ISSN 1512-1232

A New PAPR Reduction in OFDM Systems Using SLM and Orthogonal Eigenvector Matrix

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Abstract

OFDM is an attractive modulation technique for next generation of communication and used in many wireless standards. OFDM systems are known to have a high PAPR (Peak-to-Average Power Ratio) compared with single-carrier systems. In fact, the high PAPR is one of the most detrimental aspects in the OFDM system, as it can cause power degradation (Inband distortion) and spectral spreading (Out-of-band radiation) by being clipped passing through a power amplifier in the transmitter. In this paper, from the foundation of the PAPR analysis an effective method of PAPR reduction has been proposed which combines SLM (Selective Mapping) and Orthogonal Eigenvector Matrix (OEM) transform. Extensive computer simulations show that a PAPR reduction of up to 5.6 dB can be obtained where conventional SLM can reduce 2.1 dB

Keywords: *Orthogonal frequency division multiplexing (OFDM), peak-to-average power ratio (PAPR), Selective Mapping (SLM), Orthogonal Eigenvector Matrix (OEM).*

INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) is one of the most attractive multicarrier modulation schemes for high bandwidth efficiency and strong immunity to multipath fading [1]. OFDM offers a considerable high spectral efficiency, multipath delay spread tolerance, immunity to the frequency selective fading channels and power efficiency [1], [2]. As a result, OFDM has been chosen for high data rate communications and has been widely deployed in many wireless communication standards such as Digital Video Broadcasting (DVB) and worldwide interoperability for microwave access (mobile WiMAX) based on OFDM access technology [3].

One of the major challenge in OFDM is high PAPR of transmitted OFDM signals. Therefore, the OFDM receiver’s detection efficiency is very sensitive to the nonlinear devices used in its signal processing loop, such as Digital-to-Analog Converter (DAC) and High Power Amplifier (HPA), which may severely impair system performance due to induced spectral regrowth and detection efficiency degradation. For example, most radio systems employ the HPA in the transmitter to obtain sufficient transmits power and the HPA is usually operated at or near the saturation region to achieve the maximum output power efficiency, and thus the memory-less nonlinear distortion due to high PAPR of the input signals will be introduced into the communication channels. If the HPA is not operated in linear region with large power back-off, it is impossible to keep the out-of-band power below the specified limits. This situation leads to very inefficient amplification and expensive transmitters.

To overcome above mentioned serious drawbacks, various approaches have been proposed recently and some of the techniques have been summarized in [5] including clipping, filtering, coding schemes, phase optimization, nonlinear companding transforms, Tone Reservation (TR) and Tone Injection (TI), constellation shaping, Partial Transmission Sequence (PTS) and Selective Mapping (SLM) [6], [7]. The selected mapping method (SLM) provides good performance for PAPR reduction [7], and this requirement usually results in high computational complexity. Several techniques have been proposed based on low-complexity selected mapping schemes for PAPR reduction in OFDM Systems [7], [8]. There are techniques based on combining the SLM with various transforms for reducing the PAPR of OFDM systems. SLM requires the transmission of several side information bits for each data block, which results in some data rate loss. These bits must generally be channel-encoded because they are particularly critical to the error performance of the system. This increases the system complexity and transmission delay, and decreases the data rate. A novel SLM method for which no side information needs to be sent is proposed in [7], [8].

In this paper, an efficient PAPR reduction technique based on joint selective mapping and orthogonal eigenvector matrix method is proposed. This method proposes two different schemes. Performances of the proposed methods will be compared with the original OFDM system with and without selective mapping technique. The rest of the paper is organized as follows: A brief overview of PAPR in OFDM, selective mapping and orthogonal eigenvector matrix is given in Section II. Section III introduces two different schemes of the proposed method. The simulation results for the proposed schemes are shown in section IV. Section V concludes the article.

Overview Of OFDM and PAPR Problem

Peak-to-Average Power in OFDM Signals

In this section, we review the basic of OFDM transmitter and the PAPR definition. Consider an OFDM consisting of N subcarriers.

