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COMPLEX-VALUED HOPFIELD NEURAL NETWORK FOR AMPLITUDE ESTIMATION OF SINUSOIDAL SIGNALS

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Abstract

Recently models of neural networks that can directly deal with complex numbers, complex-valued neural networks, have been proposed and several studies on their abilities of information processing have been done. In this paper, the problem of amplitude estimation of sinusoidal signals from observations corrupted by colored noise using Hopfield neural network (HNN) is considered. We have introduce a complex Hopfield neural networks which can be expressed as an equivalent real valued networks by expanding its real and imaginary parameters separatly. To prove the efficient of the proposed method, it has been compared with various amplitude estimator cited in [4]. Simulation results show that the calculation precision of the amplitudes estimation improves when the mean-squared error is used.

Keywords: *Amplitude estimation, matched-filterbank, spectral analysis, Hopfield neural networks.*

I. INTRODUCTION

The need to estimate the complex amplitudes of sinusoidal signals in a noisy environment is encountered in many signal processing applications .Least square (LS) based amplitude estimators, have been most widely used due to their conceptual and computational simplicity, they are also known to be optimum when the observation noise is white and have a Gaussian distribution. However, when the observation noise is *colored* and, particularly, when the size of the observed data is relatively small, these estimators are failed [4,5]. To solve this problem, Stoica et al [4] investigated alternative techniques for amplitude estimation, including weighted-least-squares (WLS) and MAtched-FIlterbank (MAFI) approaches which are very desirable when the observation noise is colored. However, these techniques may perform poorly for cases of very small data length or small SNR that occur in practice.

In recent years, there have been increasing research interests of artificial neural networks and many efforts have been made on application of neural networks to various fields. As applications of the neural networks spread more widely, developing neural networks models which can directly deal with complex numbers is desired in various fields. Several models of Hopfield complex-valued neural networks have been proposed in spectral estimation and their abilities have been investigated [1].

Considering the justified performances of the Hopfield neural network in the problems of optimization, it was largely used in the field of the spectral estimation. We fined in the literature the Hopfield net for AR spectral estimation [3],and the estimation of the position of the spectral rays [2]… all these methods are interested in the estimate of the frequencies without no knowledge a priori on the vector of observations.

In this paper we try to apply the Hopfield neural network for the amplitudes estimation of sinusoidal signals corrupted by additive noise. The idea behind this technique is to relate the penalty function of the least square error with the Lyapunov energy function of the HNN.

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The remainder of this paper is organized as follows: In section 2 we present the problem definition. Proposed Hopfield network for amplitude estimation ²is described in section 3. Section 4 shows the experimental results obtained using our method. Finally, section 5 contains the conclusion.

II. PROBLEM DEFINITION

Consider the noise-corrupted observations of K complex-valued sinusoids

$$x(n) = \sum_{k=1}^{K} a_k e^{jw_k n} + v(n), \quad n = 0,1,\dots,N-1, \tag{1}$$

Where

a_k Complex amplitude of the kth sinusoid to be estimated;

N Number of available data samples;

w_k Frequency of the kth sinusoid which are known;

$v(n)$ observation noise, which is complex valued and assumed to be stationary with mean zero and finite unknown power spectral density.

Let us rewrite expression (1) in matrix form

$$\begin{bmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-1) \end{bmatrix} = \begin{bmatrix} 1 & \cdots & 1 \\ e^{jw_1} & \cdots & e^{jw_k} \\ \vdots & \vdots & \vdots \\ e^{j(N-1)w_1} & \cdots & e^{j(N-1)w_k} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_k \end{bmatrix} + \begin{bmatrix} v(0) \\ v(1) \\ \vdots \\ v(N-1) \end{bmatrix} \tag{2}$$

Or, with obvious definitions

$$x = Aa + V \tag{3}$$

By dividing this expression in real and imaginary parts separately

$$x = x_{\text{Re}}(n) + jx_{\text{Im}}(n)$$

$$= \sum_{k=1}^{K} \left(a_k^{\text{Re}} + ja_k^{\text{Im}}\right)\left(\cos(nw_k) + j\sin(nw_k)\right) + v(n), \quad n = 0,1,\dots,N-1. \tag{4}$$

