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A FAST MESSAGE PACKET FORWARDING USING FIBONACCI METHOD

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Abstract

The high performance router to forward hundreds of millions of message packets per second due to rapid growth of the internet and explosively increasing traffic over the internet, call for a fast message packets forwarding using Fibonacci method. Message packets of varying length were generated and simulated using linear homogeneous recurrence relations with constant coefficients of degree two and three. The results of linear homogeneous recurrence relations with constant coefficients of degree two and three were compared and it was found that more message packets can be sent at a given time by coefficients of degree three than coefficient of degree two. It was also observed that performance of each constant coefficients of either degree two or three depends on their initial conditions.

Keyword: *recurrence relations, message packets, Fibonacci methods, initial conditions, signals*

1. Introduction

Linear homogeneous recurrence relations were studied for modelling message packets that are transmitted from one node to another. The linear possess characteristic roots that can be used for an explicit formula for all the solutions of the recurrence relation. The work developed results that deal with linear homogeneous recurrence relations with constant coefficients of degree two and three. Data of message packets was obtained by reading the contents of a message packets from the local file system which later served into constant coefficients of degrees. The receiver process attempted to be ready to accept another message packet and writes data received for message packets as fast as possible to the file system. This involves that attention must be made to designing the fastest means of sending message packets among nodes.

2. Recurrence Relations

Let c_1 and c_2 be real numbers. Suppose that $r_2 - c_1 r - c_2 = 0$ has two distinct roots r_1 and r_2. Then the sequence $\{a_n\}$ is a solution of the recurrence relation

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} \text{ if and only if}$$

$$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n \text{ for } n = 0, 1, 2, \ldots \text{ where } \alpha_1 \text{ and } \alpha_2 \text{ are constants.}$$

since r_1 and r_2 are roots of

$$r^2 - c_1 r - c_2 = 0$$

it follows that

$$r_1^2 = c_1 r_1 + c_2$$

$$r_2^2 = c_1 r_2 + c_2$$

from these equations, we see that

$$c_1 a_{n-1} + c_2 a_{n-2} = c_1(\alpha_1 r_1^{n-1} + \alpha_2 r_2^{n-1}) + c_2(\alpha_1 r_1^{n-2} + \alpha_2 r_2^{n-2})$$

$$= \alpha_1 r_1^{n-2}(c_1 r_1 + c_2) + \alpha_2 r_2^{n-2}(c_1 r_2 + c_2) = \alpha_1 r_1^{n-2} r_1^2 + \alpha_2 r_2^{n-2} r_2^2 = \alpha_1 r_1^n + \alpha_2 r_2^n = a_n$$

This shows that the sequence $\{a_n\}$ with $a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$ is a solution of recurrence relation.

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Suppose that $\{a_n\}$ *is a solution of the recurrence relation and the initial conditions*

$$a_0 = c_0 \text{ and } a_1 = c_1 \text{ hold}.$$

It will be shown that there are constants α_1 *and* α_2 *so that the sequence* $\{a_n\}$ *with* $a_n = \alpha_1 r_1^n + \alpha_2 r_2^n$ *satisfies these same initial conditions. This requires that*

$$a_0 = c_0 = \alpha_1 + \alpha_2 \qquad (i)$$

$$a_1 = c_1 = \alpha_1 r_1 + \alpha_2 r_2 \qquad (ii)$$

we can solve these two equations for α_1 *and* α_2.
From the first equation it follows that

$$c_0 = \alpha_1 + \alpha_2 \Rightarrow \alpha_2 = c_0 - \alpha_1$$

inserting this expression into the second equation gives

$$c_1 = \alpha_1 r_1 + \alpha_2 r$$

$$c_1 = \alpha_1 r_1 + (c_0 - \alpha_1) r_2 \Rightarrow c_1 = \alpha_1 r_1 + c_0 r_2 - \alpha_1 r_2 \Rightarrow c_1 = \alpha_1 r_1 - \alpha_1 r_2 + c_0 r_2$$

