

# DEADLOCK INDEX ANALYSIS OF MULTI-LEVEL QUEUE SCHEDULING IN OPERATING SYSTEM USING DATA MODEL APPROACH

D. Shukla<sup>1</sup>, Shweta Ojha<sup>2</sup>

<sup>1</sup>Dept. of Mathematics and Statistics, Dr. H.S. Gour University, Sagar (M.P.), 470003, INDIA

e-mail:diwakarshukla@radiffmail.com

<sup>2</sup>Deptt. of Computer Sc. and Applications, Dr. H.S. Gour University, Sagar (M.P.) ,470003, INDIA

e-mail:meshwetaojha@gmail.com

## **Abstract**

*In the multiprocessor environment the number of jobs arriving to the processor of CPU at a time is very large which causes a long waiting queue. In the processor when any conflict arises due to shared resources or overlap of instructions or any logical error, the deadlock state appears where processing of jobs is blocked completely. As the scheduler has jumps from one job to another in order to perform the processing work the transition mechanism appears. This paper presents a general transition scenario for the functioning of CPU scheduler in the presence of deadlock condition. A data model based Markov chain model is proposed to study the transition phenomenon and a general class of scheduling scheme is designed. Some specific schemes are treated as its particular cases and are compared under the setup of model through a proposed deadlock index measure. Simulation study is performed to evaluate the comparative merits of specific schemes of the class designed with the help of varying values of  $\alpha$  and  $d$ .*

**Keywords:** Process scheduling, Markov chain model, Data model, State of system, Rest State, Deadlock State, Process queue, Multi-level queue scheduling, Transition probability matrix, Deadlock index.

## **1.0 INTRODUCTION**

Operating system plays a major role in managing processes arriving in the form of multiple queues. Arrival of a process is random along with their different categories and types. All these require scheduling algorithms to work over real time environment with special reference to task, control and efficiency (see Stankovic (1984)). The randomization involved in scheduling procedure leads to perform a probabilistic study. Demer et al. (1989) presented an analysis of Fair Queuing algorithm whereas Cobb et al. (1998) picked up fair scheduling of flaros with the consideration of time shifting approach in the area of high-speed networks. Goyal,Guo,Vin (1996) derieved the Hierarchical CPU scheduler in the environment where the multimedia operating system is used. In the similar lines, Hieh and Lam (2003) discussed smart schedulers for multimedia users. A time driven scheduling model is proposed by Janson, Locke and Tokuda (1985) attracted attention of researchers for the model formation over functioning and procedure on operating systems. Katcher et al. (1993) proposed an analysis of fixed priority schedulers and David (1994) given a successful contribution over the study of real time and conventional scheduling with a comparative analysis.

Medhi (1991) given an elaborate study of a variety of stochastic processes and their applications in various fields. Naldi (2002) presented Markov chain model for understanding the internet traffic sharing among various operators in a competitive market. Shukla et al. (2007) derived a Markov chain model for the study of transition probabilities in space division switches in computer networks. Shukla and Jain (2007) have a discussion on the use of Markov chain model for

multilevel queue scheduler in an operating system. Some other useful contributions over detailed description of operating system are due to Silberschatz and Galvin (1999), Stalling (2004) and Tanenbaum and Woodhull (2000).

The transition mechanism of scheduler over queues motivates to think over the system phenomenon in the form of a stochastic study. Naldi (2002), Shukla et.al. (2007), Shukla and Jain (2007) have shown the utility of Markov chain model explaining the system properties. Deriving a motivation from these, a class of scheduling schemes is designed in this paper for performing an integrated approach of efficiency comparison under the assumption of Markov chain model and using a deadlock index measure.

### **1.1 DEADLOCK BASED GENERAL CLASS OF MULTI-LEVEL QUEUE SCHEDULING**

Suppose a multi-level queue scheduling with four queues  $Q_1, Q_2, Q_3, Q_4$  each having large number of processes  $P_j, P'_j, P''_j, P'''_j$  ( $j=1, 2, 3, \dots$ ) respectively waiting for processing. Define  $Q_i$  ( $i=1, 2, 3, 4$ ) like states of scheduling system with two other specific states  $Q_5$  and  $Q_6$ . First four states are related to arrival and input of processes while the last two associate with resting and waiting of scheduler. A quantum is a small pre-defined slot of time given for processing, to waiting processes in queues. Symbol  $n$  denotes the  $n^{\text{th}}$  quantum allotted by the scheduler to a process for execution ( $n=1, 2, 3, 4, \dots$ ). Using above, the structure of given class is:

- (1) All the first four queues  $Q_1, Q_2, Q_3, Q_4$  are allowed to accept a new process with initial probabilities  $pr_1, pr_2, pr_3, pr_4$   $\left( \sum_{i=1}^4 pr_i = 1 \right)$
- (2) Scheduler has a random movement over all states  $Q_s$  ( $s=1, 2, 3, 4, 5, 6$ ) on quantum variation.
- (3) Scheduler starts processing of any  $Q_i$  with probability  $pr_i$  ( $i=1, 2, 3, 4$ ), then picks up the first process of that queue and allot a quantum for processing.
- (4) Process remains with processor until the quantum is over. If it completes within that, then gets out of  $Q_i$ .
- (5) Within quantum, if a process did not complete, scheduler assigns next quantum to the next process of the same queue and so on. The earlier incomplete process moves to next queue  $Q_{i+1}$  ( $(i+1) \leq 4$ ) and waits until next quantum to be allotted for its processing.
- (6) States  $Q_5$  and  $Q_6$  are used as resting the transition system like idle state or deadlock state.
- (7) Specific conditions over resting (or restricting) transition shall be undertaken within using this class.
- (8) Quantum allotment procedure, within  $Q_i$ , by scheduler, continues until  $Q_i$  is empty. The scheduler jumps from any state to any other state at the end of a quantum. When  $Q_1, Q_2, Q_3, Q_4$  are empty, scheduler moves towards states  $Q_5$  or  $Q_6$ . The characters of  $Q_5$  and  $Q_6$  are different and to be defined under the different cases of the system.
- (9) Scheduler attempts processing in queue  $Q_4$  on “first come first serve” basis. Any incomplete process or new process, if appears in  $Q_4$ , remains with  $Q_4$  only until processed completely.

### **2.0 MARKOV CHAIN MODEL**

Let  $\{x^{(n)}, n \geq 1\}$  be a Markov chain where  $x^{(n)}$  denotes the state of the scheduler at the  $n^{\text{th}}$  quantum of time. The state space for  $x^{(n)}$  is  $\{Q_1, Q_2, Q_3, Q_4, Q_5, Q_6\}$  where scheduler  $X$  moves stochastically over these in different quantum. Predefined initial selection probabilities of states are:

$$\begin{aligned}
 P[X^{(0)}=Q_1] &= pr_1 \\
 P[X^{(0)}=Q_2] &= pr_2 \\
 P[X^{(0)}=Q_3] &= pr_3 \\
 P[X^{(0)}=Q_4] &= pr_4 \\
 P[X^{(0)}=Q_5] &= pr_5 \\
 P[X^{(0)}=Q_6] &= pr_6
 \end{aligned}$$

with  $pr_1+pr_2+pr_3+pr_4+pr_5+pr_6=\sum_{i=1}^6 pr_i = 1$ , where  $pr_5=pr_6=0$ .

