# HEAVY QUARKS AND ASSOCIATED LIGHT JET PRODUCTION IN THE POLARIZED AND UNPOLARIZED PHOTOPRODUCTION 

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#### Abstract

: We calculate differential cross sections to the production of heavy-quark pairs with the associated light jet in the polarized and unpolarized photoproduction. Our method calculation consists of a mixture of analytical and numerical recipes, that one allow to implement various limiting behaviour in the total and differential cross sections, apply any relevant kinematical cuts and obtain various experimentally measurable quantities numerically. The results of this work are of importance for studies by COMPASS collaboration and possibly for the future Large Hadron Collider experiments at CERN


Keywords: Quantum Chromodynamics,Collider Physics, Next-to-leading order corrections, Polarized processes.

## I. INTRODUCTION

It has been already 20 years since the next-to-leading-order (NLO) corrections to the hadroproduction of heavy flavors were first presented in the seminal work [1]. These results were confirmed yet in another seminal work [2].

The importance of the calculation of the NLO corrections is due to the fact that the scale dependence of the theoretical prediction is expected to be considerably reduced when NLO partonic amplitudes are folded with the available NLO parton distributions.

In the past few years there was much progress in describing the experimental results on photoproduction of heavy flavors (see e.g. [3]). The improvement in the theoretical prediction is mainly due to advances in the analysis of parton distribution functions and the QCD coupling constant. In this regard, we point out the progress in dealing with numerically large mass logarithms that spoil the convergence of the perturbative expansion in the high energy (or small mass) asymptotic domain.

Similar studies are also important for the determination of the longitudinally polarized parton distribution functions, as a further test of the Standard Model. Deep inelastic scattering of longitudinally polarized particles has provided important information on the spin structure of the nucleon. However, the size and shape of the polarized gluon distribution $\Delta g$ in the proton remains an essential problem. Significant progress requires experiments on reactions with longitudinally polarized particles dominated by subprocesses with initial gluons [4]. One such reaction is a longitudinally polarized photoproduction of heavy flavors. The case of polarized incoming particles was considered in [5-7]. However, in all these calculations, polarized and unpolarized ones, too many process variables were integrated out analytically, leaving no possibility to determine meaningful differential cross sections as well as making it imposible to introduce the experimental cuts that are being imposed in the various realistic experimental setups.

The most interesting experimental observables for studying heavy quark features and checking theoretical predictions are various differential jet cross sections. In fact, the associated production of jets including b-hadrons and W vector bosons has been used for the detection of the t -quark. Several signals for new physics, such as an intermediate mass Higgs or the supersymmetric partners of the top quark, could manifest themselves via the presence in the final state, among other things, of jets
containing b-quarks. A close study of the production properties of b-jets in QCD is therefore an important phenomenological input for many of these and other future searches.

It is the aim of this work to develop such a program that will allow not only calculate the above mentioned observables and apply experimental cuts but also enable to apply effective structure function renormalization along the lines described in [8].

This paper is organized as follows. Section II contains an outline of our general approach, as well as discusses results of our calculation for the total cross sections. Section III presents our final expressions for the physical cross sections for the 2-body and 3-body kinematics. Our results are summarized in IV.


FIG.1: LO and loop graphs. In the loop graphs $p_{1} \leftrightarrow p_{2}$ crossed ones are not shown. Note that graph (i) represents gluon, quark and ghost loops]

## II. NOTATIONS AND TOTAL CROSS SECTION

Photoproduction of heavy flavors proceeds through two partonic subprocesses: photon-gluon fusion as well as photon and light-quark (antiquark) collisions. The first subprocess

$$
\begin{equation*}
\gamma\left(\mathrm{p}_{1}\right)+g\left(\mathrm{p}_{2}\right) \rightarrow Q\left(\mathrm{p}_{3}\right)+\bar{Q}\left(\mathrm{p}_{4}\right)+g(k) \tag{2.1}
\end{equation*}
$$

enters the calculation already at the leading order (LO) Born level with the two production topologies. The Born and the loop contributions to this subprocess are shown in Fig. 1. The second subprocess

$$
\begin{equation*}
\gamma\left(\mathrm{p}_{1}\right)+q\left(\mathrm{p}_{2}\right) \rightarrow Q\left(\mathrm{p}_{3}\right)+\bar{Q}\left(\mathrm{p}_{4}\right)+q(k) \tag{2.2}
\end{equation*}
$$