This can be easily expressed as:

$$x_{\text{Re}}(n) = \sum_{k=1}^{K} (a_k^{\text{Re}} \cos(nw_k) - a_k^{\text{Im}} \sin(nw_k)) + v^{\text{Re}}(n)$$

And

$$x_{\text{Im}}(n) = \sum_{k=1}^{K} (a_k^{\text{Re}} \sin(nw_k) + a_k^{\text{Im}} \cos(nw_k)) + v^{\text{Im}}(n) \tag{5}$$

Which are equivalent to

$$x_{\text{Re}}(n) = \left[\cos(nw_k) - \sin(nw_k)\right] \begin{bmatrix} a_k^{\text{Re}} \\ a_k^{\text{Im}} \end{bmatrix} + v^{\text{Re}}(n),$$

$$x_{\text{Im}}(n) = \left[\sin(nw_k)\cos(nw_k)\right] \begin{bmatrix} a_k^{\text{Re}} \\ a_k^{\text{Im}} \end{bmatrix} + v^{\text{Im}}(n) \tag{6}$$

for $k = 1,\dots,K$.

We assume that the complex vector of observations is written as a new real vector where the real and imaginary components are ordering as follow

$$x(n) = \begin{bmatrix} x^{\text{Re}}(n) \\ x^{\text{Im}}(n) \end{bmatrix} \ . \tag{7}$$

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That yield immediately

$$\begin{bmatrix} x^{\mathrm{Re}}(n) \\ x^{\mathrm{Im}}(n) \end{bmatrix} = \begin{bmatrix} \mathrm{Re}\{A\} & -\mathrm{Im}\{A\} \\ \mathrm{Im}\{A\} & \mathrm{Re}\{A\} \end{bmatrix} \begin{bmatrix} a_k^{\mathrm{Re}} \\ a_k^{\mathrm{Im}} \end{bmatrix} + \begin{bmatrix} v^{\mathrm{Re}}(n) \\ v^{\mathrm{Im}}(n) \end{bmatrix}.$$

Or, with obvious definitions

$$\bar{x} = \bar{A}\bar{a} + \bar{V} . \qquad (8)$$

III. PROPOSED HOPFIELD AMPLITUDE ESTIMATOR

The idea behind the Hopfield amplitude estimator is to relate the penalty function of the amplitude estimation using the trivial Ls method with the Lyapunov function of the HNN which is given by [1,3]

$$E_{hop} = -\frac{1}{2}\sum_{i=1}^{K}\sum_{j=1}^{K} w_{ij} v_i v_j - \sum_{i=1}^{K} I_i v_i \quad . \qquad (9)$$

Or in the matrix form

$$E_{Hop} = -\frac{1}{2} V^T W V - I^T V . \qquad (10)$$

It has been shown by Hopfield that the HNN always achieves a local minimum of the lyapunov energy function [1]. Hence if a penalty function of an optimisation problem can be related with the lyapunov function, we can obtain at least a good solution for the problem.

For our optimisation problem, the objective function to minimise is given by

$$j(\bar{a}) = \left\| \bar{x} - \bar{A}\bar{a} \right\|^2 \quad . \qquad (11)$$

This is equivalent to (12) by ignoring the constant term $x^T x$

$$j(\hat{a}) = \frac{1}{2} \bar{a}^T \bar{A}^T \bar{A}\bar{a} - \bar{x}^T \bar{A}\bar{a} \quad . \qquad (12)$$

By relating Eq. 10 with Eq. 12, we can easily obtain the interconnection weights and the thresholds of the HNN

$$\begin{cases} W = -\bar{A}^T \bar{A} \\ I = \bar{A}^T \bar{x} \\ V = \bar{a} \end{cases} \qquad (13)$$

The dynamic equation of the HNN is given by [1,3]

$$\frac{\partial u_i}{\partial t} = -\frac{u_i}{\tau} + \sum_{i=1}^{K} T_{ij} v_j + b_i \ . \qquad (14)$$

Since the convergence to a local minimum is guaranteed only if $\frac{\partial v_i}{\partial u_i} \geq 0$, a modified sigmoid function has been adapted for the problem in consideration. In our application, we have choose the activation function given by Eq. (15)

$$f(u_i) = \frac{2\alpha}{1 + e^{-\beta . u_i}} - \alpha \quad . \qquad (15)$$

The choice of α is arbitrary with the condition that all value of amplitudes must verify $|a_k| \leq \alpha$. The parameter β represents the slop of the sigmoid function. It must be choose carefully to obtain a good result.