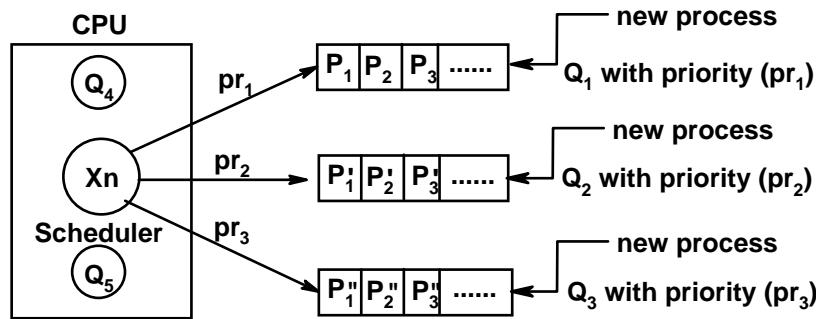


Fig. 2.1 (System Diagram)

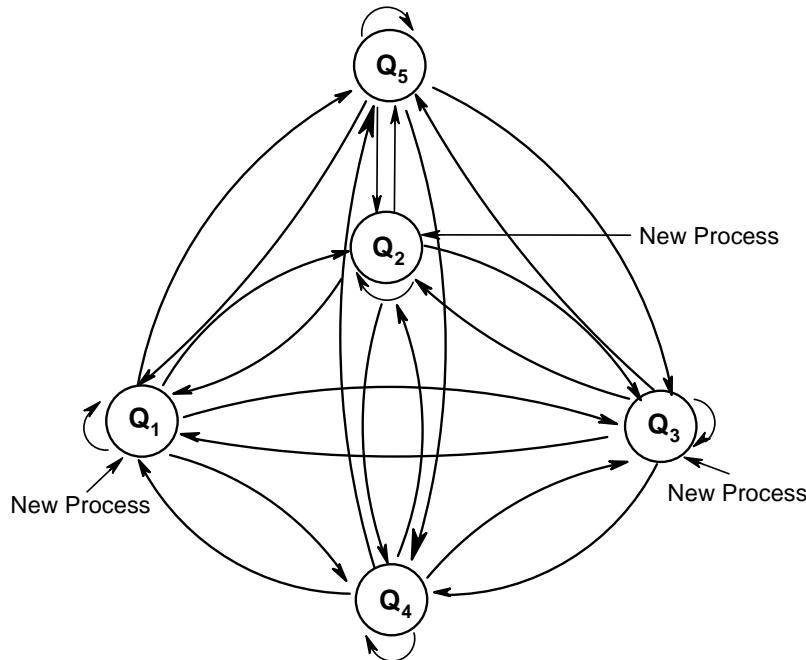


Fig. 2.2 (Unrestricted Transition Diagram)

Let  $s_{ij}$  ( $i,j=1,2,3,4,5,6$ ) be the transition probabilities of scheduler over six states then unit-step transition probability matrix for  $X^{(n)}$  is  $s_{ij}=P[X^{(n)}=Q_i / X^{(n-1)}=Q_j], i+j;$

		$X^{(n)}$					
		Q <sub>1</sub>	Q <sub>2</sub>	Q <sub>3</sub>	Q <sub>4</sub>	Q <sub>5</sub>	Q <sub>6</sub>
$X^{(n-1)}$	Q <sub>1</sub>	S <sub>11</sub>	S <sub>12</sub>	S <sub>13</sub>	S <sub>14</sub>	S <sub>15</sub>	S <sub>16</sub>
	Q <sub>2</sub>	S <sub>21</sub>	S <sub>22</sub>	S <sub>23</sub>	S <sub>24</sub>	S <sub>25</sub>	S <sub>26</sub>
	Q <sub>3</sub>	S <sub>31</sub>	S <sub>32</sub>	S <sub>33</sub>	S <sub>34</sub>	S <sub>35</sub>	S <sub>36</sub>
	Q <sub>4</sub>	S <sub>41</sub>	S <sub>42</sub>	S <sub>43</sub>	S <sub>44</sub>	S <sub>45</sub>	S <sub>46</sub>
	Q <sub>5</sub>	S <sub>51</sub>	S <sub>52</sub>	S <sub>53</sub>	S <sub>54</sub>	S <sub>55</sub>	S <sub>56</sub>
	Q <sub>6</sub>	S <sub>61</sub>	S <sub>62</sub>	S <sub>63</sub>	S <sub>64</sub>	S <sub>65</sub>	S <sub>66</sub>

subject to condition  $s_{15} = \left(1 - \sum_{i=1}^5 s_{1i}\right)$ ,  $s_{25} = \left(1 - \sum_{i=1}^5 s_{2i}\right)$ ,  $s_{35} = \left(1 - \sum_{i=1}^5 s_{3i}\right)$ ,

$$s_{45} = \left(1 - \sum_{i=1}^5 s_{4i}\right), s_{55} = \left(1 - \sum_{i=1}^5 s_{5i}\right), s_{65} = \left(1 - \sum_{i=1}^5 s_{5i}\right) \text{ and } 0 \leq s_{ij} \leq 1.$$

The state probabilities, after first quantum can be obtained by a simple relationship:

$$\begin{aligned} P[X^{(1)} = Q_1] &= p[X^{(0)} = Q_1]p[X^{(1)} = Q_1 / X^{(0)} = Q_1] + p[X^{(0)} = Q_2]p[X^{(1)} = Q_1 / X^{(0)} = Q_2] \\ &\quad + p[X^{(0)} = Q_3]p[X^{(1)} = Q_1 / X^{(0)} = Q_3] + p[X^{(0)} = Q_4]p[X^{(1)} = Q_1 / X^{(0)} = Q_4] \\ &\quad + p[X^{(0)} = Q_5]p[X^{(1)} = Q_1 / X^{(0)} = Q_5] + p[X^{(0)} = Q_6]p[X^{(1)} = Q_1 / X^{(0)} = Q_6] \\ &= \sum_{i=1}^6 pr_i s_{i1} \end{aligned}$$

$$P[X^{(1)} = Q_2] = \sum_{i=1}^6 pr_i s_{i2}$$

$$P[X^{(1)} = Q_3] = \sum_{i=1}^6 pr_i s_{i3}$$

$$P[X^{(1)} = Q_4] = \sum_{i=1}^6 pr_i s_{i4}$$

$$P[X^{(1)} = Q_5] = \sum_{i=1}^6 pr_i s_{i5}$$

$$P[X^{(1)} = Q_6] = \sum_{i=1}^6 pr_i s_{i6}$$

Similarly, the state probabilities after the second quantum could be obtained by simple relationship:

$$\left. \begin{aligned}
 P[X^{(2)} = Q_1] &= \sum_{j=1}^6 (\sum_{i=1}^6 pr_i s_{ij}) s_{j1} \\
 P[X^{(2)} = Q_2] &= \sum_{j=1}^6 (\sum_{i=1}^6 pr_i s_{ij}) s_{j2} \\
 P[X^{(2)} = Q_3] &= \sum_{j=1}^6 (\sum_{i=1}^6 pr_i s_{ij}) s_{j3} \\
 P[X^{(2)} = Q_4] &= \sum_{j=1}^6 (\sum_{i=1}^6 pr_i s_{ij}) s_{j4} \\
 P[X^{(2)} = Q_5] &= \sum_{j=1}^6 (\sum_{i=1}^6 pr_i s_{ij}) s_{j5} \\
 P[X^{(2)} = Q_6] &= \sum_{j=1}^6 (\sum_{i=1}^6 pr_i s_{ij}) s_{j6}
 \end{aligned} \right\} \quad (2.4)$$

**Remark 2.1** In the similar fashion, for n quantum, the generalized expression is

$$\left. \begin{aligned}
 P[X^{(n)} = Q_1] &= \sum_{m=1}^6 \dots \sum_{t=1}^6 \sum_{k=1}^6 \left\{ \sum_{j=1}^6 \left( \sum_{i=1}^6 pr_i s_{ij} \right) s_{jk} \right\} s_{kt} \dots s_{m1} \\
 P[X^{(n)} = Q_2] &= \sum_{m=1}^6 \dots \sum_{t=1}^6 \sum_{k=1}^6 \left\{ \sum_{j=1}^6 \left( \sum_{i=1}^6 pr_i s_{ij} \right) s_{jk} \right\} s_{kt} \dots s_{m2} \\
 P[X^{(n)} = Q_3] &= \sum_{m=1}^6 \dots \sum_{t=1}^6 \sum_{k=1}^6 \left\{ \sum_{j=1}^6 \left( \sum_{i=1}^6 pr_i s_{ij} \right) s_{jk} \right\} s_{kt} \dots s_{m3} \\
 P[X^{(n)} = Q_4] &= \sum_{m=1}^6 \dots \sum_{t=1}^6 \sum_{k=1}^6 \left\{ \sum_{j=1}^6 \left( \sum_{i=1}^6 pr_i s_{ij} \right) s_{jk} \right\} s_{kt} \dots s_{m4} \\
 P[X^{(n)} = Q_5] &= \sum_{m=1}^6 \dots \sum_{t=1}^6 \sum_{k=1}^6 \left\{ \sum_{j=1}^6 \left( \sum_{i=1}^6 pr_i s_{ij} \right) s_{jk} \right\} s_{kt} \dots s_{m5} \\
 P[X^{(n)} = Q_6] &= \sum_{m=1}^6 \dots \sum_{t=1}^6 \sum_{k=1}^6 \left\{ \sum_{j=1}^6 \left( \sum_{i=1}^6 pr_i s_{ij} \right) s_{jk} \right\} s_{kt} \dots s_{m6}
 \end{aligned} \right\} \quad (2.5)$$

### 3.0 SCHEDULING SCHEME IN GENERAL CLASS

By imposing restrictions and conditions over ways and procedures, one can generate various scheduling schemes from the generalized version of class stated in section 1.1.