FIG. 2:A) Gluon Brems graphs; $p_{1} \leftrightarrow p_{2}$ crossed ones are not shown. B) Graphs of the subprocess $\gamma q \rightarrow Q \bar{Q} . q$
starts to contribute at the next-to-leading order. Therefore, the tree contributions for both subprocesses are depicted in Fig.2.
Irrespective of the partons involved, the general kinematics is, of course, the same in both processes. With the 4 -momenta $p_{i}, i=1, \ldots, 4$ as indicated and with m the heavy quark mass we define:

$$
\begin{gather*}
\mathrm{s} \equiv\left(\mathrm{p}_{1}+\mathrm{p}_{2}\right)^{2}, t=\left(\mathrm{p}_{1}-\mathrm{p}_{3}\right)^{2}-m^{2} \\
u \equiv\left(\mathrm{p}_{2}-\mathrm{p}_{3}\right)^{2}-m^{2} \tag{2.3}
\end{gather*}
$$

so that one has the energy-momentum conservation relation $\mathrm{s}+\mathrm{t}+\mathrm{u}=0$ in the case of a 2-body kinematics, i.e. for the graphs depicted in Fig.1. The momentum flow directions correspond to the physical configuration, e.g. $p_{1}$ and $p_{2}$ are ingoing whereas $p_{3}$ and $p_{4}$ are outgoing. At the next-to-leading order, when additional real gluon is being emitted with a momentum $k$, the energymomentum conservation relation is apparently modified. In our further analysis for the phase space integration for a 3-body kinematics we will closely follow the methodology developed in the paper
[9] that has worked out for the case of hadroproduction of heavy quarks. We will modify and adapt the above mentioned procedure for our case of photoproduction of heavy flavors.

It will be convenient to introduce variables

$$
\begin{equation*}
s_{2}=\left(k+p_{4}\right)^{2}-m^{2}, u_{k}=\left(p_{2}-k\right)^{2}=-\frac{1}{2} s(1-x)(1+y), \quad x=s_{2} / s \tag{2.4}
\end{equation*}
$$

and y - the cosine of the angle between vectors $\vec{p}_{1}$ and $\vec{k}$ in the center of mass system of the incoming partons (e.g. partonic frame).

The contribution of the 3-particle final state diagrams to the cross section in given by

$$
\begin{equation*}
\mathrm{d} \sigma_{r}=\mathrm{M}_{\mathrm{r}} d \Phi_{3} ; \quad M_{r}=\frac{1}{2 s} \Sigma_{\text {color }, \text { spin }}\left|A_{r}\right|^{2} \tag{2.5}
\end{equation*}
$$

where $A_{r}$ stands for the invariant amplitude and we sum over the final and average over initial color and spin. The 3-body phase space in $n=4-2 \varepsilon$ dimensions is expressed as

$$
\begin{equation*}
d \Phi_{3}=H N d \Phi_{2}^{(x)} \frac{s^{1-\varepsilon}}{2 \pi}(1-x)^{1-2 \varepsilon}\left(1-y^{2}\right)^{-\varepsilon} d y \sin ^{-2 \varepsilon} \theta_{2} d \theta_{2} \tag{2.6}
\end{equation*}
$$