The simulation of the continuous Hopfield network can be done with the Euler method [1]

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$$u_i(t+1) = u_i(t) + \Delta t(\sum_{i=1}^{K} T_{ij} v_j + b_i) . \quad (16)$$

Where Δt is the time step of the Euler method and $u_i(t)$ is the input signal of the neuron i in time moment t. A number of trials are performed and best results are chosen.

IV. SIMULATION RESULTS

In this section we present some examples to illustrate the performance of the HNN in the problem of amplitudes estimation and compare our method (HOP) with the three methods (LSEK, APESK, and MAFI1) cited in [4].

In order to test the robustness of the proposed method, we have considered the white noise and the colored one.

In the first example of simulation that we have done, the signal which we will estimate its parameters is composed of three sinusoids with parameters $a_1 = e^{j\pi/4}$, $a_2 = e^{j\pi/3}$, $a_3 = e^{j\pi/4}$, $f_1 = 0.1$, $f_2 = 0.11$, *and* $f_3 = 0.3$ corrupted by colored noise. Such noise is generated by an autoregressive model where the input is a white Gaussian noise with zero mean and variance σ^2 . The correlation between samples is given by [4]

$$v(n) = 0.99v(n-1) + e(n) . \quad (17)$$

For the SNR, we have use the local SNR which the expression is given by [4]

$$SNR_k \approx 10\log_{10} \frac{N|a_k|^2}{\phi(f_k)} , \quad (18)$$

where $\phi(f_k)$ is the psd of the noise, it's expressed by

$$\phi(f_k) = \frac{b^2\sigma^2}{1 + a^2 - 2a\cos(2\pi f_k)} \quad , \quad (19)$$

where a and b are the parameter's model.

In the second example, we will show the performance of HOP when the observation noise is white, to do this, we have considered an example that is similar to the previous one, except that $v(n)$ is replaced by a zero mean complex white Gaussian noise.

We have generated 200 Monte-Carlo simulations for different local SNR and the mean squared error was used to test the performances of estimators

$$MSE\{a_k\} = \frac{1}{200}\sum|\hat{a}_k(i) - a_k|^2 \quad , \quad (20)$$

where $\hat{a}_k(i)$ is the amplitude estimated at the ieme simulation.

In all our simulations, the time step of Euler method is fixed to 0.01.

A. *Choosing α and β*

The choice of the Hopfield net parameters α and β denotes the tradeoff between the fastness of the convergence and the best estimation of the signal amplitudes. Obviously, the value of α should be chosen larger than the maximum possible value of the real and imaginary parts of a_i's. In our simulations, the value of β can be reduced to a value less then one for better convergence. It depends on the magnitude of T_{ij} and b_i's. to show the effect of the choice of the parameter β on the quality of the estimation, we have simulate our net for different value of β. Fig. 1.a and b show the effect of the parameter β on the estimates $\hat{a}_3$ and $\hat{a}_1$ respectively. For $\hat{a}_3$, all value of β less then one gives the same MSE, however, for $\hat{a}_1$, all value greater then 0.5 gives poor estimation for higher SNR.

We propose $\alpha = 4$ and $\beta \in [0.01 \quad 0.2]$ ($\beta = 0.02$ was used for the rest of simulations) as a possible set of parameters.