#### 3.1 SCHEME- I:

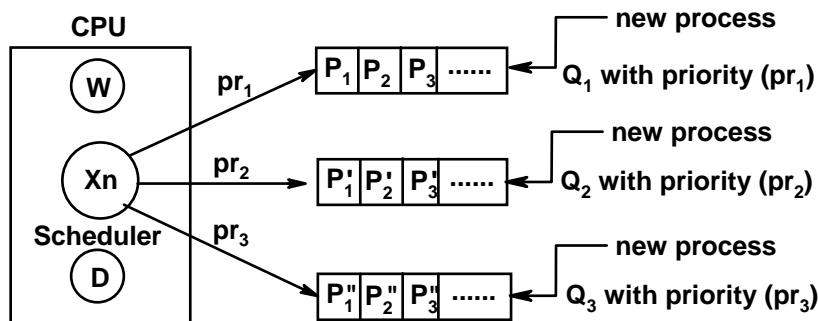
(a) Define  $Q_5$  and  $Q_6$  specifically as

$$\begin{aligned}
 Q_5 &= D \text{ (waiting state)} \\
 Q_6 &= W \text{ (deadlock state)}
 \end{aligned}$$

The deadlock state  $D$  is where the return back transition is not possible whereas the waiting state is one where system can reach in any quantum during processing to a job but can also transit back to the same queue in any quantum.

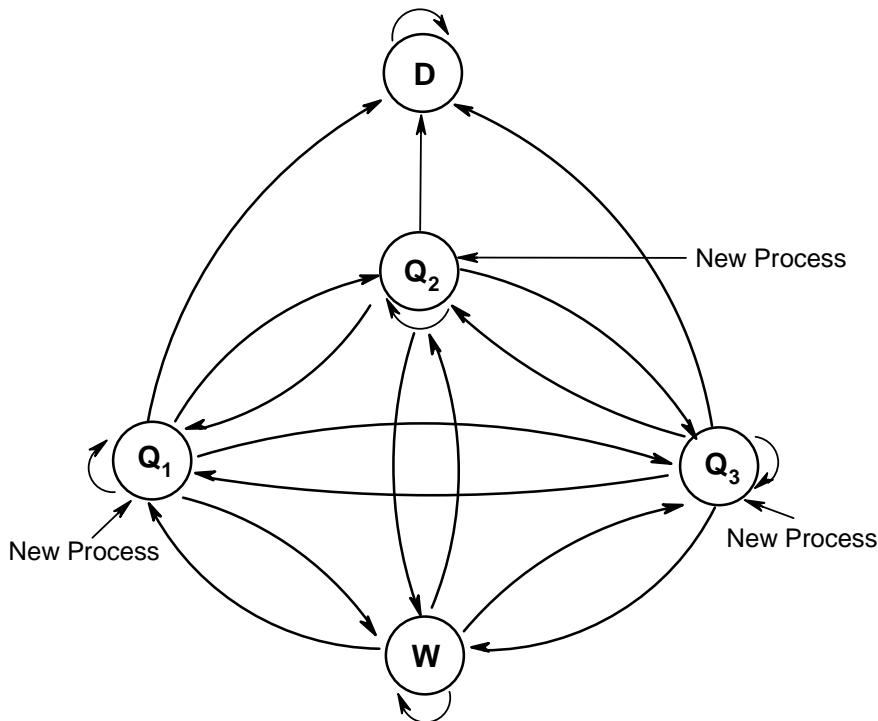
- (b) A new process enters to  $Q_1, Q_2, Q_3$  and  $Q_4$  queues.
- (c) Transition between  $W$  and  $D$  is restricted.
- (d) The  $D$  is absorbing state.

The diagrammatic form of this scheme is in fig 3.1.



**Fig. 3.1.1 (System Diagram of Scheme-I)**

The transition diagram for Scheme-I is in fig 3.1.2.



**Fig. 3.1.2 (Transition Diagram of Scheme-I)**

The unit-step transition probability matrix for  $X^{(n)}$  under scheme I is

	$X^{(n)}$					
	Q <sub>1</sub>	Q <sub>2</sub>	Q <sub>3</sub>	Q <sub>4</sub>	W	D
Q <sub>1</sub>	S <sub>11</sub>	S <sub>12</sub>	S <sub>13</sub>	S <sub>14</sub>	S <sub>15</sub>	S <sub>16</sub>
Q <sub>2</sub>	S <sub>21</sub>	S <sub>22</sub>	S <sub>23</sub>	S <sub>24</sub>	S <sub>25</sub>	S <sub>26</sub>
Q <sub>3</sub>	S <sub>31</sub>	S <sub>32</sub>	S <sub>33</sub>	S <sub>34</sub>	S <sub>35</sub>	S <sub>36</sub>
Q <sub>4</sub>	S <sub>41</sub>	S <sub>42</sub>	S <sub>43</sub>	S <sub>44</sub>	S <sub>45</sub>	S <sub>46</sub>
W	S <sub>51</sub>	S <sub>52</sub>	S <sub>53</sub>	S <sub>54</sub>	S <sub>55</sub>	S <sub>56</sub>
D	0	0	0	0	0	1

(3.1.1)

Define an indicator function

$$l_{ij} = \begin{cases} 0, & \text{if } (i=6, j=1, 2, 3, 4, 5) \\ 1, & \text{otherwise.} \end{cases}$$

The initial probabilities are

$$\left. \begin{aligned} P[X^{(0)}=Q_1] &= pr_1 \\ P[X^{(0)}=Q_2] &= pr_2 \\ P[X^{(0)}=Q_3] &= pr_3 \\ P[X^{(0)}=Q_4] &= pr_4 \\ P[X^{(0)}=W] &= 0 \\ P[X^{(0)}=D] &= 0 \end{aligned} \right\} \quad (3.1.2)$$

**Remark 3.1.1** The state probabilities, over the imposed condition, after the first quantum are:

$$\begin{aligned} P[X^{(1)}=Q_1] &= pr_1 s_{11} l_{11} + pr_2 s_{21} l_{21} + pr_3 s_{31} l_{31} + pr_4 s_{41} l_{41} \\ &= \sum_{i=1}^4 pr_i s_{i1} l_{i1} \\ P[X^{(1)}=Q_2] &= \sum_{i=1}^4 pr_i s_{i2} l_{i2} \\ P[X^{(1)}=Q_3] &= \sum_{i=1}^4 pr_i s_{i3} l_{i3} \\ P[X^{(1)}=Q_4] &= \sum_{i=1}^4 pr_i s_{i4} l_{i4} \\ P[X^{(1)}=W] &= \sum_{i=1}^4 pr_i s_{i5} l_{i5} \\ P[X^{(1)}=D] &= \sum_{i=1}^4 pr_i s_{i6} l_{i6} \end{aligned} \quad (3.1.3)$$

**Remark 3.1.2** The state probabilities after the second quantum are:

$$\begin{aligned}
 P[X^{(2)}=Q_1] &= \sum_{j=1}^6 \left[ \sum_{i=1}^4 pr_i s_{ij} l_{ij} \right] s_{j1} l_{j1} \\
 P[X^{(2)}=Q_2] &= \sum_{j=1}^6 \left[ \sum_{i=1}^4 pr_i s_{ij} l_{ij} \right] s_{j2} l_{j2} \\
 P[X^{(2)}=Q_3] &= \sum_{j=1}^6 \left[ \sum_{i=1}^4 pr_i s_{ij} l_{ij} \right] s_{j3} l_{j3} \\
 P[X^{(2)}=Q_4] &= \sum_{j=1}^6 \left[ \sum_{i=1}^4 pr_i s_{ij} l_{ij} \right] s_{j4} l_{j4} \\
 P[X^{(2)}=W] &= \sum_{j=1}^6 \left[ \sum_{i=1}^4 pr_i s_{ij} l_{ij} \right] s_{j5} l_{j5} \\
 P[X^{(2)}=D] &= \sum_{j=1}^6 \left[ \sum_{i=1}^4 pr_i s_{ij} l_{ij} \right] s_{j6} l_{j6}
 \end{aligned} \tag{3.1.4}$$