where

$$
\begin{align*}
H & =\frac{\Gamma(1-\varepsilon)}{\Gamma(1+\varepsilon) \Gamma(1-2 \varepsilon)}=1-\frac{\pi^{2}}{3} \varepsilon^{2}+\mathrm{O}\left(\varepsilon^{3}\right)  \tag{2.7}\\
N & =\frac{4 \pi^{\varepsilon}}{(4 \pi)^{2}} \Gamma(1+\varepsilon)  \tag{2.8}\\
d \Phi_{2}^{(x)} & =\frac{2^{2 \varepsilon}}{\Gamma(1-\varepsilon)}\left(\frac{4 \pi}{s x}\right)^{\varepsilon} \frac{1}{16 \pi} \beta_{x}^{1-2 \varepsilon} \sin ^{-2 \varepsilon} \theta_{1} d \cos \theta_{1} d x \tag{2.9}
\end{align*}
$$

angles $\theta_{1}, \theta_{2} \in[0, \pi]$ are correspondingly polar and azimuthal angles of the heavy quark in the quark-antiquark rest frame, while $\beta_{x}=\sqrt{1-4 m^{2} / s x}$. The quantity $d \Phi_{2}^{(x)}$ transforms into the 2body phase space when $x \rightarrow 1$.

The real emission (so called "hard") cross section $M_{r}$ has soft and collinear singularities that manifest themselves as poles in $\varepsilon$. Our aim is to isolate those singularities from the $M_{r}$, calculate them analytically, and the finite part evaluate numerically. As the collinear pole can only arise from the initial light particle with momentum $p_{2}$ being collinear to the emitted gluon, the function

$$
\begin{equation*}
f_{r}=-2 s(1-x) u_{k} M_{r} \tag{2.10}
\end{equation*}
$$

has to be regular for $y=-1$ and $x=-1$. Therefore, we get

$$
\begin{equation*}
M_{r}=\frac{f_{r}}{s^{2}(1-x)^{2}(1+y)} \tag{2.11}
\end{equation*}
$$

Then our cross section can be written as

$$
\begin{equation*}
d \sigma_{r}=H N d \Phi_{2}^{(x)} \frac{s^{-1-\varepsilon}}{2 \pi} d y \sin ^{-2 \varepsilon} \theta_{2} d \theta_{2}(1-x)^{-1-2 \varepsilon}(1-y)^{-\varepsilon}(1+y)^{-1-\varepsilon} f_{r} \tag{2.12}
\end{equation*}
$$

next we Taylor expand first $(1-x)^{-1-2 \varepsilon}$ and then $(1+y)^{-1-\varepsilon}$ to obtain

$$
\begin{equation*}
d \sigma_{r}=d \sigma_{s}+d \sigma_{-}+d \sigma_{f} \tag{2.13}
\end{equation*}
$$

The first term in the Eq. (2.13) contains all the soft poles of our cross section

$$
\begin{align*}
& d \sigma_{s}=H N d \Phi_{2}^{(x)} \frac{s^{-1-\varepsilon}}{2 \pi} d y \sin ^{-2 \varepsilon} \theta_{2} d \theta_{2}\left[-\frac{\beta^{-4 \varepsilon}}{2 \varepsilon} \delta(1-x)\right](1-y)^{-\varepsilon}(1+y)^{-1-\varepsilon} f_{r},  \tag{2.14}\\
& \beta=\sqrt{1-\rho}, \quad \rho=\frac{4 m^{2}}{s}
\end{align*}
$$

As it can be seen from the Eq. (2.14), the soft part of the cross section $d \sigma_{s}$ is determined by the $\delta(1-x)$ function, i.e. leading infrared behaviour $\mathrm{x} \rightarrow 1$, and can be evaluated without calculation of the full real matrix element. The relatively uncomplicated analytical structure of such a matrix element in conjunction with a simplified phase space one allows to analytically integrate Eq.(2.14) over all the variables but $\theta_{1}$. Leaving out the details of such a calculation, we present below the end result for the soft part:

$$
\left.\begin{array}{c}
d \sigma_{s}=H N d \Phi_{2} \frac{1}{4 \pi \varepsilon} s^{-1-\varepsilon} \beta^{-4 \varepsilon} f_{\gamma g}^{(s)} \\
f_{g g}^{(s)}=8 \pi g^{2} d \sigma_{L O}\left\{\begin{array}{l}
-n_{C} s\left(\ln (s)-\ln \left(\frac{-t}{m^{2}}\right)-\ln \left(\frac{-u}{m^{2}}\right)\right)+ \\
-\frac{1}{\varepsilon} n_{C} s-2 c_{F} s+\left(2 c_{F} s-n_{C}\right) \frac{2 m^{2}-s}{\beta} \ln (x)- \\
{\left[\begin{array}{l}
\left(2 c_{F}-n_{C} \frac{2 m^{2}-s}{\beta}\left(L i\left(\frac{2 \beta}{\beta-1}\right)-L i\left(\frac{2 \beta}{\beta+1}\right)\right)+2 c_{F} \frac{s}{\beta} \ln (x)-n_{C} s \frac{\ln (x)^{2}}{2}-\right. \\
+\varepsilon\left[\begin{array}{l}
-n_{C} s
\end{array} \ln \left(\frac{-2 t}{s(1-\beta)}\right) \ln \left(\frac{-2 t}{s(1+\beta)}\right)+L i\left(1+\frac{2 t}{s(1-\beta)}\right)+L i\left(1+\frac{2 t}{s(1+\beta)}\right)+(t \leftrightarrow s)\right.
\end{array}\right)}
\end{array}\right] \tag{2.16}
\end{array}\right\}
$$

with $g$ being a QCD strong coupling constant. The second term in the Eq. (2.13) contains only collinear poles of our cross section:
$d \sigma_{-}=N d \Phi_{2}^{(x)} \frac{s^{-1-\varepsilon}}{2 \pi} d y \sin ^{-2 \varepsilon} \theta_{2} d \theta_{2}\left[\left(\frac{1}{1-x}\right)_{\rho}-2 \varepsilon\left(\frac{\ln (1-x)}{1-x}\right)_{\rho}\right]\left[-\frac{4^{-\varepsilon}}{\varepsilon} \delta(1+y)\right](1-y)^{-\varepsilon} f_{r}$
where the $\rho$-distributions in round brackets abogve are defined according to the following prescriptions:

$$
\begin{align*}
& \int_{\rho}^{1} f(x)\left(\frac{1}{1-x}\right)_{\rho} d x=\int_{\rho}^{1} \frac{f(x)-f(1)}{1-x} d x  \tag{2.18}\\
& \int_{0}^{1} f(x)\left(\frac{\ln (1-x)}{1-x}\right)_{\rho} d x=\int_{\rho}^{1}[f(x)-f(1)] \frac{\ln (1-x)}{1-x} \tag{2.19}
\end{align*}
$$

Integration over the variables $y$ and $\theta_{2}$ renders the Eq. (2.17) to the form

$$
\begin{gather*}
d \sigma_{-}=-N \frac{s^{-1-\varepsilon}}{2 \varepsilon} d \Phi_{2}^{(x)}\left[\left(\frac{1}{1-x}\right)_{\rho}-2 \varepsilon\left(\frac{\ln (1-x)}{1-x}\right)_{\rho}\right] f_{i j}\left(x, \theta_{1}\right)  \tag{2.20}\\
f_{i j}\left(x, \theta_{1}\right)=16 \pi g^{2} s_{i j} d \sigma_{L O}^{(i j)}\left(p_{1}, x p_{2}\right) \tag{2.21}
\end{gather*}
$$