$$c_1 = \alpha_1 (r_1 - r_2) + c_0 r_2$$

$$\alpha_1 (r_1 - r_2) + c_0 r_2 = c_1 \Rightarrow \alpha_1 (r_1 - r_2) + = c_1 - c_0 r_2$$

$$\alpha_1 = \frac{c_1 - c_0 r_2}{r_1 - r_2}$$

$$c_0 = \alpha_1 + \alpha_2 \Rightarrow c_0 = \frac{c_1 - c_0 r_2}{r_1 - r_2} + \alpha_2 \Rightarrow \frac{c_1 - c_0 r_2}{r_1 - r_2} + \alpha_2 = c_0$$

$$\alpha_2 = c_0 - \frac{c_1 - c_0 r_2}{r_1 - r_2} \Rightarrow \alpha_2 = \frac{c_0 (r_1 - r_2) - c_1 + c_0 r_2}{r_1 - r_2} \Rightarrow \alpha_2 = \frac{c_0 r_1 - c_0 r_2 - c_1 + c_0 r_2}{r_1 - r_2}$$

$$\alpha_2 = \frac{c_0 r_1 - c_1}{r_1 - r_2}$$

where these expressions for α_1 *and* α_2 *depend on the fact that* $r_1 \neq r_2$
hence with these values for α_1 *and* α_2, *the sequence* $\{a_n\}$ *with*
$\alpha_1 r_1^n + \alpha_2 r_2^n$ *satisfies these same initial conditions.*
since this recurrence relation and these initial conditions uniquely determine the sequence. It follows that

$$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n.$$

3. Fibonacci Methods with Two Signals

The recurrence relations $f_n = f_{n-1} + f_{n-2}$ is a linear homogeneous recurrence relation of degree two. It also satisfies the initial conditions of $f_0 = 0$ and $f_1 = 1$. The roots of the characteristic equation $r^2 - r - 1 = 0$ are

$$r_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \qquad r_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

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$$r_1 = \frac{-(-1) + \sqrt{(-1)^2 - 4(1(-1))}}{2(1)} \qquad r_2 = \frac{-(-1) - \sqrt{(-1)^2 - 4(1(-1))}}{2(1)}$$

$$r_1 = \frac{1+\sqrt{5}}{2} \qquad r_2 = \frac{1-\sqrt{5}}{2}$$

From section 2, it follows that Fibonacci numbers are given by

$$f_n = \alpha_1\left(\frac{1+\sqrt{5}}{2}\right)^n + \alpha_2\left(\frac{1-\sqrt{5}}{2}\right)^n$$

for some constants α_1 *and* α_2. The initial conditions $f_0 = 0$ and $f_1 = 1$ can be used to find the constants
we have

$$f_{(0)} = \alpha_1 + \alpha_2 = 0 \qquad (i)$$

$$f_{(1)} = \alpha_1\left(\frac{1+\sqrt{5}}{2}\right) + \alpha_2\left(\frac{1-\sqrt{5}}{2}\right) \qquad (ii)$$

$$\alpha_1 = -\alpha_2$$

inserting this value in equation (ii)

$$= -\alpha_2\left(\frac{1+\sqrt{5}}{2}\right) + \alpha_2\left(\frac{1-\sqrt{5}}{2}\right) = 1$$

$$= -\alpha_2\left(1+\sqrt{5}\right) + \alpha_2\left(1-\sqrt{5}\right) = 2$$

$$= -\alpha_2 - \alpha_2\sqrt{5} + \alpha_2 - \alpha_2\sqrt{5} = 2$$

$$= -2\alpha_2\sqrt{5} = 2 \quad \Rightarrow \quad -\alpha_2 = \frac{2}{2\sqrt{5}} \quad \Rightarrow \quad \alpha_2 = -\frac{1}{\sqrt{5}}$$