**Remark 3.1.3** For n quantum the generalized expression is:

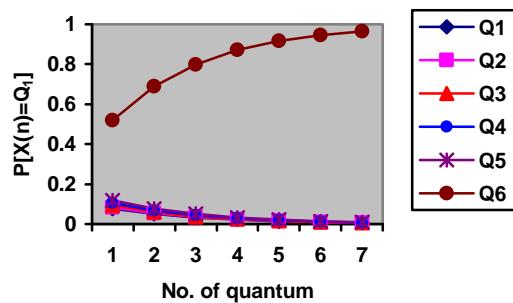
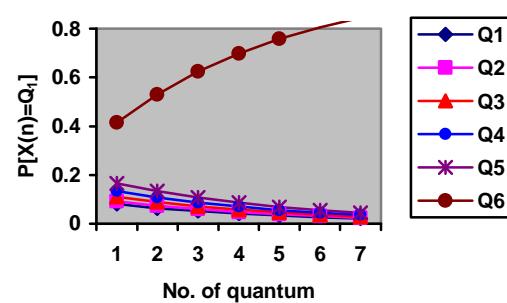
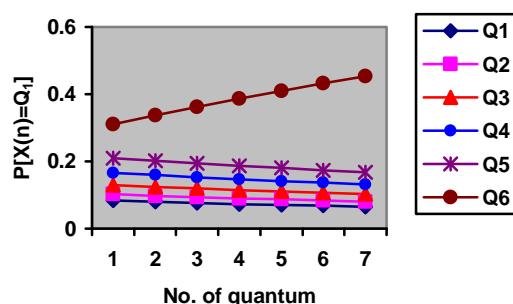
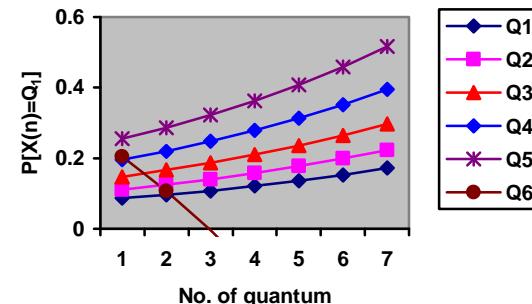
$$\begin{aligned}
 P[X^{(n)}=Q_1] &= \sum_{m=1}^6 \dots \sum_{t=1}^6 \left[ \sum_{k=1}^6 \left\{ \sum_{j=1}^6 \left( \sum_{i=1}^4 pr_i s_{ij} l_{ij} \right) s_{jk} l_{jk} \right\} \right] s_{kt} l_{kt} \dots s_{m1} l_{m1} \\
 P[X^{(n)}=Q_2] &= \sum_{m=1}^6 \dots \sum_{t=1}^6 \left[ \sum_{k=1}^6 \left\{ \sum_{j=1}^6 \left( \sum_{i=1}^4 pr_i s_{ij} l_{ij} \right) s_{jk} l_{jk} \right\} \right] s_{kt} l_{kt} \dots s_{m2} l_{m2} \\
 P[X^{(n)}=Q_3] &= \sum_{m=1}^6 \dots \sum_{t=1}^6 \left[ \sum_{k=1}^6 \left\{ \sum_{j=1}^6 \left( \sum_{i=1}^4 pr_i s_{ij} l_{ij} \right) s_{jk} l_{jk} \right\} \right] s_{kt} l_{kt} \dots s_{m3} l_{m3} \\
 P[X^{(n)}=Q_4] &= \sum_{m=1}^6 \dots \sum_{t=1}^6 \left[ \sum_{k=1}^6 \left\{ \sum_{j=1}^6 \left( \sum_{i=1}^4 pr_i s_{ij} l_{ij} \right) s_{jk} l_{jk} \right\} \right] s_{kt} l_{kt} \dots s_{m4} l_{m4} \\
 P[X^{(n)}=W] &= \sum_{m=1}^6 \dots \sum_{t=1}^6 \left[ \sum_{k=1}^6 \left\{ \sum_{j=1}^6 \left( \sum_{i=1}^4 pr_i s_{ij} l_{ij} \right) s_{jk} l_{jk} \right\} \right] s_{kt} l_{kt} \dots s_{m5} l_{m5} \\
 P[X^{(n)}=D] &= \sum_{m=1}^6 \dots \sum_{t=1}^6 \left[ \sum_{k=1}^6 \left\{ \sum_{j=1}^6 \left( \sum_{i=1}^4 pr_i s_{ij} l_{ij} \right) s_{jk} l_{jk} \right\} \right] s_{kt} l_{kt} \dots s_{m6} l_{m6}
 \end{aligned} \tag{3.1.5}$$

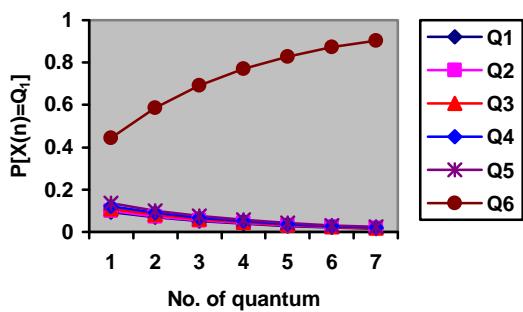
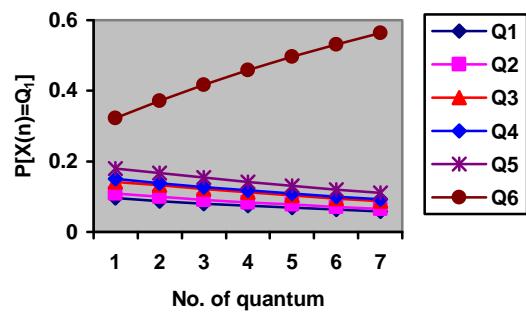
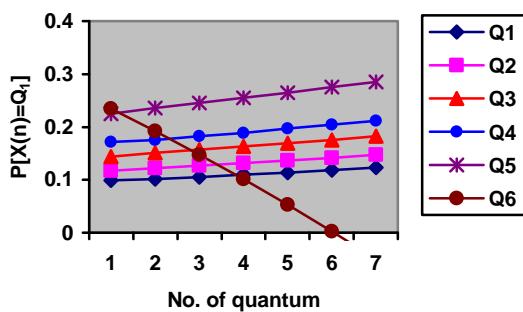
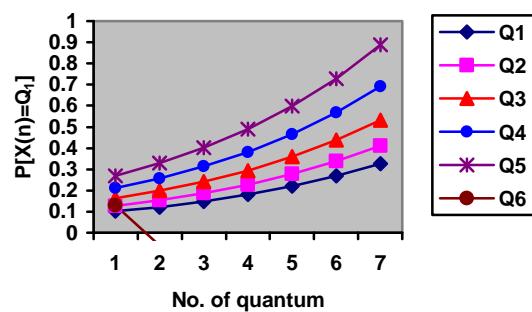
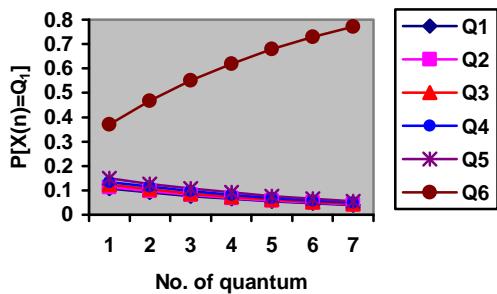
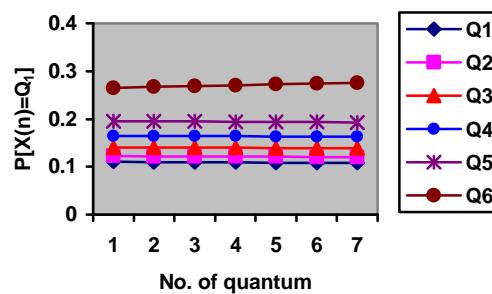
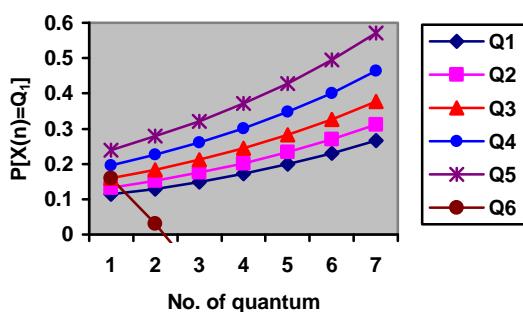
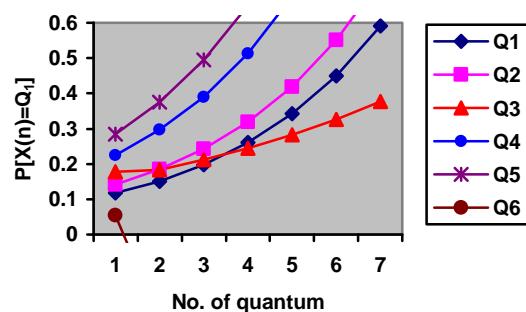
#### 4.0 NUMERICAL ILLUSTRATIONS USING DATA SET CREATED WITH THE HELP OF MATHEMATICAL MODEL