$\mathrm{P}_{i j}$ 's are the Altarelli-Parisi split functions depending on the subprocess under consideration, $d \sigma_{L O}^{(i j)}\left(p_{1}, x p_{2}\right)$ are the corresponding leading order born amplitudes with the rescaled momentum $p_{2}$ (in one instance for the $\gamma$-quark subprocess the momentum $p_{1}$ is rescaled). Finally, for the hard part of the cross section we obtain:

$$
\begin{equation*}
d \sigma_{f}=N \frac{s^{-1}}{64 \pi^{2}} \beta_{x} d \cos \theta_{1} d \theta_{2} d y d x\left(\frac{1}{1-x}\right)_{\rho}\left(\frac{1}{1+y}\right)_{\omega} 2 f_{r}\left(x, y, \theta_{1}, \theta_{2}\right) \tag{2.22}
\end{equation*}
$$

$$
\begin{equation*}
\int_{-1}^{-1+\omega} f(y)\left(\frac{1}{1+y}\right)_{\omega} d y=\int_{-1}^{-1+\omega} \frac{f(y)-f(-1)}{1+y} d y \tag{2.23}
\end{equation*}
$$

where the hard amplitudes $f_{r}\left(x, y, \theta_{1}, \theta_{2}\right)$ can be deduced from the Eqs. (4.2) and (5.1) of [7] for the subprocesses $\gamma$-gluon and $\gamma$-quark, respectively.

With the above notations in place, we analytically regulated ultraviolet, infrared, and collinear singularities using dimensional regularization [10] in the unpolarized case and dimensional reduction [11] for the polarized one. To get the total cross sections, we had to integrate over the phase space of generally five variables, e.g. Feynman parameters $x_{b}$, variables x , y and azimuthal and polar angles of a heavy quark of momentum $p_{3}$ in the rest frame of the heavy quark pair. After plugging in virtual and real matrix elements, for the two contributing subprocesses, that were obtained in our previous publication [7], we found a complete agreement with the numerical results for the total cross sections of heavy quark production that were also presented in [7].

## III. DIFFERENTIAL CROSS SECTIONS

The quantity of interest for the experimental measurements is the so called transverse energy of a heavy quark jet which in the laboratory center of mass system of incoming hadrons is defined as follows:

$$
\begin{equation*}
E_{\perp}=p_{3}^{0} \sin \theta_{3} \tag{3.1}
\end{equation*}
$$

Here $p_{3}^{0}$ is the energy of the heavy quark and $\sin \theta_{3}$ is its polar angle with the beam direction, all in the laboratory frame. Therefore, the next step is to obtain differential cross sections in the transverse energy $E_{\perp}$ from the analytical expressions we got previously. Mathematically, this task is equivalent to changing and shuffling integration variables. However, one has to take into account that one has to deal with various physical center of mass systems involved in the process. While for a $2 \rightarrow 2$ kinematics this task can be readily done analytically, things get very complicated in the $2 \rightarrow 3$ case.

Starting with the $2 \rightarrow 2$ phase space, which is relevant to the leading order, soft and virtual contributions, we note that in this case the rest frame coincides with the partonic one. After some algebraic manipulations, for the physical cross section we obtain the following end result:

$$
\begin{equation*}
\frac{d \sigma}{d E_{\perp} d \cos \theta_{3}}=J \frac{-T}{(S+U)^{2}} f\left(x_{b}^{\min }\right) \frac{d \hat{\sigma}}{d t d u} \tag{3.2}
\end{equation*}
$$

with

$$
\begin{gather*}
J=\frac{2 S}{\sin ^{2} \theta_{3}} \sqrt{E_{\perp}^{2}-m^{2} \sin ^{2} \theta_{3}}, \quad x_{b}^{\min }=\frac{-T}{S+U} \\
T=-\sqrt{S}\left(p_{3}^{0}-\left|\vec{p}_{3}\right| \cos \theta_{3}\right)=-\sqrt{S}\left(\frac{E_{\perp}}{\sqrt{\sin ^{2} \theta_{3}}}-\sqrt{\frac{E_{\perp}^{2}}{\sin ^{2} \theta_{3}}-m^{2}} \cos \theta_{3}\right)  \tag{3.3}\\
U=-\sqrt{S}\left(p_{3}^{0}+\left|\vec{p}_{3}\right| \cos \theta_{3}\right)=-\sqrt{S}\left(\frac{E_{\perp}}{\sqrt{\sin ^{2} \theta_{3}}}+\sqrt{\frac{E_{\perp}^{2}}{\sin ^{2} \theta_{3}}-m^{2}} \cos \theta_{3}\right)
\end{gather*}
$$