Let a block of N symbols $\boldsymbol{X} = \{X_k, k = 0,1,\ldots,N-1\}$ is formed with each symbol modulating one of a set of subcarriers $\{f_k, k = 0,1,\ldots,N-1\}$. The N subcarriers are chosen to be orthogonal, i.e., $f_k = k\Delta f$, where $\Delta f = 1/(NT)$ and T is the original symbol period. Therefore, the complex baseband OFDM signal can be written as

$$x(t) = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k e^{j(2\pi f_k t)}, 0 \le t \le NT \qquad (1)$$

Where X_k is the symbol carried by the k_{th} sub-carrier, Δf is the frequency difference between sub-carriers. In the transmitter, the signal or sequence may be generated by the Inverse Fast Fourier Transform (IFFT) of the *N*-point $\{X_k\}$ sequence, and at the receiver, the Fast Fourier Transform (FFT) is employed to restore the signal.

An OFDM signal consists of an *N* number of independently modulated Sub carriers, which can give a large peak-to-average power (PAP) ratio when added up coherently. When *N* signals are added with the same phase, they produce a peak power that is *N* times the average power. PAPR is the ratio between the maximum power and the average power of the complex signal. The PAPR can be expressed for the time domain OFDM signal $x(t)$ as

$$PAPR = 10\log_{10}\left[\frac{max|x(t)|^2}{E\{|x(t)|^2\}}\right] \qquad (2)$$

Where $E\{\cdot\}$ denotes the expectation operation. As more subcarriers are added, higher peak values may occur, hence the PAPR increases proportionally with the number of subcarriers. Reducing $max|x(t)|$ is the principle goal of PAPR reduction techniques. In practice, most systems deal with a discrete-time signal, therefore, we have to sample the continuous time signal $x(t)$.

Complementary Cumulative Distribution Function (CCDF) is the most common way to evaluate the statistic properties of PAPR by estimating the probability of PAPR when it exceeds a certain level $PAPR_0$. When the number of the subcarriers N is relatively small, the CCDF expression of the PAPR of OFDM signals can be written as [5]

$$CCDF = P(PAPR > PAPR_0) = 1 - (1 - \exp(PAPR_0))^N$$

This equation can be interpreted as the probability that the PAPR of a symbol block exceeds some clip level $PAPR_0$, sometimes referred as symbol clip probability.

Selective Mapping (SLM) PAPR reduction

In SLM, the input data sequences are multiplied by each of the phase sequences to generate alternative input symbol sequences. Each of these alternative input data sequences is made the IFFT operation, and then the one with the lowest PAPR is selected for transmission [9].

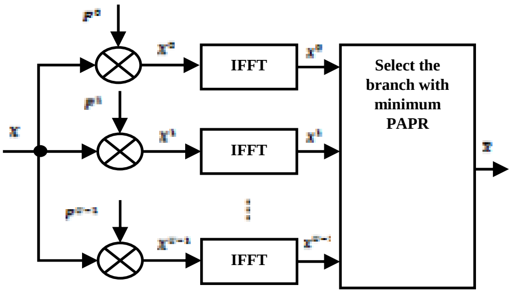

Figure 01: Block diagram of Selective Mapping (SLM).

Here, the input data block $X = [X(0), X(1), \dots X(N-1)]$ is multiplied with U different phase sequences $P^u = [P_0^u, P_1^u, P_2^u, \dots, P_{V-1}^u]^T$ Where, $P_v^u = e^{j\varphi_v^u}$ and $\varphi_v^u \in [0, 2\pi]$ for $v = 0, 1, 2, \dots, N-1$ and $u = 1, 2, 3, \dots, U$ which produce a modified data block $X^u = \left[X^u[0], X^u[1], \dots, X^u[N-1]\right]^T$. IFFT of U independent sequences $X^u$are taken to produce the sequences $x^u = \left[x^u[0], x^u[1], \dots, x^u[N-1]\right]^T$among which the one with the lowest PAPR is selected for transmission and the corresponding selected phase factors P_v^u also should be transmitted to receiver as Side Information (SI). For implementation of SLM OFDM systems, the SLM technique needs U IFFT operation and the $\lfloor \log_2 U \rfloor$ number of required bits as side information is for each data block. Therefore, the ability of PAPR reduction in SLM depends on the number of phase factors U and the design of the phase factors. Some extension of SLM also have been proposed to reduce the computational complexity and number of the bits for side information transmission [4].