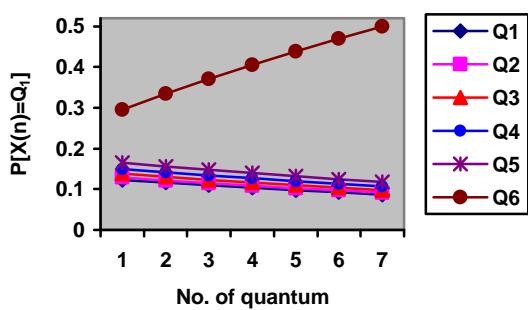
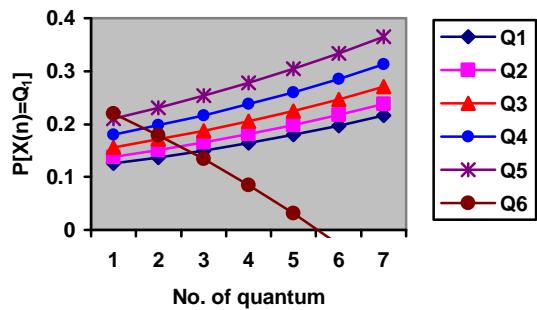
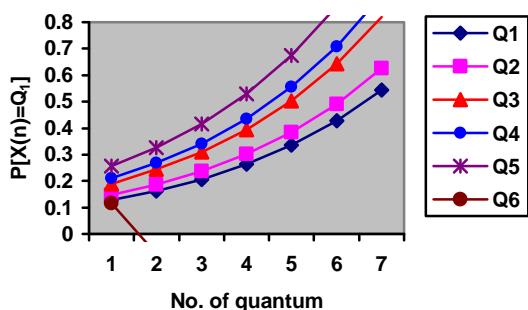
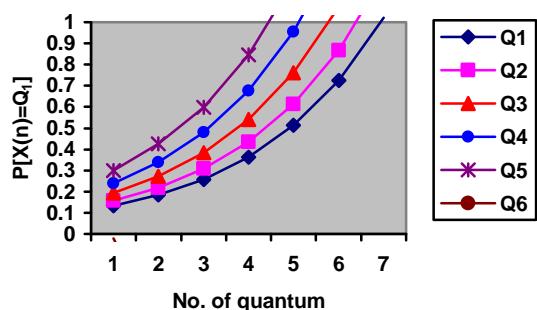
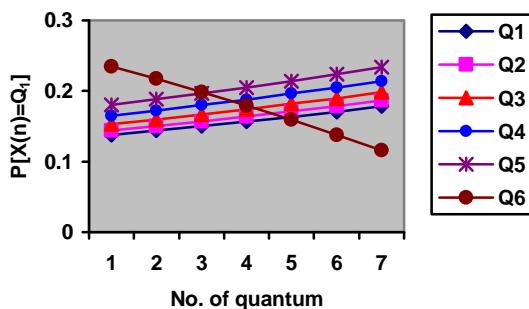
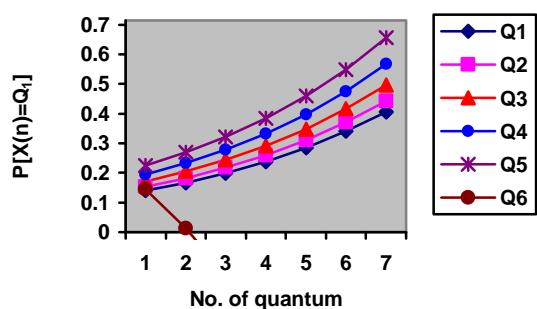
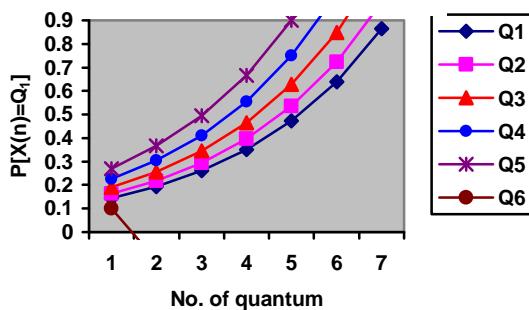
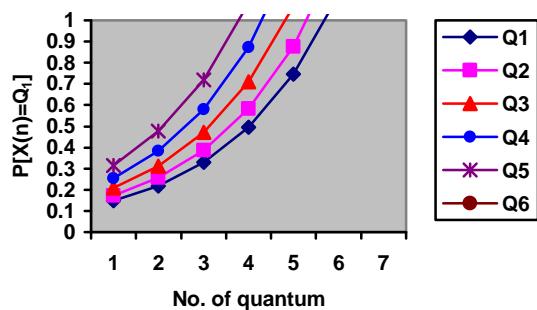
The basic and scientific approach for data analysis related to state transition probabilities is managed by a data model with two parameters  $\alpha$  and  $d$ . The  $i$  stands for number of queues.

	$Q_1$	$Q_2$	$Q_3$	$X^{(n)}$	$Q_4$	$Q_5$	$Q_6$
$Q_1$	$\alpha$	$\alpha + d.i$	$\alpha + 2d.i$	$\alpha + 3d.i$	$\alpha + 4d.i$	$1 - (5\alpha + 10d.i)$	
$Q_2$	$\alpha + d.i$	$\alpha + 2d.i$	$\alpha + 3d.i$	$\alpha + 4d.i$	$\alpha + 5d.i$	$1 - (5\alpha + 15d.i)$	
$Q_3$	$\alpha + 2d.i$	$\alpha + 3d.i$	$\alpha + 4d.i$	$\alpha + 5d.i$	$\alpha + 6d.i$	$1 - (5\alpha + 20d.i)$	
$Q_4$	$\alpha + 3d.i$	$\alpha + 4d.i$	$\alpha + 5d.i$	$\alpha + 6d.i$	$\alpha + 7d.i$	$1 - (5\alpha + 25d.i)$	
$Q_5$	$\alpha + 4d.i$	$\alpha + 5d.i$	$\alpha + 6d.i$	$\alpha + 7d.i$	$\alpha + 8d.i$	$1 - (5\alpha + 30d.i)$	
$Q_6$	$\alpha + 5d.i$	$\alpha + 6d.i$	$\alpha + 7d.i$	$\alpha + 8d.i$	$\alpha + 9d.i$	$1 - (5\alpha + 35d.i)$	

#### Case I with $\alpha=0.1$

Fig1.1 ( $\alpha=0.1, d=0.002$ )Fig1.2 ( $\alpha=0.1, d=0.004$ )Fig1.3 ( $\alpha=0.1, d=0.006$ )Fig1.4 ( $\alpha=0.1, d=0.008$ )

**Case II with  $\alpha=0.12$** Fig. 2.1 ( $\alpha=0.12$ ,  $d=0.002$ )Fig. 2.2 ( $\alpha=0.12$ ,  $d=0.004$ )Fig. 2.3 ( $\alpha=0.12$ ,  $d=0.006$ )Fig. 2.4 ( $\alpha=0.12$ ,  $d=0.008$ )**Case II with  $\alpha=0.14$** Fig. 3.1 ( $\alpha=0.14$ ,  $d=0.002$ )Fig. 3.2 ( $\alpha=0.14$ ,  $d=0.004$ )Fig. 3.3 ( $\alpha=0.14$ ,  $d=0.006$ )Fig. 3.4 ( $\alpha=0.14$ ,  $d=0.008$ )