where S is a total energy of the physical process in the laboratory center of mass system, $f_{i} \equiv F_{i} / x_{i}$ is the corresponding hadron structure function, and $d \hat{\sigma} / d t d u$ is the partonic cross section. The
differential cross section over $E_{\perp}$ is obtained from Eq. (3.2) by integrating its right hand side over the polar angle (in the laboratory center of mass system) of a heavy quark

$$
\begin{equation*}
\cos \theta_{3} \in\left\{-\sqrt{1-\frac{4 E_{\perp}^{2}}{S}}, \sqrt{1-\frac{4 E_{\perp}^{2}}{S}}\right\} \tag{3.4}
\end{equation*}
$$

Turning to the $2 \rightarrow 3$ kinematics, we were able to derive analytical transformations for the collinear configurations. In our case of $y=-1$ we obtain:

$$
\begin{equation*}
\frac{d \sigma}{d E_{\perp} d \cos \theta_{3}}=J \int_{x^{\min }}^{1} d x x_{b} f\left(x_{b}\right) \frac{-2}{S \beta_{x} T} \frac{d \hat{\sigma}}{d \cos \theta_{1} d x} \tag{3.5}
\end{equation*}
$$

with

$$
\begin{equation*}
x^{\min }=\frac{-T}{S+U}, u \rightarrow \widetilde{u}=x x_{b} U, x_{b}=-\frac{1}{x} \frac{T}{S+U} \tag{3.6}
\end{equation*}
$$

In the above expression $\cos \theta_{1}$ is a polar angle of a heavy quark with mimentum $p_{3}$ in the rest frame of a heavy quark pair, and $s=x_{b} S$ is the collision energy in the partonic center of mass system. Differential cross section in $E_{\perp}$ is obtained by integrating the right hand side of the Eq.(3.5) over the heavy quark polar angle $\cos \theta_{3}$ similarly to the 2 -body case.

What concerns a general case of $2 \rightarrow 3$ kinematics, e.g. hard gluon bremsstrahlung, we were unable to find a satisfactory analytical solution to reshufling five integration variables. Therefore, we adopted the following numerical procedure:

We seek a differential cross section

$$
\begin{equation*}
\frac{d \sigma}{d E_{\perp}} \equiv f\left(E_{\perp}\right) \tag{3.7}
\end{equation*}
$$

where function $f\left(E_{\perp}\right)$ is a heavy quark production cross section integrated over all the variables except $E_{\perp}$ (note that in this approach instead of a transverse energy $E_{\perp}$ one could have any other variable). In fact, we need its value at some given $E_{\perp}^{\prime}\left(\equiv E^{\prime}\right)$, i.e.

$$
\begin{equation*}
\left.\frac{d \sigma}{d E_{\perp}}\right|_{E_{\perp}=E^{\prime}} \equiv f\left(E^{\prime}\right) \tag{3.8}
\end{equation*}
$$

Now consider the following integral

$$
\begin{equation*}
\delta \sigma \equiv \int_{E^{\prime}-\frac{\delta E}{2}}^{E^{\prime}+\frac{\delta E}{2}} d E_{\perp} f\left(E_{\perp}\right) \tag{3.9}
\end{equation*}
$$

where we chose $\delta E$ to be a small enough increment of the variable $E_{\perp}$. Clearly, if one draws the function $f\left(E_{\perp}\right)$, on the plane $f\left(E_{\perp}\right)--E_{\perp}$, the value of $\delta \sigma$ above is a AREA of a rectangle under function $f\left(E_{\perp}\right)$ with a central value around $E_{\perp}$. From the other side, one can write an approximate relation for the area of this rectangle:

$$
\begin{equation*}
\delta \sigma \approx \delta E f\left(E^{\prime}\right) \Rightarrow \frac{\delta \sigma}{\delta E} \approx f\left(E^{\prime}\right) \tag{3.10}
\end{equation*}
$$