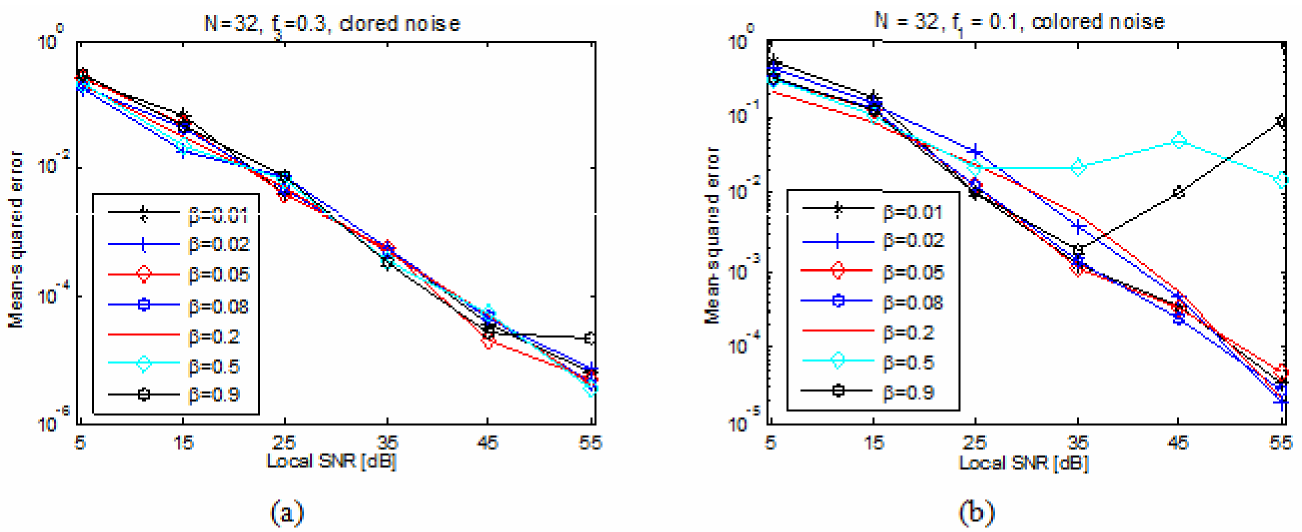

Fig. 1. Mean-squared error versus local SNR for the estimate $\hat{a}_3$ (a) and $\hat{a}_1$ (b) using different value of β

B. ***Estimation performance Versus SNR***

To see the effect the length of the vector of observations on the quality of the estimates, we consider a data sequence of length $N = 32$ and 16. We note here that the limit of Fourier is not checked for f_1 and f_2 which are closely spaced frequencies that cannot be resolved by Fourier-based processing ($|f_2 - f_1| = 0.01 < \frac{1}{N}$).

In the first example we consider that the signal is corrupted by colored noise. fig. 2(a) and (b) show the MSE's of the amplitude estimates of $\hat{a}_3$ and $\hat{a}_1$ respectively for N=32.as we can see that all estimators (LSEK, APESK, MAFI and neuronal) of $\hat{a}_1$ are asymptotically efficient except that the APES method has a great instability when the signal is strongly disturbed. Our estimator gives a better performance and robustness for all the SNR. The results for $\hat{a}_2$ are omitted because they resemble those for $\hat{a}_1$. For $\hat{a}_3$ where the limit of Fourier is checked, all the estimators have the same performance (fig. 2(a)) but our method still the best estimator.

When N=16, the performance degrades for the three methods (LSEK, APESK, MAFI) however our method offer a best performance for the two estimates (fig. 3(a) and (b)). As we can see in fig. 3(b), our method give a remarkable result when the two sinusoids are closely spaced and with a few number of observation.

In the second example, when the signal is corrupted by white noise, it is clear from the fig. 4 and 5, our method give a best performance for the two estimates in the two cases (N=32,16) which confirm the robustness of the neural estimator.