**Case II with  $\alpha=0.16$** Fig. 4.1 ( $\alpha=0.16, d=0.002$ )Fig. 4.2 ( $\alpha=0.16, d=0.004$ )Fig. 4.3 ( $\alpha=0.16, d=0.006$ )Fig. 4.4 ( $\alpha=0.16, d=0.008$ )**Case II with  $\alpha=0.18$** Fig. 5.1 ( $\alpha=0.18, d=0.002$ )Fig. 5.2 ( $\alpha=0.18, d=0.004$ )Fig. 5.3 ( $\alpha=0.18, d=0.006$ )Fig. 5.4 ( $\alpha=0.18, d=0.008$ )

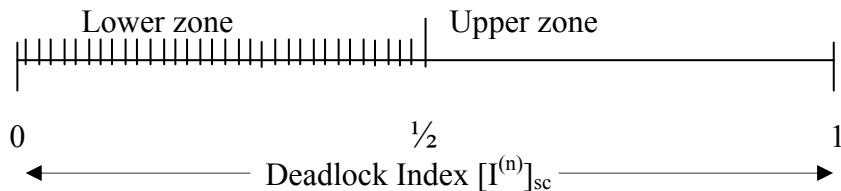
## 5.0 WAITING INDEX ANALYSIS

For the state  $Q_6=D$ , a deadlock index  $[I^{(n)}]$  is defined below:

$$[I^{(n)}]_{sc} = P[x^{(n)}=Q_4] / [P[x^{(n)}=Q_4] + P[x^{(n)}=Q_5]] \quad (5.1)$$

where 'sc' denotes different scheduling schemes [ $sc= I, II, III, IV$ ].

The (5.1) is a relative measure of scheduler probability towards the chances of being on the deadlock state. The  $Q_5 (=W)$  is like an idle state where scheduler reaches in most of the time when no process in the queue left or otherwise and  $Q_6 (=D)$  is an absorbing state where deadlock of the transition system occurs. The deadlock index measures the intensity of chances towards deadlock transition by the scheduler under specified values of  $\alpha$  and  $d$ . As special, if  $P[x^{(n)}=Q_5]=0$  then  $[I^{(n)}]_{sc}=1$  which shows the scheduling scheme highly suffers from deadlock. If  $P[x^{(n)}=Q_6]=0$  then  $[I^{(n)}]_{sc}=0$  which reveals the high efficiency of the scheme because the scheme is independent of the deadlock fear. Therefore,  $0 \leq [I^{(n)}]_{sc} \leq 1$  and  $P[x^{(n)}=Q_6]=P[x^{(n)}=Q_5]=1/2$  provides index  $[I^{(n)}]_{sc}=1/2$ . The  $0 \leq [I^{(n)}]_{sc} \leq 1/2$  is the lower zone of index while  $1/2 < [I^{(n)}]_{sc} \leq 1$  is the upper zone as shown in fig 5.1.



**Fig 5.1.** The lower zone reflects for better operation and efficiency of scheduling scheme.

### 5.1 Calculation of Index

#### CASE 1

**ConditionI** ( $\text{Alpha}=0.10, d=0.002$ )

quantum	$P[x^{(n)}=Q_5]$	$P[x^{(n)}=Q_6]$	$P[I^{(n)}]_I$
<b>n=1</b>	0.120000	0.520000	0.1875
<b>n=2</b>	0.077850	0.689650	0.101433
<b>n=3</b>	0.050349	0.799297	0.059259
<b>n=4</b>	0.032561	0.870205	0.036068
<b>n=5</b>	0.021057	0.916061	0.02247
<b>n=6</b>	0.013618	0.945717	0.014195
<b>n=7</b>	0.008807	0.964895	0.009045

**ConditionII** ( $\text{Alpha}=0.10, d=0.004$ )

quantum	$P[x^{(n)}=Q_5]$	$P[x^{(n)}=Q_6]$	$P[I^{(n)}]_{II}$
<b>n=1</b>	0.165000	0.415000	0.284483
<b>n=2</b>	0.132900	0.531100	0.200151
<b>n=3</b>	0.106602	0.623926	0.145925
<b>n=4</b>	0.085500	0.698372	0.109074
<b>n=5</b>	0.068575	0.758080	0.082955
<b>n=6</b>	0.055000	0.805969	0.063882
<b>n=7</b>	0.044113	0.844378	0.049649

**ConditionIII** ( $\text{Alpha}=0.10, d=0.006$ )

quantum	$P[x^{(n)}=Q_5]$	$P[x^{(n)}=Q_6]$	$P[I^{(n)}]_{III}$
<b>n=1</b>	0.210000	0.310000	0.403846
<b>n=2</b>	0.202650	0.336850	0.375626
<b>n=3</b>	0.194943	0.362119	0.349948
<b>n=4</b>	0.187518	0.386415	0.326725
<b>n=5</b>	0.180376	0.409786	0.305638
<b>n=6</b>	0.173505	0.432267	0.28642
<b>n=7</b>	0.166897	0.453891	0.268847

**ConditionIV** ( $\text{Alpha}=0.10, d=0.008$ )

quantum	$P[x^{(n)}=Q_5]$	$P[x^{(n)}=Q_6]$	$P[I^{(n)}]_{IV}$
<b>n=1</b>	0.255000	0.205000	0.554348
<b>n=2</b>	0.287100	0.106900	0.72868
<b>n=3</b>	0.322806	-0.004142	1.012998
<b>n=4</b>	0.362946	-0.129003	1.551429
<b>n=5</b>	0.408077	-0.269391	2.942453
<b>n=6</b>	0.458820	-0.427235	14.52652
<b>n=7</b>	0.515872	-0.604707	-5.80708

**CASE 2****ConditionI** (Alpha=0.12, d=0.002)

<b>quantum</b>	P[x <sup>(n)</sup> =Q <sub>5</sub> ]	P[x <sup>(n)</sup> =Q <sub>6</sub> ]	P[I <sup>(n)</sup> ] <sub>I</sub>
<b>n=1</b>	0.135000	0.445000	0.232759
<b>n=2</b>	0.100950	0.586150	0.146922
<b>n=3</b>	0.075291	0.691357	0.098208
<b>n=4</b>	0.056151	0.769818	0.067982
<b>n=5</b>	0.041877	0.828333	0.048123
<b>n=6</b>	0.031231	0.871973	0.034578
<b>n=7</b>	0.023292	0.904519	0.025104

**ConditionII** (Alpha=0.12, d=0.004)

<b>quantum</b>	P[x <sup>(n)</sup> =Q <sub>5</sub> ]	P[x <sup>(n)</sup> =Q <sub>6</sub> ]	P[I <sup>(n)</sup> ] <sub>II</sub>
<b>n=1</b>	0.180000	0.322000	0.358566
<b>n=2</b>	0.166200	0.371200	0.309267
<b>n=3</b>	0.153276	0.416278	0.269116
<b>n=4</b>	0.141354	0.457844	0.235905
<b>n=5</b>	0.130359	0.496178	0.208063
<b>n=6</b>	0.120219	0.531529	0.184456
<b>n=7</b>	0.110868	0.564131	0.164249

**ConditionIII** (Alpha=0.12, d=0.006)

<b>quantum</b>	P[x <sup>(n)</sup> =Q <sub>5</sub> ]	P[x <sup>(n)</sup> =Q <sub>6</sub> ]	P[I <sup>(n)</sup> ] <sub>III</sub>
<b>n=1</b>	0.225000	0.235000	0.48913
<b>n=2</b>	0.235980	0.192340	0.550943
<b>n=3</b>	0.245219	0.147623	0.624218
<b>n=4</b>	0.254764	0.101163	0.715776
<b>n=5</b>	0.264679	0.052895	0.83344
<b>n=6</b>	0.274980	0.002748	0.990105
<b>n=7</b>	0.285682	-0.049350	1.208816

**ConditionIV** (Alpha=0.12, d=0.008)

<b>quantum</b>	P[x <sup>(n)</sup> =Q <sub>5</sub> ]	P[x <sup>(n)</sup> =Q <sub>6</sub> ]	P[I <sup>(n)</sup> ] <sub>IV</sub>
<b>n=1</b>	0.270000	0.130000	0.675
<b>n=2</b>	0.330000	-0.059600	1.220414
<b>n=3</b>	0.402288	-0.291632	3.635483
<b>n=4</b>	0.490389	-0.574498	-5.8304
<b>n=5</b>	0.597784	-0.919312	-1.8592
<b>n=6</b>	0.728698	-1.339639	-1.19275
<b>n=7</b>	0.888282	-1.852018	-0.92171