Symmetric, Orthogonal Eigenvector Matrix

A symmetric matrix is a square matrix that is equal to its transpose $A = A^T$. Where ^{T} represents the transpose of the matrix. The entries of a symmetric matrix are symmetric with respect to the main diagonal. So if the entries are written as $A = (a_{ij})$, then $a_{ij} = a_{ji}$, where i, j being the coordinates.

In linear algebra, *symmetry* means real eigenvalues and perpendicular eigenvectors. A matrix with these properties must be symmetric; every symmetric matrix has these properties. Diagonal matrices are included as a special case, they are obviously symmetric. Their eigenvalues are on the main diagonal and their eigenvectors lie along the coordinate axes. These directions are certainly orthogonal. For other symmetric matrices the eigenvectors point in other directions, but the key property remains true: the eigenvectors are *perpendicular*, [12, p. 60].

A is an orthogonal matrix if its transpose is equal to its inverse, i.e., $A^T = A^{-1}$ which entails $A^T A = A A^T = I$ where I is the identity matrix. An orthogonal matrix **A** is necessarily invertible $(A^{-1} = A^T)$, unitary$(A^{-1} = A^*)$, and normal $A^* A = A A^*$. Where * represents the conjugate of the matrix. As a linear transformation, an orthogonal matrix preserves the dot product of vectors, and therefore acts as an isometry of Euclidean space, such as a rotation or reflection. In other words, it is a unitary transformation. An orthogonal matrix is always a square matrix with real entries whose columns and rows are orthogonal unit vectors.

According to the Theorem 2.1 in [11, p. 157], a symmetric orthogonal eigenvector matrix can be given as

$$S = \sqrt{\frac{2}{n+1}}\left(\left(\sin\frac{ij\pi}{n+1}\right)\right)^{n}_{i,j=1}, \rho_n(S) \le \frac{n+1}{2}$$

S is a symmetric, orthogonal eigenvector matrix for the second difference matrix and ρ_n is the *growth factor* [12]. For the simplicity of our analysis we define this Symmetric, Orthogonal Eigenvector Matrix (OEM) $\boldsymbol{S}$ as an $N \times N$ square matrix:

$$S(i,j) = \sqrt{\frac{2}{n+1}}\left(\sin\frac{ij\pi}{n+1}\right) \qquad (3)$$

For example $N = 3$, we obtain the OEM matrix as

$$S = \begin{bmatrix} \frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2} \\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}} \\ \frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2} \end{bmatrix}_{3\times3}$$

Surely the matrix is symmetric and can be designed for any dimension using (3). Mathematically it can also be proved that $S = S^T = S^{-1}$ and $S^T S = SS^T = I$, which proves that the matrix is symmetric as well as orthogonal.

System Model of The Proposed Scheme

The main idea of the proposed scheme(s) is to perform an orthogonal transformation on the data stream to reduce autocorrelation coefficients of IFFT input, so that PAPR of OFDM signal could also be reduced. Using the combination of orthogonal transformation and selective mapping two methods can be proposed:

Proposed Method 1 (OEM and SLM)

The block diagram of the transmitter is shown in the Fig 2(a) for the proposed method 1. Firstly, the serial data sequence is divided into block of length ‘N’, then mapped to constellation points by PSK to produce the modulated symbols $m_0, m_1, \dots, m_{N-1}$. Then each block of modulated symbols is multiplied by $N \times N$ OEM matrix and each transformed output block is multiplied by ‘U’ different phase sequence $P_v^u = e^{j\varphi_v^u}$. After computing the IFFT of the each branch there are ‘U’ different OFDM signals with the same information. The transmitter selects and transmits the branch with minimum PAPR. SLM takes advantage of the fact that the PAPR of an OFDM signal is very sensitive to *phase shifts* in the frequency domain data.

