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LEPTON FAMILY NUMBER VIOLATION VIA INTERMEDIATE BLACK HOLE

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Abstract
We analyzed lepton flavor violation processes via intermediate black hole. We considered $\mu \to e\gamma$, $\mu \to ee\bar{e}$ decays and muonium(μ^+e^-)-antimuonium(μ^-e^+) oscillation. We concluded that three body decays and oscillation seem more favorable then radiative one. The limits on effective coupling constants are extracted from several experiments on the lepton flavor violation processes.

Keywords: *Oscillation, three body decays, lepton.*

Charged lepton flavor violation (LFV) processes first arise at the one loop level in the Standard Model (SM) with neutrino mixing and have typical suppression factor m_ν^2 / M_W^2 [1-5]. These processes are very sensitive to the New Physics (NP) effects beyond the SM (BSM), because of some possible mechanism which enhance them. In some extensions of the SM the rates of LFV processes enhance and become close to the modern experimental limitations. The NP can manifest themselves directly through the production of new quanta or the topology of events or indirectly by inducing forces that modify rare LFV processes. Such indirect searches are not a luxury. To differ various NP scenarios, we are in need to investigate their influence on the flavor dynamics. Any experimental observation of charged LFV process would be a manifestation of NP effects beyond the SM.

The goal of this note is just the analysis of LFV processes via intermediate black hole in the large extra dimension scenario. Namely, our attention will be devoted to $\mu \to e\gamma$, $\mu \to ee\bar{e}$ decays and muonium (μ^+e^-) to antimuonium (μ^-e^+) oscillation (Fig.1). Black holes of the effective Planck range $M_{Pl} \sim 1\ \ TeV$ naturally arise in extra dimension theories [6-11]. One should mention that, LHC is considered as a factory for TeV scale black holes ($M_{bh} \sim 1\ \ TeV$)[6-11].

LFV processes are intensively investigated in large extra dimension scenarios. As these studies show in case when theoretical approaches are not enriched other way than simply adding extra dimension to the SM, there is hard to get theoretical predictions close to experimental bounds.

It is expected from common theoretical sense that LFV processes would be possibly enhanced in case when particles running in the appropriate loops have close masses [12]. Loop amplitudes with comparable masses of inter mediate particles running in the loop seem to be quite large because the generic quadratic suppression factor is changed to a linear one. Such a situation with comparable masses in principle is realizable in the models with extra dimensions. It is not obvious without specific calculations how would be changed the SM estimate of the above processes in the models with extra dimensions. Some details of the models can enhance suitable amplitudes and others can cause suppression. On general grounds, one expects an enhancement of the amplitudes, but this expectation is not fulfilled because of the almost degeneracy of the massive neutrino towers modes from different generations. This is not necessarily the last word, though; the black hole can inspire LFV processes and enhance them [10-16].

We accept the conjecture that black holes violate global symmetries [13-16] including lepton family number. So, black holes could manifest themselves in LFV processes as intermediate

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states and enhance them. We assume that black holes with mass lighter than effective Planck mass have a zero charges (electric, color) and zero angular moment in the classical case and this feature is adopted by quantum gravity too. So, one can write the effective Lagrangian describing interactions between charged leptons and black hole in the following way [15, 16]

$$L = g_{ij}\bar{l}_{Li} l_{Rj} \Phi_{bh} + h.c. \qquad (1)$$

where g_{ij} are dimensionless effective coupling constants. The black hole can produce LFV process $\mu \to ee\bar{e}$ and muonium (μ^+e^-) to antimuonium (μ^-e^+) oscillation on the tree level and radiative LFV decay $\mu \to e\gamma$ on the one loop level (Fig.1). The branching ratios of radiative decay $\mu \to e\gamma$ in frame of the SM with neutrino mixing are suppressed. The typical suppression factor is m_ν^2 / M_W^2 [1-5];

$$Br(\mu \to e\gamma) = \frac{3\alpha}{32\pi}\left(V_{ei}V_{\mu i}^* \frac{m^2(\nu_i)}{M_W^2} \right)^2 \qquad (2)$$

and the branching ratio bounded by neutrino oscillation experiments data $Br(\mu \to e\gamma) \le 10^{-54}$, which is out of reach experimentally.