**CASE 3****ConditionI** (Alpha=0.14, d=0.002)

<b>quantum</b>	P[x <sup>(n)</sup> =Q <sub>5</sub> ]	P[x <sup>(n)</sup> =Q <sub>6</sub> ]	P[I <sup>(n)</sup> ] <sub>I</sub>
<b>n=1</b>	0.150000	0.370000	0.288462
<b>n=2</b>	0.127050	0.467650	0.213637
<b>n=3</b>	0.107373	0.550117	0.163307
<b>n=4</b>	0.090740	0.619808	0.127704
<b>n=5</b>	0.076683	0.678704	0.101515
<b>n=6</b>	0.064804	0.728476	0.081691
<b>n=7</b>	0.054766	0.770537	0.066359

**ConditionII** (Alpha=0.14, d=0.004)

<b>quantum</b>	P[x <sup>(n)</sup> =Q <sub>5</sub> ]	P[x <sup>(n)</sup> =Q <sub>6</sub> ]	P[I <sup>(n)</sup> ] <sub>II</sub>
<b>n=1</b>	0.195000	0.265000	0.423913
<b>n=2</b>	0.195300	0.267100	0.422362
<b>n=3</b>	0.194838	0.268906	0.420141
<b>n=4</b>	0.194360	0.270701	0.417924
<b>n=5</b>	0.193883	0.272492	0.415723
<b>n=6</b>	0.193407	0.274278	0.413541
<b>n=7</b>	0.192932	0.276059	0.411377

**ConditionIII** (Alpha=0.14, d=0.006)

<b>quantum</b>	P[x <sup>(n)</sup> =Q <sub>5</sub> ]	P[x <sup>(n)</sup> =Q <sub>6</sub> ]	P[I <sup>(n)</sup> ] <sub>I</sub>
<b>n=1</b>	0.240000	0.160000	0.6
<b>n=2</b>	0.278250	0.030850	0.900194
<b>n=3</b>	0.321279	-0.118901	1.587519
<b>n=4</b>	0.370929	-0.291810	4.688242
<b>n=5</b>	0.428250	-0.491440	-6.777118
<b>n=6</b>	0.494430	-0.721920	-2.17341
<b>n=7</b>	0.570837	-0.988018	-1.36832

**ConditionIV** (Alpha=0.14, d=0.008)

<b>quantum</b>	P[x <sup>(n)</sup> =Q <sub>5</sub> ]	P[x <sup>(n)</sup> =Q <sub>6</sub> ]	P[I <sup>(n)</sup> ] <sub>II</sub>
<b>n=1</b>	0.285000	0.055000	0.838235
<b>n=2</b>	0.375900	-0.241100	2.788576
<b>n=3</b>	0.494130	-0.631322	-3.60174
<b>n=4</b>	0.649504	-1.144271	-1.31275
<b>n=5</b>	0.853733	-1.818510	-0.8849
<b>n=6</b>	1.122178	-2.704756	-0.70908
<b>n=7</b>	1.475034	-3.869671	-0.61597

**CASE 4****ConditionI** (Alpha=0.16, d=0.002)

<b>quantum</b>	P[x <sup>(n)</sup> =Q <sub>5</sub> ]	P[x <sup>(n)</sup> =Q <sub>6</sub> ]	P[I <sup>(n)</sup> ] <sub>I</sub>
<b>n=1</b>	0.165000	0.295000	0.358696
<b>n=2</b>	0.156150	0.334150	0.318478
<b>n=3</b>	0.147495	0.371077	0.284425
<b>n=4</b>	0.139315	0.405955	0.255497
<b>n=5</b>	0.131589	0.438899	0.23066
<b>n=6</b>	0.124292	0.470016	0.209137
<b>n=7</b>	0.117399	0.499408	0.190333

**ConditionII** (Alpha=0.16, d=0.004)

<b>quantum</b>	P[x <sup>(n)</sup> =Q <sub>5</sub> ]	P[x <sup>(n)</sup> =Q <sub>6</sub> ]	P[I <sup>(n)</sup> ] <sub>I</sub>
<b>n=1</b>	0.210000	0.220000	0.488372
<b>n=2</b>	0.231000	0.179200	0.56314
<b>n=3</b>	0.253176	0.134296	0.653405
<b>n=4</b>	0.277459	0.085081	0.76532
<b>n=5</b>	0.304070	0.031145	0.907089
<b>n=6</b>	0.333233	-0.027964	1.091604
<b>n=7</b>	0.365194	-0.092742	1.340398

**ConditionIII** (Alpha=0.16, d=0.006)

<b>quantum</b>	P[x <sup>(n)</sup> =Q <sub>5</sub> ]	P[x <sup>(n)</sup> =Q <sub>6</sub> ]	P[I <sup>(n)</sup> ] <sub>I</sub>
<b>n=1</b>	0.255000	0.115000	0.689189
<b>n=2</b>	0.325650	-0.074450	1.296377
<b>n=3</b>	0.415083	-0.316361	4.204564
<b>n=4</b>	0.529057	-0.624707	-5.53118
<b>n=5</b>	0.674325	-1.017719	-1.96371
<b>n=6</b>	0.859481	-1.518645	-1.3039
<b>n=7</b>	1.095478	-2.157114	-1.03188

**ConditionIV** (Alpha=0.16, d=0.008)

<b>quantum</b>	P[x <sup>(n)</sup> =Q <sub>5</sub> ]	P[x <sup>(n)</sup> =Q <sub>6</sub> ]	P[I <sup>(n)</sup> ] <sub>I</sub>
<b>n=1</b>	0.300000	-0.020000	1.071429
<b>n=2</b>	0.424800	-0.437600	-33.1875
<b>n=3</b>	0.599232	-1.027712	-1.39851
<b>n=4</b>	0.845222	-1.860101	-0.83283
<b>n=5</b>	1.192192	-3.034193	-0.64723
<b>n=6</b>	1.681596	-4.690258	-0.55892
<b>n=7</b>	2.371902	-7.026148	-0.50962

**CASE 5****ConditionI** (Alpha=0.18, d=0.002)

<b>quantum</b>	P[x <sup>(n)</sup> =Q <sub>5</sub> ]	P[x <sup>(n)</sup> =Q <sub>6</sub> ]	P[I <sup>(n)</sup> ] <sub>I</sub>
<b>n=1</b>	0.180000	0.235000	0.433735
<b>n=2</b>	0.188250	0.217300	0.464184
<b>n=3</b>	0.196557	0.198787	0.497118
<b>n=4</b>	0.205225	0.179457	0.533493
<b>n=5</b>	0.214276	0.159275	0.573619
<b>n=6</b>	0.223726	0.138202	0.618151
<b>n=7</b>	0.233593	0.116200	0.667804