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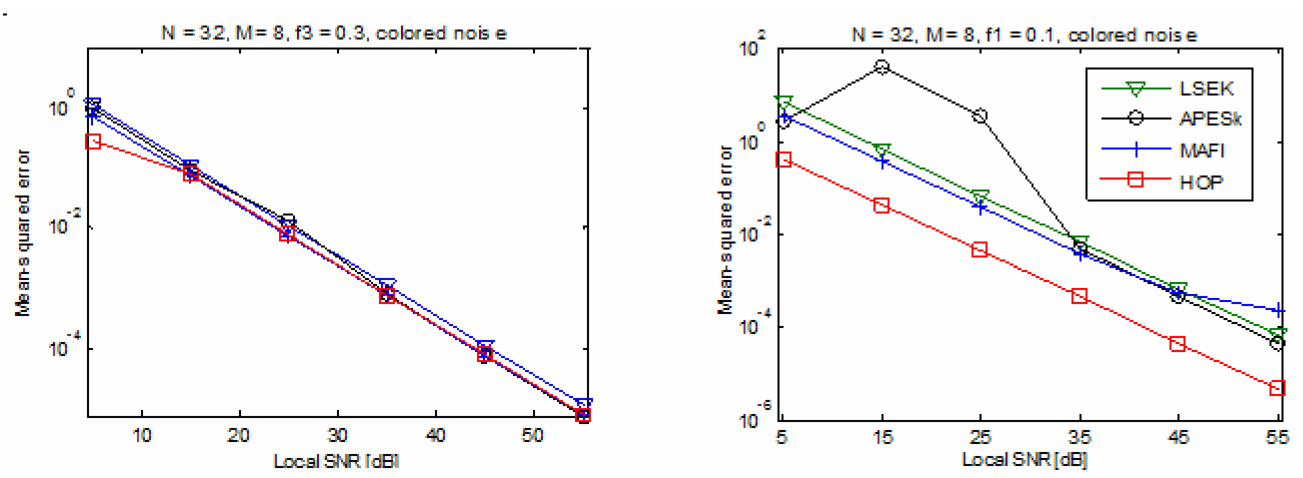
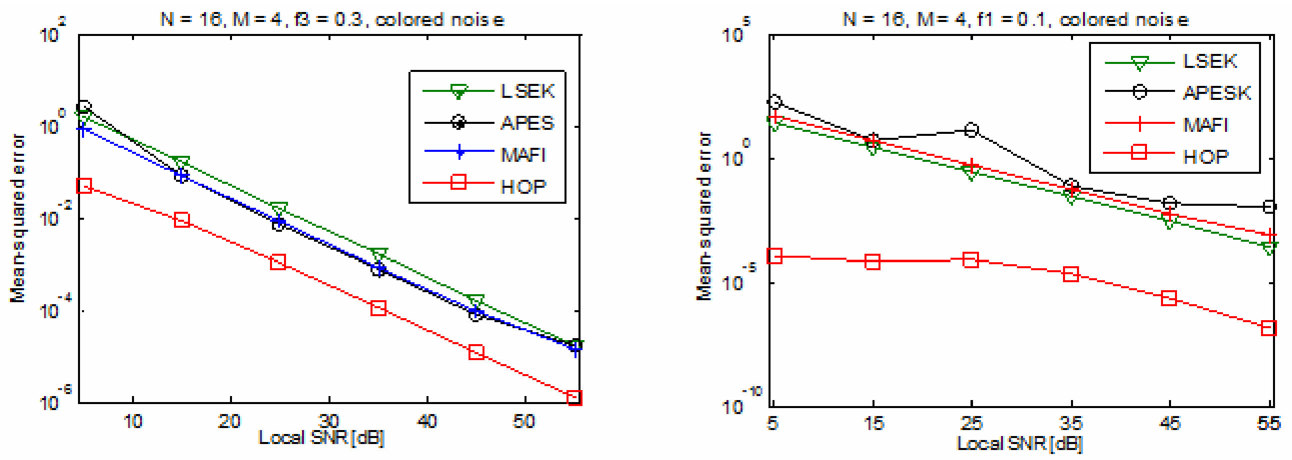


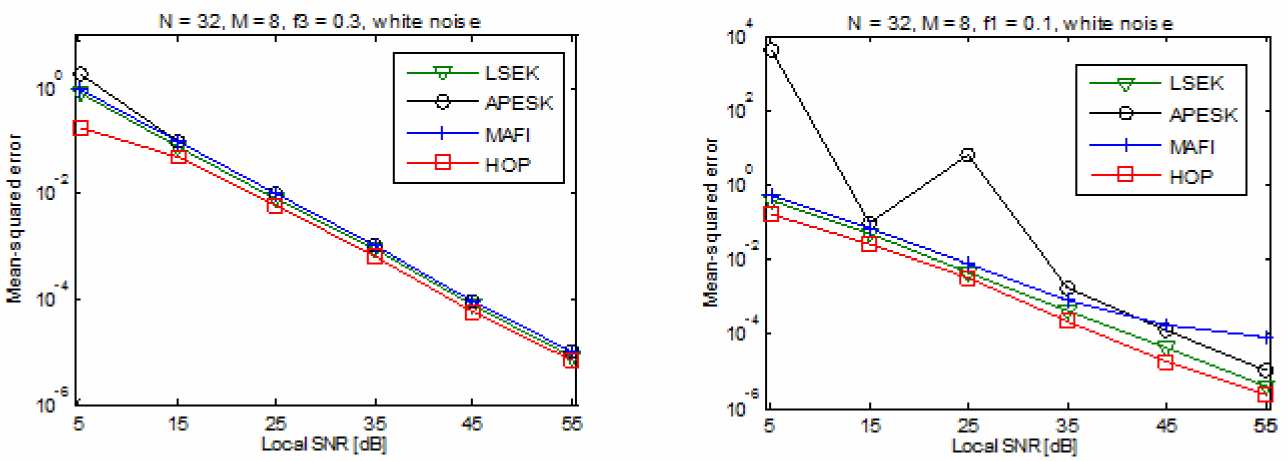
(a) (b)