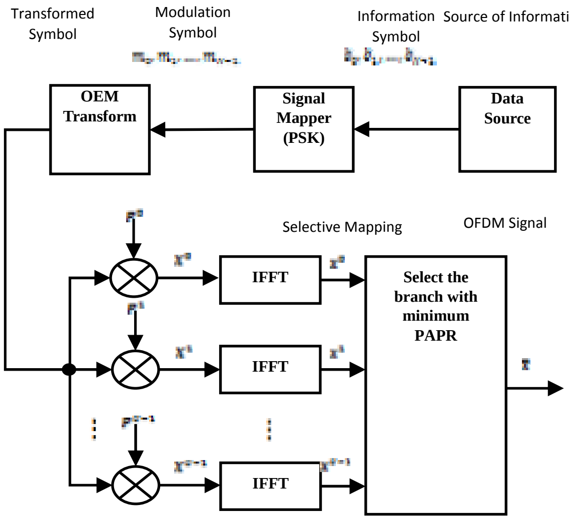

Figure 02 (a): OFDM transmitter with OEM and SLM. (Proposed Method 1)

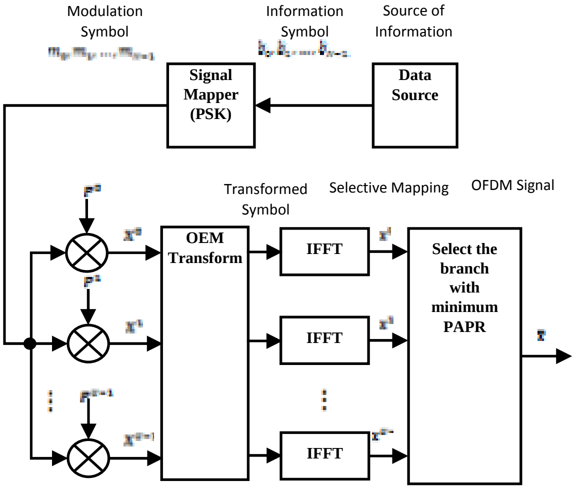

Figure 02 (b): OFDM transmitter with SLM and OEM. (Proposed Method 2)

Proposed Method 2 (SLM and OEM)

In this Method, PAPR reduction is achieved by multiplying independent phase sequences with the original data and then transformed by OEM matrix. Here, we perform OEM transformation after the phase modification to further reduce the PAPR of the signal. In this fashion, the autocorrelation of the signal modified by phase vector is reduced and hence the PAPR. The branch with the lowest PAPR is selected and transmitted.

Simulation

In this section, we present simulations for a complex baseband OFDM system with N = 64 number of subcarrier employing a BPSK modulation by using 10^7 randomly generated OFDM symbols. In numerical simulations, we have selected the system parameters to be compatible with some recent works on PAPR reduction mentioned in [5].

Figure 3 shows the CCDF performance of proposed method 1, labeled as '*OEM+SLM S-1*' in Fig 03 and Fig 04. In this method the OEM transform is followed by selective mapping and the phase sequence varies as 2, 4, 8, 16 and 32 (side information bits = 1, 2, 3 and 4 respectively). With this method, at $CCDF = 10^{-3}$ the peak power is reduced by 4.9 dB when compared with the case of original OFDM system and 2.8 dB when compared with the conventional SLM technique. This improvement can be achieved by using U=2, i.e., only one bit side information in 64 bits IFFT block. Simulations show that by employing more redundant bits the PAPR can be further reduced.