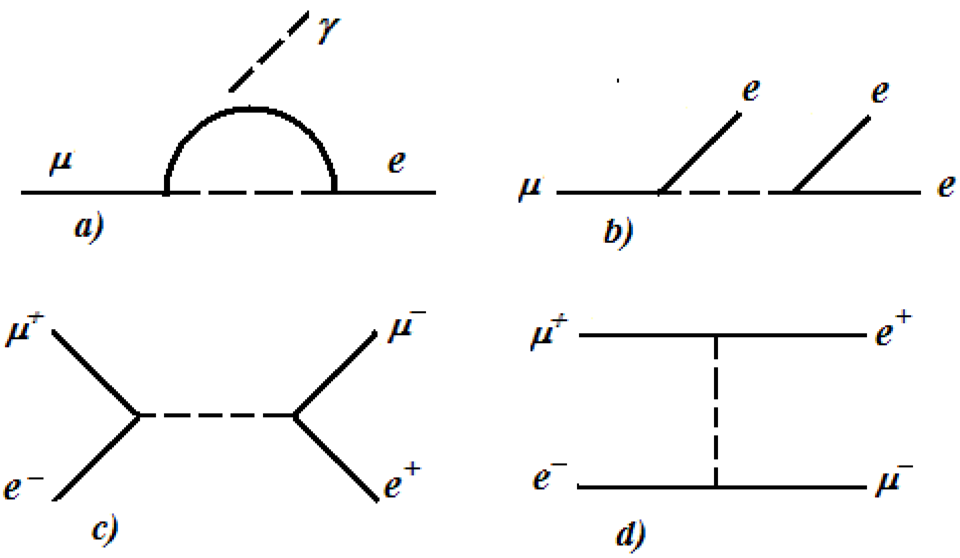

FIG. 1: LFV processes via intermediate black hole (dashed intermediate line): a) LFV one-photon radiative decay $\mu \to e\gamma$; b) three lepton LFV decay $\mu \to ee\bar{e}$; c,d) muonium (μ^+e^-) to antimuonium (μ^-e^+) oscillation.

Virtual black hole can induce LFV process $\mu \to e\gamma$ at the one loop level (Fig.1). Through direct calculation we get following expression for branching ratio

$$Br(\mu \to e\gamma) = \frac{3\alpha}{4\pi G_F^2 M^4}\left(g_{ei} g_{\mu i}^* f(x_i) \right)^2 \qquad (3)$$

where M is mass of the intermediate black hole and we have introduced following notation

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$$f(x) = \frac{1}{24(1-x)^4}(2+3x-6x^2+x^3+6x\ln x), \qquad x_i = \frac{m^2(l_i)}{M^2} \qquad (4)$$

The current experimental limit on the $\mu \to e\gamma$ decay rate is [17]

$$Br(\mu \to e\gamma) < 2.4 \cdot 10^{-12} \qquad (5)$$

The MEG collaboration continues data-taking and is expected to improve the current constraint on the lepton flavor violation decay $\mu \to e\gamma$ to a level 10^{-13} in the next few years. Using formulae (3-5) we get constraints on effective coupling constants g and mass of black hole M

$$\frac{\left|g_{ei} g^*_{\mu i}\right|}{M^2} < 3.7 \cdot 10^{-5} G_F \qquad (6)$$

Let us consider now $\mu \to ee\bar{e}$ process via intermediate black hole (fig.1). The following experimental constraint for the branching ratio was obtained by SINDRUM collaboration [18]

$$Br(\mu \to ee\bar{e}) < 1.0 \cdot 10^{-12} \qquad (7)$$

Through direct calculation we obtain for the branching ratios of the LFV process $\mu \to ee\bar{e}$ the following expression

$$Br(\mu \to ee\bar{e}) = \frac{g^2_{\mu e} g^2_{ee}}{16 G_F^2 M^4} \quad . \qquad (8)$$

Using formulae (7) and (8) we get constraints on effective coupling constant g and mass of black hole M

$$\frac{g_{\mu e} g_{ee}}{M^2} < 4 \cdot 10^{-6} G_F \qquad (9)$$

The limit is presented graphically in Fig.2.

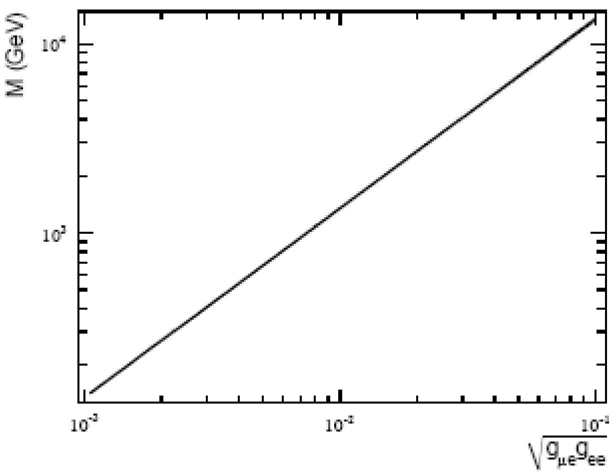

FIG. 2: Constraints on M and $\sqrt{g_{\mu e} g_{ee}}$ obtained from the process $\mu \to ee\bar{e}$. Allowed area for the parameters M and $\sqrt{g_{\mu e} g_{ee}}$ below the line on the fig.2.

