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CALCULATION OF THE GRAVITOELECTROMAGNETISM FORCE FOR THE BIANCHI TYPE IX SPACETIME

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Abstract: *The gravitoelectromagnetism force acting on the test particle in the Bianchi type IX metric is calculated.*

Keywords: *gravitoelectromagnetism.*

1. INTRODUCTION

The slicing and threading points of view today are introduced, respectively, by Misner, Thorne and Wheeler [1] in 1973 and , Landau and Lifshitz [2] 1n 1975. Both points of view can be traced back when Landau and Lifshitz [3] in 1941 introduced the threading point of view splitting of the space-time metric. After them, Lichnerowicz [4] introduced the beginning of slicing point of view. In threading point of view, splitting of space-time is introduced by a family of time-like congruencies with unit tangent vector field, may be interpreted as the world-lines of a family of observers, and it defines a local time direction plus a local space through its orthogonal subspace in the tangent space. Let[1] $\left(M, g_{\alpha\beta}\right)$ be a 4-dim manifold of a stationary space-time. We now can construct a 3-dim orbit manifold as $\tilde{M} = \frac{M}{G}$ with projected metric tensor γ_{ij} by the smooth map $\varsigma: M \to \tilde{M}$ where $\varsigma\left(p\right)$ denotes the orbit of time-like Killing vector $\frac{\partial}{\partial t}$ at the point $p \in M$ and G is 1-dim group of transformations generated by the time-like Killing vector of the space-time under consideration, [5,6]. The threading decomposition leads to the following line element, [2, 6]:

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta = h\left(dt - g_i dx^i\right)^2 - \gamma_{ij} dx^i dx^j, \qquad (1)$$

where $\gamma_{ij} = -g_{ij} + h g_i g_j$, in which $g_i = -\frac{g_{0i}}{h}$ and $h = g_{00}$. In a space-time with time dependent metric (1), the gravitoelectromagnetism force acting on a relativistic test particle whose mass m due to time dependent gravitoelectromagnetism[2] fields as measured by threading observers is described by the following equation[3] , we use gravitational units with $c = 1$, [9]:

[1]The Greek indices run fron 0 to 3, the Latin indices take values 1 to 3.

[2]For more details about gravitoelectromagnetism see references [7,8]

[3]The vector $\boldsymbol{C} = \boldsymbol{A} \times \boldsymbol{B}$ has components as $C^i = \frac{e^{ijk}}{\sqrt{\gamma}} A_j B_k$ in which $\gamma = \det\left(\gamma_{ij}\right)$ and 3-dim Levi-Civita tensor ε_{ijk} is antisymmetric in any exchange of indices while $\varepsilon^{123} = \varepsilon_{i23} = 1$. [2]

$$^*\mathbf{F} = \frac{^*d^*\mathbf{p}}{dt} - \frac{m}{\sqrt{1 - {^*v}^2}}\left\{^*\mathbf{E} + {^*\boldsymbol{v}} \times {^*\mathbf{B}} + \mathbf{f}\right\}, \qquad (2)$$

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where ${}^{*}\boldsymbol{p}^{i}=\frac{m^{*}\upsilon^{i}}{\sqrt{1-{}^{*}\upsilon^{2}}}$ such that ${}^{*}\upsilon^{2}=\gamma_{ij}{}^{*}\upsilon^{i*}\upsilon^{j}$ in which ${}^{*}\upsilon^{i}=\frac{\upsilon^{i}}{\sqrt{h\left(1-g_{k}\upsilon^{k}\right)}}$ with $\upsilon^{i}=\frac{dx^{i}}{dt}$ and starry total derivative with respect to time is defined as $\frac{{}^{*}d}{dt}=\frac{{}^{*}\partial}{\partial t}+{}^{*}\upsilon^{i*}\partial_{i}$ where $\frac{{}^{*}\partial}{\partial t}=\frac{1}{\sqrt{h}}\frac{\partial}{\partial t}$ and ${}_{*i}={}^{*}\partial_{t}+g_{i}\frac{\partial}{\partial t}$. In equation (2), the last term is defined as

$$f^{i}=-\left({}^{*}\lambda_{jk}^{i}\upsilon^{j}+2D_{k}^{i}\right)^{*}\upsilon^{k}, \tag{3}$$

where the 3-dim starry Christoffel symbols are defined with the following form

$${}^{*}\lambda_{jk}^{i}=\frac{1}{2}\gamma^{il}\left(\gamma_{jl*k}+\gamma_{kl*j}-\gamma_{jk*l}\right), \tag{4}$$

and deformation rates of the reference frame with respect to the observer are represented by tensors $D_{ij}=\frac{1}{2}\frac{*\partial\gamma_{ij}}{\partial t}$ and $D^{ij}=-\frac{1}{2}\frac{*\partial\gamma^{ij}}{\partial t}$. Finally, time dependent gravitoelectromagnetism fields are defined in terms of gravioelectric potential $\phi=\ln\sqrt{h}$ and graviomagnetic vector potential $\boldsymbol{g}=\left(g_{1},g_{2},g_{3}\right)$ as follows[4]

$${}^{*}\boldsymbol{E}=-{}^{*}\nabla\phi-\frac{\partial g}{\partial t};\qquad {}^{*}E_{i}=-\phi_{*i}-\frac{\partial g_{i}}{\partial t}, \tag{5}$$

$$\frac{{}^{*}\boldsymbol{B}}{\sqrt{h}}={}^{*}\nabla\times\boldsymbol{g};\qquad \frac{{}^{*}\boldsymbol{B}^{i}}{\sqrt{h}}=\frac{\varepsilon^{ijk}}{2\sqrt{\gamma}}g_{[k*j]} \tag{6}$$