$$\alpha_1 + \alpha_2 = 0$$

$$\alpha_1 + \left(-\frac{1}{\sqrt{5}}\right) = 0 \quad \Rightarrow \quad \alpha_1 - \frac{1}{\sqrt{5}} = 0 \quad \Rightarrow \quad \alpha_1 = \frac{1}{\sqrt{5}}$$

$$\alpha_1 = \frac{1}{\sqrt{5}} \qquad \alpha_2 = -\frac{1}{\sqrt{5}}$$

Consequently the Fibonacci numbers with two roots are given by

$$f_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n \quad [5].$$

This proof of $f_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$ concludes that different message packets can be transmitted in any <u>n</u> times using two routes. Looking at the formula for f_n, we can see that

$$\left| f_n - \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n \right| = \left| \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right) \right| < \frac{1}{\sqrt{5}} < \frac{1}{2}$$

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This means that f_n is the integer that is closest to $\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n$. When n is even, f_n is less and greater when n is odd.

4. Fibonacci Methods with Three Signals

The recurrence relations $f_n = f_{n-1} + f_{n-2}\ f_{n-3}$ is a linear homogeneous recurrence relation of degree three. It also satisfies different initial conditions.

$$r^3 - r^2 - r - 1 = 0$$
$$= r^3 - r^2 - r = 0 + 1$$
$$= r(r^2 - r - 1) = 0$$

for a cubic equation the roots should be taken as $\alpha - \beta,\ \alpha\ \ \alpha + \beta$

let $s_1 = -\frac{a_1}{a_0},\ s_2 = \frac{a_2}{a_0},\ s_3 = -\frac{a_3}{a_0}$

$$s_1 = (\alpha - \beta) + \alpha + (\alpha - \beta) = -(-1)/1 = 1$$
$$s_1 = 3\alpha = 1 \Rightarrow \alpha = \frac{1}{3},\ \ hence\ (\frac{1}{3} - \beta),\ \frac{1}{3},\ (\frac{1}{3} + \beta)$$
$$s_1 = -(-1)/1 = (\frac{1}{3} - \beta) * \frac{1}{3} * (\frac{1}{3} + \beta)$$
$$= (1/9 - \beta^2)\frac{1}{3} = 1$$
$$= 1/27 - \beta^2/3 = 1 \Rightarrow -\beta^2/3 = 1 - 1/27 \Rightarrow \beta^2 = 26/27 * -3$$
$$= \beta^2 = -26/9 \Rightarrow \beta = \sqrt{-26}/3 \ \ where\ \ i = \sqrt{-1}\ \ hence\ r_1 = 1/3$$
$$r_2 = (\alpha - \beta) \Rightarrow (1 - i\sqrt{26}/3\ )$$
$$r_3 = (\alpha + \beta) = (1/3 + i\sqrt{26}/3 = (1 + i\sqrt{26}/3$$
$$f_n = \alpha_1 + \alpha_2 + \alpha_3 = 0$$
$$\alpha_1 + \alpha_2(1 + i\sqrt{26}/3 + \alpha_3(1 - i\sqrt{26}/3 = 1$$
$$1/3 + \alpha_2 + \alpha_3 = 0 \qquad\qquad (I)$$
$$= 1/3 + \alpha_2(1 + i\sqrt{26}/3 + \alpha_3(1 - i\sqrt{26}/3 = 1\ \ multiply\ by\ 3$$
$$= 1 + \alpha_2(1 + i\sqrt{26}\ ) + \alpha_3(1 - i\sqrt{26}) = 3 \Rightarrow \alpha_2(1 + i\sqrt{26}\ ) + \alpha_3(1 - i\sqrt{26}) = 2$$

from equation I

$$\alpha_2 = -(1/3 + \alpha_3)$$
$$= -(1/3 + \alpha_3)\ (1 + i\sqrt{26}\ ) + \alpha_3(1 - i\sqrt{26}) = 2$$
$$= -1/3 - i\sqrt{26}/3 - \alpha_3 - \alpha_3 i\sqrt{26} + \alpha_3 - \alpha_3 i\sqrt{26} = 2$$
$$= -1 - i\sqrt{26}/3 - 2\alpha_3 i\sqrt{26} = 2 \Rightarrow -1 - i\sqrt{26} - 6\alpha_3 i\sqrt{26} = 6$$
$$= -6\alpha_3 i\sqrt{26} = 6 + 1 + i\sqrt{26} \Rightarrow -6\alpha_3 i\sqrt{26} = 7 + i\sqrt{26}$$