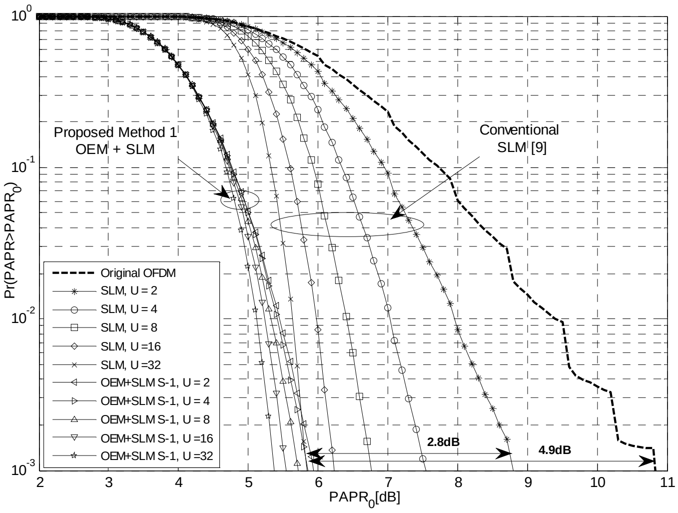

Figure 03: PAPR characteristics of proposed method 1 (OEM+SLM S-1), Original OFDM with and without SLM [9].

Figure 4 shows the CCDF performance of proposed method 2, labeled as *'SLM+OEM S-2'*. At $CCDF = 10^{-3}$, the proposed method 2 reduces the PAPR by 3.5 dB over the conventional SLM and 5.6 dB over the original OFDM system. So, a 5.6 dB PAPR reduction is achieved where conventional SLM can reduce 2.1 dB only.

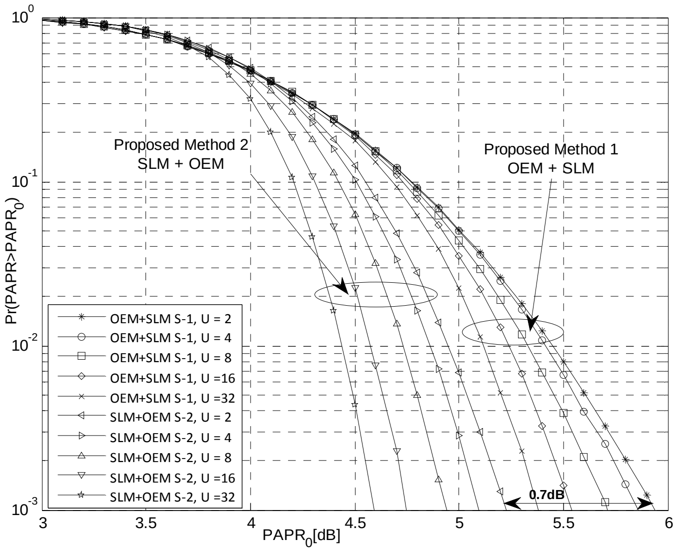

Figure 04: PAPR performances of proposed method 1 (OEM+SLM S-1) and proposed method 2 (OEM+SLM S-1).

Conclusion

OFDM is a very attractive technique for communications due to its spectrum efficiency and channel robustness. One of the serious drawbacks of OFDM systems is that the composite transmit signal can exhibit a very high peak power when the input sequences are highly correlated. In this paper, a new method of PAPR reduction technique has been proposed based on orthogonal eigenvector transformation to reduce the autocorrelation coefficients of IFFT input. Two different schemes of the proposed method have been proposed in this article. The PAPR reduction performances are evaluated by MATLAB simulation. It was shown that proposed method 1 reduces the PAPR by 4.9 dB and method 2 reduces 5.6 dB over the original system. Whereas a conventional selective mapping can reduce 2.1 dB only. Mathematical analysis, including the distribution of the PAPR has been provided. Simulation results state that the PAPR reduction performance is greatly improved compared to conventional SLM.

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Article received: 2012-11-04