**ConditionII** (Alpha=0.18, d=0.004)

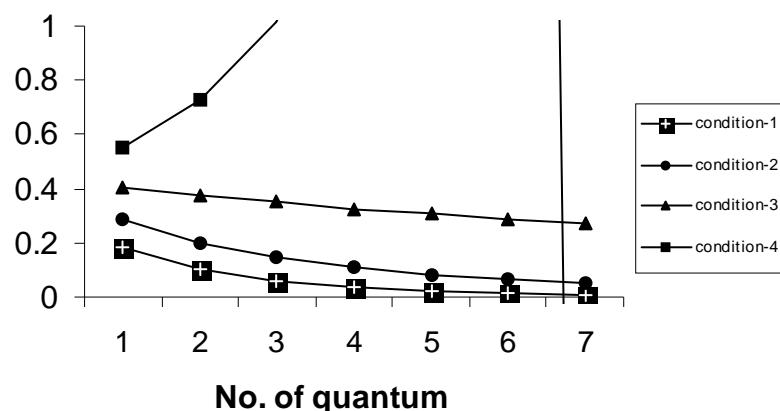
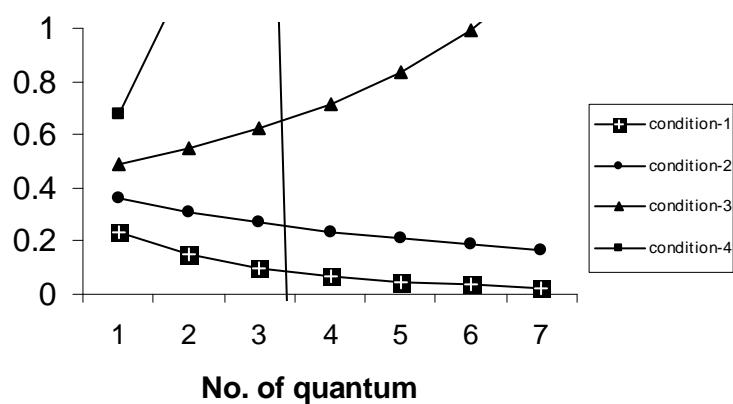
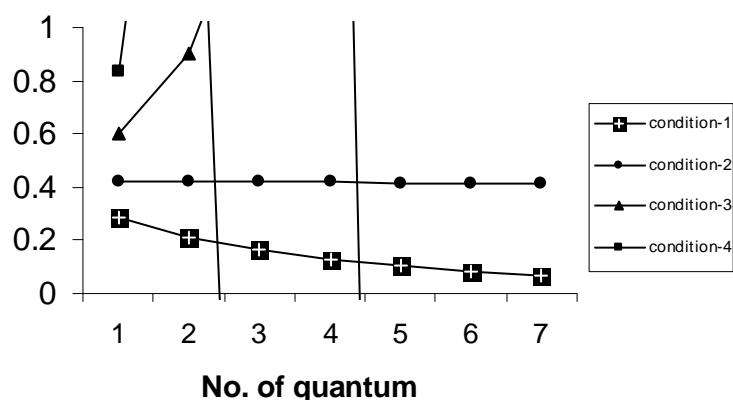
<b>quantum</b>	P[x <sup>(n)</sup> =Q <sub>5</sub> ]	P[x <sup>(n)</sup> =Q <sub>6</sub> ]	P[I <sup>(n)</sup> ] <sub>I</sub>
<b>n=1</b>	0.225000	0.145000	0.608108
<b>n=2</b>	0.269700	0.012700	0.955028
<b>n=3</b>	0.322194	-0.145514	1.823602
<b>n=4</b>	0.384878	-0.334513	7.641775
<b>n=5</b>	0.459756	-0.560282	-4.5735
<b>n=6</b>	0.549202	-0.829975	-1.95604
<b>n=7</b>	0.656050	-1.152137	-1.32245

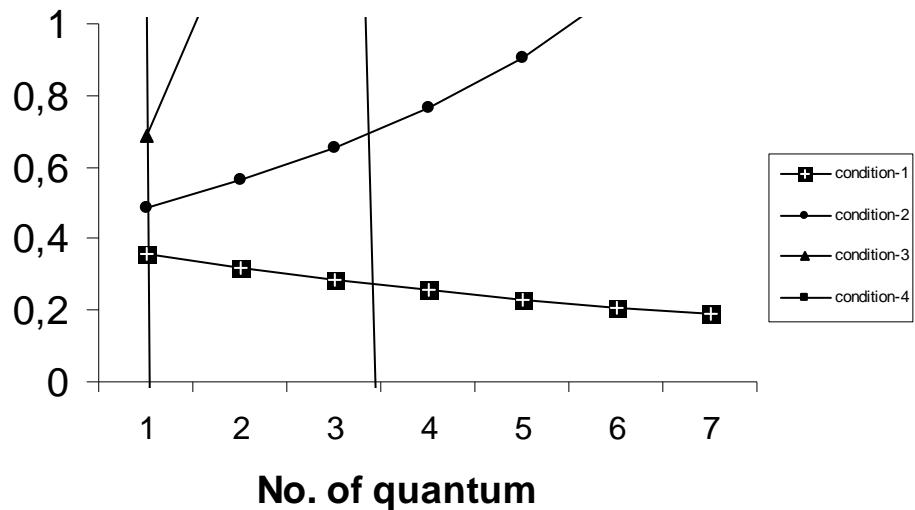
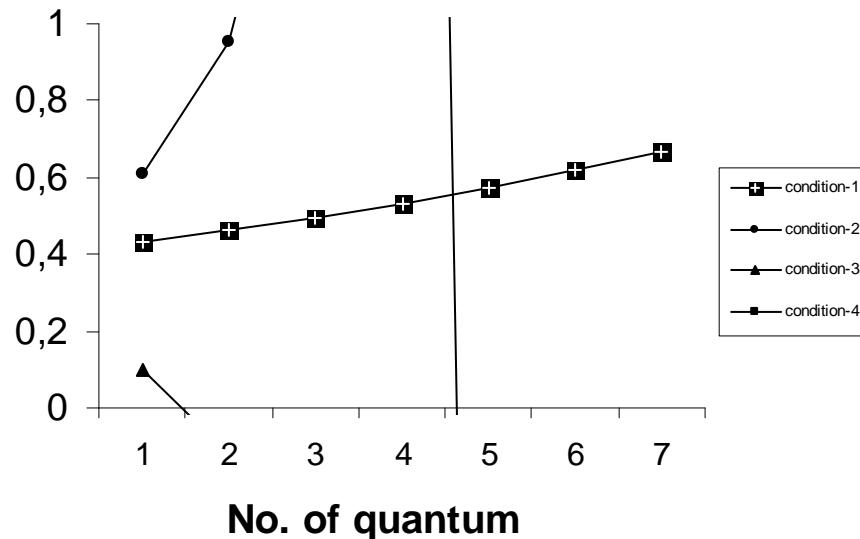
**ConditionIII** (Alpha=0.18, d=0.006)

<b>quantum</b>	P[x <sup>(n)</sup> =Q <sub>5</sub> ]	P[x <sup>(n)</sup> =Q <sub>6</sub> ]	P[I <sup>(n)</sup> ] <sub>I</sub>
<b>n=1</b>	0.270000	0.100000	0.72973
<b>n=2</b>	0.366030	-0.120086	1.488266
<b>n=3</b>	0.494173	-0.417740	6.46544
<b>n=4</b>	0.667116	-0.819577	-4.37565
<b>n=5</b>	0.900581	-1.362043	-1.95158
<b>n=6</b>	1.215748	-2.094350	-1.38373
<b>n=7</b>	1.641212	-3.082935	-1.13837

**ConditionIV** (Alpha=0.18, d=0.008)

<b>quantum</b>	P[x <sup>(n)</sup> =Q <sub>5</sub> ]	P[x <sup>(n)</sup> =Q <sub>6</sub> ]	P[I <sup>(n)</sup> ] <sub>I</sub>
<b>n=1</b>	0.315000	-0.095000	1.431818
<b>n=2</b>	0.476562	-0.648926	-2.76486
<b>n=3</b>	0.718053	-1.484591	-0.93675
<b>n=4</b>	1.081820	-2.743640	-0.65099
<b>n=5</b>	1.629868	-4.640523	-0.54137
<b>n=6</b>	2.455556	-7.498363	-0.48694
<b>n=7</b>	3.699536	-11.80398	-0.45648

**Index Graph for Case I****Index Graph for Case II****Index Graph for Case III**

**Index Graph for Case IV****Index Graph for Case V**

By fig 5.1, the deadlock for scheme-III is constantly lower than any other schemes over the increasing quantum. This shows superiority of this scheme over I, II and IV. The same remark is obtained with Equal transition probability matrix. Therefore, in terms of deadlock index viewpoint, the scheme-III is in lower zone in quantum up to n=7 and so superior and recommendable over others.

## 6.0 CONCLUDING REMARKS

The general class of multi-level queue scheduling contains four schemes, as members, which are compared using a Markov chain model. Every scheme has terms, conditions and restrictions over the general class. Markov chain model considered in Section 2.0 is a common platform to compare the properties of these schemes. Scheme-I suffers a high chance for system reaching to deadlock state. Unequal transition probabilities are like a beneficial approach for scheme-I. In Scheme-II, the chance for deadlock is little lower than other schemes and unequal transition element matrix provides better results. Scheme-III is having lowest amount of deadlock probability for scheduler and unequal case is efficient over equal. Last scheme-IV bears moderately high deadlock probability than scheme-III. Moreover this one is also better for unequal transition elements. Deadlock index analysis also supports the above fact. These indices are in lower zone for scheme-III constantly over all seven quantum. Scheme-IV has poorest deadlock index, showing the low performance in terms of index measure.

Overall, in the setup of Markov chain model and under deadlock index as a performance measure, scheme-III is better than scheme-I, II, IV. The unequal value elements of transition probability matrix provide betterment over equal in all the simulated groups. Deadlock index is proved as a strong measure of performance evaluation using Markov chain model setup, while comparing different scheduling schemes.

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**Article received: 2010-11-18**