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$$\alpha_3 = \frac{-7 + i\sqrt{26}}{6i\sqrt{26}}$$

$$\alpha_2 = -\left(\frac{1}{3} - \frac{7 + i\sqrt{26}}{6i\sqrt{26}}\right) \Rightarrow \frac{-2i\sqrt{26} + 7 + i\sqrt{26}}{6i\sqrt{26}} \Rightarrow \frac{7 - i\sqrt{26}}{6i\sqrt{26}}$$

hence the Fibonacci numbers with three roots are given by

$$f_n = \left(\frac{1}{3}\right)^n + \left(\frac{7 - i\sqrt{26}}{6i\sqrt{26}}\right)\left(\frac{1 + \sqrt{26}}{3}\right)^n + \left(\frac{7 - i\sqrt{26}}{6i\sqrt{26}}\right)\left(\frac{1 - \sqrt{26}}{3}\right)^n$$

The Fibonacci numbers with two and three roots determine messages that can be transmitted in *n* microseconds using either two or three signals with different initial conditions.

5. Implementation and Results

The Fibonacci numbers with two and three roots were coded in Java programming language in Linus platforms. The message packets were encapsulated in an daemon program that forked on start up in order to provide the receiver and transmitter processes. The signals were sent via the kill system call. A buffering scheme was used for the message packets in the disk file. If message packet sizes are larger than the natural file system size, then the buffering is discarded. The whole process was simulated with different number of packets that can be transmitted in microseconds. The sample result is shown in table 1. Looking at the table 1, using two signal with initial values of 1 and 2, 89 packets can be sent in 10 microseconds while 1346269 packets can be transmitted in 30 microseconds. When one of the signals would require 2 microseconds for packet transmission and the transmittal of the second signal would require 3 microseconds, 114 packets would be sent in 20 microseconds while 533456 packets would be sent in 50 microseconds.

Applying three different signals of which one signal would require 1 microsecond for packet transmittal and the other two signals would require 2 microseconds each for packet transmission, 291 packets would be sent in 10 microseconds and 269890678 packets would be sent in 30 microseconds. By modifying the initial conditions whereby one signal would require 1 microsecond, second signal would require 2 microseconds and third signal would require 3 microseconds for packet transmission, 81 packets could be sent in 10 microseconds while 36234 packets could also be sent in 20 microseconds. The constraint is that each signal of a packet is followed by the next signal.

Table 1: Results of Packet Transmission using different Signals

time ms	packets	two signals using 1ms and 2ms transmittal	two signals using 2ms and 3ms transmittal	three signals using 1ms, 2ms and 2ms transmittal	three signals using 1ms, 2ms and 3ms transmittal
10	number	89	7	291	81
20	number	10946	114	256210	36234
30	number	1346269	1887	269895678	16055415
50	number	20509560074	533456	2.84716E+14	1.28138E+12

Moreso, three signals with initial conditions of 1, 2 and 2 performed better than with initial conditions of 1, 2 and 3. This is also observed in two signals with initial conditions of 1 and 2 as against 2 and 3. The three signals still performed better in packet transmission than two signals as shown in table 1.

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6. Conclusion

Fibonacci method was proofed for different message packets forwarding in any n times using either two or three routes. Both methods were implemented and compared on linus platform. It was observed that initial conditions play a vital role on the number of message packets that can be transmitted in *n* microseconds. Fibonacci method with three signals performed better than with two signals without considering delay times in the network. With this functionality being added for packet processing, it will enhance the rate at which message packets can be transferred from one node to another.

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