymptotics of gluodynamics [4-6]. Though this region is non-perturbative, in some cases it is possible to extract not only leading $\left(\vec{q}^{\,2}\right)^{-2}$, but next to leading Coulomb $\vec{q}^{\,-2}$ term, as well [7,8]. Therefore, it is relevant to ask whether these results are compatible with the potentials mentioned above.

In the paper [8] the Dyson-Schwinger equation for gluon propagator contracted by gauge vector n_μ in the light-cone gauge $\left(n^2 = 0\right)$ was studied in the most general case – when both tensor structures of the gluon propagator are taken into account. There was obtained a solution

$$\Delta_{\mu\nu}^{ab}\left(q\right) = -\frac{i\delta^{ab}}{q^2}\left(-\frac{M^2}{q^2} + b\right)\left\{g_{\mu\nu} - \frac{q_\mu n_\nu + q_\nu n_\mu}{nq} + \lambda\varepsilon q^2 \frac{n_\mu n_\nu}{\left(nq\right)^2}\right\} \qquad (6)$$

where M^2, b and λ are arbitrary constants and ε is a parameter of the dimensional regularization$\left(\varepsilon \to 0^+\right)$. In obtaining this formula, the coefficient of the second (induced) structure was set proportional to ε. It is evident that in the limit $\varepsilon \to 0$ the contribution of this term to the one-gluon exchange potential vanishes. However, it gives contribution in loop integrations and therefore the parameter still remains in the right-hand side of Dyson-Schwinger equation. In order to satisfy this equation in the limit $q^2 \to 0$, we must make use of an additional condition for b, which for $SU\left(3\right)_C$ group takes the form [8]

$$1 = \frac{g^2 b}{48\pi^2}\left(17 + 60\lambda - 9\lambda^2\right) \qquad (7)$$

Let us define a one-gluon exchange potential by the static limit $q_0 = 0$, in the propagator (which gives the leading contribution in the quasipotential formalism [9] and take the color factor into account. Consequently, we have

$$V\left(\vec{q}^{\,2}\right) = -\frac{4}{3}\left[\frac{g^2 M^2}{(\vec{q}^{\,2})^2} + \frac{g^2 b}{\vec{q}^{\,2}}\right] \qquad q_0 = 0 \qquad (8)$$

It gives the Cornell potential in coordinate space with the following values of parameters:

$$k = \frac{g^2 M^2}{6}, \qquad \alpha_S = \frac{g^2 b}{4\pi} \qquad (9)$$

With the help of (7) $g^2 b$ is expressed in terms of the parameter λ, which is arbitrary as yet and by varying the later we can obtain any given value of $g^2 b$.

There exists a physically distinguished gauge among the numerous possibilities – so-called Arbusov gauge [10]. This gauge for $n^2 = 0$ is defined by the following constraint: the leading singular part of gluon propagator in coordinate space must be transversal

$$x^\mu \tilde{\Delta}_{\mu\nu}^{Sing}\left(x\right) = 0 \qquad (10)$$

Here $\tilde{\Delta}_{\mu\nu}^{Sing}\left(x\right)$ is the Fourier transform of q^{-4} term of the propagator (6), which in the arbitrary d-dimensional space has the form [10]:

$$\tilde{\Delta}_{\mu\nu}^{Sing}\left(x\right) = -\frac{i^{-d} M^2}{16\pi^{d/2}}\left(x^2\right)^{2-d/2} \times$$
$$\times\left\{\Gamma\left(\frac{d}{2} - 2\right)\left(g_{\mu\nu} - \frac{x_\mu n_\nu + x_\nu n_\mu}{\left(nx\right)}\right) - \left(1 + \lambda\frac{4-d}{2}\right)\Gamma\left(\frac{d}{2} - 3\right)\frac{x^2 n_\mu n_\nu}{\left(nx\right)^2}\right\} \qquad (11)$$

Then

ISSN 1512-1461

$$x^{\mu}\tilde{\Delta}^{Sing}_{\mu\nu}(x) = -\frac{i^{-d}M^2}{16\pi^{d/2}}\left(x^2\right)^{3-d/2}\left(2-\frac{d}{2}\right)\Gamma\left(\frac{d}{2}-3\right)\frac{n_\nu}{(nx)}(1-\lambda) \qquad (12)$$

It is clear that the choice $\lambda = 1$ guarantees the transversality of $\Delta_{\mu\nu}$. Choosing this gauge and taking into account (7), (9) one obtains

$$\alpha_S = \frac{3\pi}{17} \approx 0.554 \qquad (13)$$

This value differs only by 3% from α_S^R (Eq. (5)). If we consider the expression (8) as the first two terms of the expansion in Richardson parameterization, the corresponding potential in our case will become

$$V\left(\vec{q}^{\,2}\right) = -\frac{4}{3}\frac{48\pi^2}{34}\frac{1}{\vec{q}^{\,2}\ln\left(1+\vec{q}^{\,2}/\Lambda^2\right)} \qquad (14)$$

where the QCD parameter Λ is determined by obvious formula:

$$k = \frac{g^2M^2}{6} = \frac{8\pi\Lambda^2}{34} \qquad (15)$$

The comparison of k and k^R shows that parameter Λ evaluated from the infrared region of gluodynamics via Richardson parameterization is in satisfactory agreement with its perturbative (i.e. determined by ultraviolet asymptotics) value. Note that if we restrict ourselves only by the first (free) structure $(\lambda = 0)$ in gluon propagator the result for α_S and k will change by factor 4 [7].

To make the picture complete we must incorporate fermions (quarks) as well. Besides, it is extremely desirable to investigate this problem in other gauges.

REFERENCES

1. E.Eichten et al. Phys. Rev., (1978)**D17**, 3090; (1980),D21, 203.
2. D.Gross, F. Wilczek. Phys.Rev.Lett**.,** (1973), **30**, 1343.
   H.D.Politzer. Phys.Rev.Lett., (1973)**30**, 1346.
3. H.D. Richardson. Phys. Lett., (1979), **82B**, 272.
4. S. Mandelstam. Phys. Rev., (1979), **D20**, 3223.
5. M. Baker et al., Nucl. Phys., (1981), **B186**, 531.
6. A.I. Alekseev. Sov. Jour. Nucl. Phys. (1981), **33**,516.
7. K.R. Natroshvili et al., Sov. Journ. TMF, (1985)**65**, 360.
8. L.G. Vachnadze et al., Sov. Journ. TMF, (1989), **80**, 264.
9. A.A. Logunov, A.N. Tavkhelidze. Nuovo Cim., (1963), **29**, 380.
10. For the full information see B.A. Arbusov's review in Sov. Journ. "Elem. Particles and Atomic Nuclei", (1988),**19**, 5.

Article received: 2010-02-07