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The black hole can induce also muonium (μ^+e^-) to antimuonium (μ^-e^+) oscillation at the tree (Fig.1) level. Muonium gives us perfect opportunity to test fundamental interactions, because it does not contain hadronic constituents. The corresponding effective Hamiltonian has the form [19, 20]

$$H_{eff} = \frac{G_{M\overline{M}}}{\sqrt{2}} \overline{\mu}\gamma^{\alpha}(1+\gamma_5)e\overline{\mu}\gamma_{\alpha}(1+\gamma_5)e + h.c. \tag{10}$$

where $G_{M\overline{M}}$ is an effective four fermion coupling constant. The present experimental constraint on the $G_{M\overline{M}}$ is [21]

$$G_{M\overline{M}} < 3\cdot 10^{-3} G_F \tag{11}$$

One can rewrite the effective four fermion coupling constant in our case in the following form

$$G_{M\overline{M}} = \frac{g^2_{\mu e}}{2\sqrt{2}M^2} \tag{12}$$

and we get bound on effective coupling constant $g_{\mu e}$ and mass of black hole M

$$\frac{g^2_{\mu e}}{M^2} < 8.5\cdot 10^{-2} G_F \tag{13}$$

This limit is presented graphically in Fig.3.

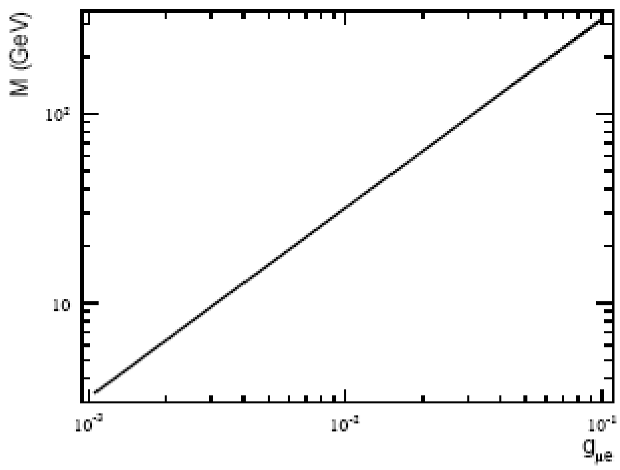

FIG. 3: Constraints on M and $g_{\mu e}$ obtained from the muonium-antimuonium oscillation. Allowed area for the parameters M and $g_{\mu e}$ below the line on the fig.3

Let us mention that usual hierarchy of LFV processes $\mu \to e\gamma$, $\mu \to ee\overline{e}$ seems like $Br(\mu \to e\gamma) > Br(\mu \to ee\overline{e})$. It is not excluded vice-versa situation in some BSM approaches, which could be interesting from the point of view NP. On the other hand, the search for three lepton LFV decays $l \to 3l'$ could be more favorable by some experimental reasons even if $Br(l \to 3l')$ is a little less than $Br(l \to l'\gamma)$. In this aspect even more intriguing would be situation with the hierarchy $Br(l \to 3l') > Br(l \to l'\gamma)$. One can conclude from formulae (3), (8) that this case is just the situation which could be predicted by the case of LFV via intermediate black hole $Br(\mu \to ee\overline{e}) > Br(\mu \to e\gamma)$. It shall be interesting to check this on experiments. The upcoming experiments aim to test LFV processes at a sensitivity of $10^{-13} \div 10^{-16}$ and they may reach 10^{-18} sensitivity [17, 22, 23, 24]. Our suggestion is to change accents from the LFV radiative decays to three lepton LFV decays and/or muonium-antimuonium oscillation. Before their direct detection NP

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effects may manifest themselves in rare processes. One can conclude that we have good chance to discover NP in LFV processes. If extra dimensions scenario realize at O(1 TeV) level, it is possible to discover their traces in LFV processes and three lepton LFV decay $\mu \to ee\bar{e}$ and muonium-antimuonium oscillation seem favorable. It should be mentioned that the expressions (6), (9) and (13) describe constrains on the black Hole mass M and effective coupling constants g_{ij}. These expressions contain 7, 3 and 2 parameters respectively. This fact means that it's easier to analyze parameters from $\mu \to ee\bar{e}$ muonium-antimuonium oscillation processes, than from the $\mu \to e\gamma$ .This last statement makes our suggestion more strength.

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