Therefore, combining Eqs. (3.9) and (3.10), we finally arrive at

$$
\begin{equation*}
f\left(E^{\prime}\right) \approx \frac{1}{\delta E} \int_{E^{\prime}-\frac{\delta E}{2}}^{E^{\prime}+\frac{\delta E}{2}} d E_{\perp} f\left(E_{\perp}\right) \tag{3.11}
\end{equation*}
$$

We will use Eq. (3.11) to obtain numerical values for the differential cross section $\frac{d \sigma}{d E_{\perp}}$ at a given value $E_{\perp}^{\prime}$. This is feasible as an integral in the right hand side of Eq. (3.11) is nothing but the TOTAL cross section of a heavy quark production except that it is integrated over interval $\delta \mathrm{E}$. To implement this idea practically, we have to introduce logical statements into our FORTRAN program, particularly in the parts that deal with the $2 \rightarrow 3$ kinematics (e.g. hard bremsstrahlung subprocesses), that will eventually cut off all the events when values of the transverse energy do not fall into the specified small area $\delta \mathrm{E}$ around some fixed value $E_{\perp}^{\prime}$. Thus, when running our FORTRAN subroutine for a particular set of integration variables, one has to check at the same time which value of a transverse energy it corresponds to. To begin with, note that our total cross section is being calculated with the variables in the rest and partonic frames. However, the transverse energy is measured in the laborary frame. Thus, to perform the above mentioned nontrivial verification, one has to perform several Laurence boosts of all the relevant kinematical variables numerically between different center of mass frames. In particular, first one has to boost kinematical variables (e.g. momenta) from the rest frame to the partonic one, where the light momentum $p_{1}$ is OFF z-axes. Secondly, one has to perform $\mathrm{SO}(3)$ rotation of the whole system by some angle which ensures that the light momentum $\mathrm{p}_{1}$ is directed along collision axes z . Third, one has to boost all the relevant kinematical variables to the laboratory frame. To ensure that all the numerical steps were done correctly, we derived several analytical relations between mainly massless kinematical variables in different center of mass frames, where possible. We then checked intermediate numerical values of the kinematical variables against those analytical ratios. Needless to say, we found a complete agreement.

To include a jet isolation criteria, we adopt the Snowmass convention [12], where particles are clustered in cones of Radius R in the pseudorapidity-azimuthal angle plane. Following work [13], the jet differential cross section is written as a sum of an open heavy quark cross section for which we already got all the necessary ingredients above, and the additional term $\mathrm{d} \Delta$ defined as:

$$
\begin{equation*}
d \Delta=\widetilde{S}_{3} M_{a, b} d \Phi_{3} d E_{J \perp} d \eta_{J} d \phi_{J} \tag{3.12}
\end{equation*}
$$

where $M_{a, b}$ are partonic matrix elements for the light parton bremsstrahlung (i.e. radiation) that we already used in our FORTRAN code, $d \Phi_{3}$ is our 3-body phase space, $d E_{J \perp} d \eta_{J} d \phi_{J}$ is the measure over jet variables (e.g. transverse energy, pseudorapidity, and azimuthal angle), and