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[4]Here, curl of an arbitrary vector in a 3-space with metric γ_{ij} is defined by $\left({}^{*}\nabla\times\boldsymbol{A}\right)^{i}=\frac{\varepsilon^{ijk}}{2\sqrt{\gamma}}A_{[k*j]}$ while the symbol [ ] represent the anticommutation over indices

1.1 **Classical motion of a test particle in the Bianchi type IX spacetime and calculation of the gravitoelectromagnetism force**

As is well known, Bianchi type cosmological models play a vital role in general relativity to discuss the early stages of evolution of universe. Also, the Bianchi models can be coupled to any gravitational Theory. The Bianchi type IX space-time is important because FRW with positive curvature. Taub-NUT and de Sitter space-times etc. correspond to this space-time . We now consider the Rianchi type IX metric in Cartesian coordinates as

$$ds^{2}=dt^{2}-a^{2}dx^{2}-b^{2}dy^{2}-\left(a^{2}\cos^{2}y+b^{2}\sin^{2}y\right)dx^{2}+2a^{2}\cos y\,dxdy \tag{7}$$

where a and b are unknown functions of t. Firstly, it is not difficult to check that all components of gravitoelectromagnetism fields are zero and also the nonzero 3-dim starry Christoffel symbols are

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$$\begin{aligned} &{}^{*}\lambda_{12}^{1} = f\cot y, \\ &{}^{*}\lambda_{23}^{1} = \frac{1+(1-2l)\cos^2 y}{2\sin y}, \\ &{}^{*}\lambda_{13}^{2} = -l\sin y, \\ &{}^{*}\lambda_{33}^{2} = (2l-1)\sin y\cos y, \\ &{}^{*}\lambda_{12}^{3} = \frac{l}{\sin y}, \\ &{}^{*}\lambda = (1-l)\cos y \end{aligned} \tag{8}$$

where $l = a^2/2b^2$. In continuation, we will determine the trajectory of a test particle with mass m that moving in the Bianchi type IX space-time by using the Hamilton-Jacobi equation, [10,11]. Therefore, this equation is of the form

$$\left(b\frac{\partial S}{\partial t}\right)^2 - \left(\frac{1}{2l}+\cot^2 y\right)\left(\frac{\partial S}{\partial x}\right)^2 - \frac{1}{\sin^2 y}\left[\left(\frac{\partial S}{\partial x}\right)^2 + 2\cos y\frac{\partial S}{\partial x}\frac{\partial S}{\partial z}\right] - \left(\frac{\partial S}{\partial y}\right)^2 - (mb)^2 = 0 \tag{9}$$

By general procedure for solving the Hamilton-Jacobi equation, a natural form will be

$$S(t,x,y,z) = S_1(t) + p_x x + S_2(y) + p_z z, \tag{10}$$

where p_x and p_z are constants and can be identified as the momentum of the test particle along x and z-directions. If we substitute the ansatz (10) in Hamilton-Jacobi equation, then we get the following integral expressions for unknown functions S_1 and S_2:

$$S_1 = \varepsilon\int \xi dt \tag{11}$$

$$S_2 = \varepsilon\int \lambda dt \tag{12}$$

where $\xi = \left[m^2 + \frac{p_z^2}{a^2} + \frac{L^2}{b^2}\right]^{1/2}$ and $\lambda = \left[L^2 - \left(\frac{p_x\cos y + p_y}{\sin y}\right)^2\right]^{1/2}$, while L is the constant of separation and $\varepsilon = \pm 1$. Now the equations for the trajectory can be obtained by considering the following conditions, [10,11]:

$$\frac{\partial S}{\partial L} = cons\tan t, \qquad \frac{\partial S}{\partial p_x} = cons\tan t, \qquad \frac{\partial S}{\partial p_y} = cons\tan t \tag{13}$$

we can take the above constants to be zero without lose of generality. Then, the set of equations (13) change to the following relations respectively

$$\int\frac{dL}{b^2\xi} = -\int\frac{dy}{\lambda}, \tag{14}$$

$$x = \varepsilon\int\frac{(p_x\cos y + p_y)\cos y dy}{\lambda\sin^2 y} - \varepsilon p_x\int\frac{dt}{a^2\xi}, \tag{15}$$

$$z = \varepsilon\int\frac{(p_x\cos y + p_y)dy}{\lambda\sin^2 y}. \tag{16}$$

Next, with employing equations (14-16), we find

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$$ {}^{*}\upsilon^{i} = -\frac{\varepsilon}{b^{2}\xi \sin^{2} y}\begin{cases} p_{x}\left(\cos^{2} y + \frac{1}{2l}\sin^{2} y\right) + p_{y}\cos y & i=1, \\ \varepsilon\lambda \sin^{2} y, & i=2 \\ p_{x}\cos y + p_{y} & i=3 \end{cases} \tag{17} $$

As a result, after some calculations, yields

$$ \frac{m}{\sqrt{1 - {}^{*}\upsilon^{2}}} = \xi \, . \tag{18} $$

Finally, with the help of equations (17-18) and after tedious calculations, we prove that

$$ {}^{*}\boldsymbol{F} = \boldsymbol{0} \tag{19} $$

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