Fig. 2. Mean-squared error versus local SNR for the estimate $\hat{a}_3$ (a) and $\hat{a}_1$ (b) for $\beta = 0.02$ and $N = 32$ and colored noise



(a) (b)

Fig. 3. Mean-squared error versus local SNR for the estimate $\hat{a}_3$ (a) and $\hat{a}_1$ (b) for $\beta = 0.02$ and $N = 16$ and colored noise



(a) (b)

Fig. 4. Mean-squared error versus local SNR for the estimate $\hat{a}_3$ (a) and $\hat{a}_1$ (b)

for $\beta = 0.02$ and $N = 32$ and white noise

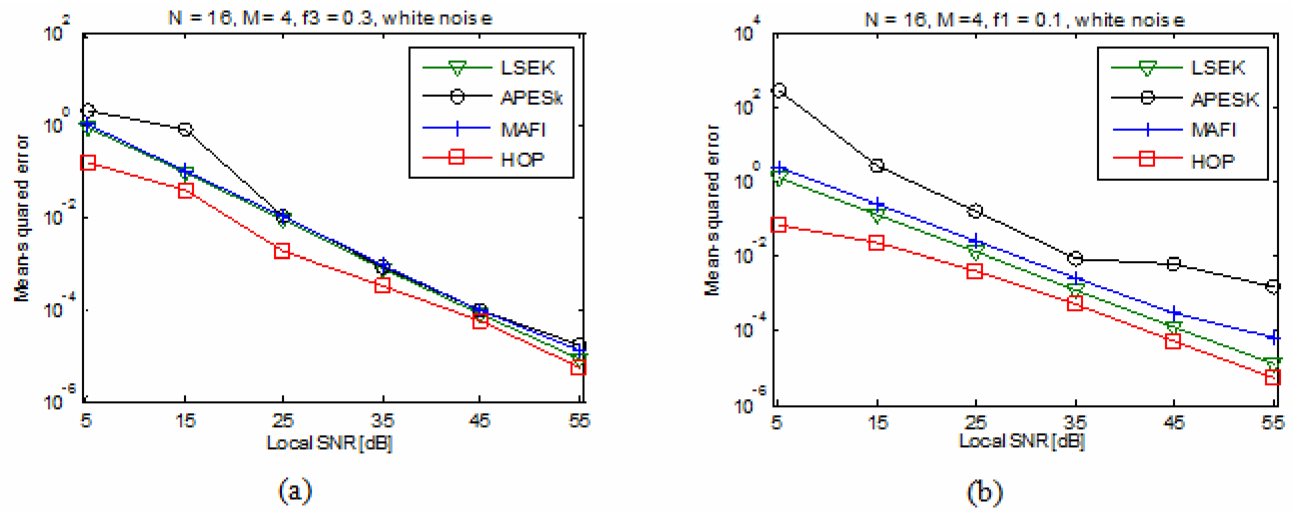


Fig. 5 Mean-squared error versus local SNR for the estimate $\hat{a}_3$ (a) and $\hat{a}_1$ (b) for $\beta = 0.02$ and $N = 32$ and white noise

V. Conclusions

In this paper, the problem of the estimate of the amplitudes of harmonics of the sinusoidal signals corrupted by colored noise was studied by using the Hopfield network. we have shown that by using this emergent technique, we can obtain results that are more better then those cited in [4] and the high efficiently is obtained when the length of data is very small and the sinusoids are closely spaced. As perspective, our method can easily be extended to spectral estimation of two-dimensional (2D) data.

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