$$
\begin{align*}
& \tilde{S}_{3}=\delta\left(E_{J \perp}-E_{1 \perp}\right) \delta\left(\eta_{J}-\eta_{1}\right) \delta\left(\phi_{J}-\phi_{1}\right)\left[\begin{array}{l}
-\theta\left(\left|\omega_{1}-\omega\right|<g\left(E_{1 \perp}, E_{\perp}\right)\right)- \\
-\theta\left(\omega_{1}-\omega_{2} \mid<g\left(E_{1 \perp}, E_{2 \perp}\right)\right)
\end{array}\right]+ \\
& +\delta\left(E_{J \perp}-E_{1 \perp}-E_{\perp}\right) \delta\left(\eta_{J}-\frac{\eta_{1} E_{1 \perp}+\eta E_{\perp}}{E_{J \perp}}\right) \delta\left(\phi_{J}-\frac{\phi_{1} E_{1 \perp}+\phi E_{\perp}}{E_{J \perp}}\right) \theta\left(\left|\omega_{1}-\omega\right|<g\left(E_{1 \perp}, E_{\perp}\right)\right)+  \tag{3.13}\\
& +\delta\left(E_{J \perp}-E_{1 \perp}-E_{2 \perp}\right) \delta\left(\eta_{J}-\frac{\eta_{1} E_{1 \perp}+\eta_{2} E_{2 \perp}}{E_{J \perp}}\right) \delta\left(\phi_{J}-\frac{\phi_{1} E_{1 \perp}+\phi_{2} E_{2 \perp}}{E_{J \perp}}\right) \theta\left(\left|\omega_{1}-\omega_{2}\right|<g\left(E_{1 \perp}, E_{2 \perp}\right)\right)
\end{align*}
$$

where

$$
\begin{gather*}
g\left(E_{i} E_{j}\right)=\frac{E_{i}+E_{j}}{\max \left(E_{i}, E_{j}\right)} R  \tag{3.14}\\
\left|\omega_{i}-\omega_{j}\right|=\sqrt{\left(\eta_{i}-\eta_{j}\right)^{2}+\left(\phi_{i}-\phi_{j}\right)^{2}} \tag{3.15}
\end{gather*}
$$

Indices 1 and 2 for the quantities in the above expression (3.13) for the function $\widetilde{S}_{3}$ indicate their connection to the heavy quark and heavy antiquark, respectively, while quantities without any index relate to the variables of the light parton. In Eq. (3.14), R is the usual jet-resolution parameter, which defines the cone size in the pseudorapidity--azimuthal angle plane.

## IV.CONCLUSIONS

We have presented calculation of some differential cross sections at the next-to-leading order accuracy for heavy quark production in association with a light jet in hadronic collisions. We have dealt with the cases when initial state particles are either unpolarized or longitudinally polarized. For the 2-body kinematics, as well as for the case of 3-body kinematics with collinear configurations, all the phase-space integrations are performed analytically. We conveniently present these solutions in a compact form. As a consequence, numerical evaluation of the corresponding part of our FORTRAN program proceeds extremely fast. For the 3-body kinematics we found partly analytical and partly numerical solution for the phase space integrations. All the analytical results in this work were obtained with the help of a symbolic manupulation program REDUCE [14]. Our method is universal and one allows to evaluate the jet differential cross sections in a general way, for instance applying kinematical cuts on various measurable observables, and for any limiting behaviour, for example one can easily apply the effective renormalization procedure for structure functions as described in [8]. The alternative, simpler option would be to use our matrix elements in conjunction with the structure functions determined in the fixed flavour number scheme [15]. The studies on the numerical results for various physical quantities of interest, including asymmetries, will be presented elsewhere.

Our results form part of the NLO description of heavy-quark pair photoproduction, the process being under extensive study by COMPASS collaboration [4] at CERN, and could be of relevance for the future experiments by LHeC collaboration [16] at CERN.

Finally we have to emphasize that our FORTRAN program should not be considered as an event generator for heavy quark production. It should rather be considered a tool capable of giving the exact NLO numerical result for the heavy quark cross section even when complicated kinematical cuts are imposed over the final-state partons.

## ACKNOWLEDGMENTS

We would like to thank B.A. Kniehl and G. Kramer for useful discussions. We are very grateful to Hannes Jung for his help with FORTRAN Libraries at DESY. Z.M. acknowledges support of the Georgian Shota Rustaveli National Science Foundation through Project No. FR/11/24.

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Article received: 2013-03-11

