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ONE APPROACH TO THE REGIME OPTIMIZATION OF WATER-POWER SYSTEM

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Abstract

Paper deals with the problem of long-term control of Hydro Power System. Actually, this is the problem of optimization of the regimes of Hydro Power Plants (HPP) with reservoirs, aiming to achieve the maximal economic effect. The problem is dynamic with traditional constraints. The model is based on the method of Markov decision process. The matrices of transition probabilities and the corresponding recurrent inequalities are composed. One practical idea is also suggested to avoid the dimension problem in the case of a large number of HPPs.

Keywords: *Hydro Energy, control, optimization, dynamic programming, strategy, Markov process.*

1. Introduction

Control issues of Energy Systems, especially of Hydro Power Plants (HPP) are well known among the problems of Operations Research. Optimal control of the system comprehends to obtain the regimes (work schedules) of synchronized work of its terms so, that the maximal economic effect is achieved. The existence of HPP-s with reservoirs is essential for the optimization of the regimes of the system. It's there basic task to balance power and energy and they cover the peak parts of the load. Other terms of energy generation, in fact do not take part in the process of control and cover basic parts of the load.

Consequently, here we only consider the control problem for HPP with reservoir. Actually, this is the problem of optimal storage control of water and fuel. It should be noted that in our region (South Caucasus) and possibly elsewhere, natural resources of the neighbor countries differ with regard to Energy Industry. Some of them have high Hydro Energy potential, while others – expensive fuel. Besides, due to the seasonality of the water flow, capacities of generation of hydro energy during the year are sharply diverse. Therefore coordinated control of energy systems of the countries at the regional level would imply economical benefits as for each country, also in general. First we consider separately regulation process for the HPP with one reservoir and then proceed to the problem of control of the system. The issue of optimal functioning of HPP with one reservoir under various constraints (such as the volume of the reservoir, the character of water flow etc.) is considered in the works [1, 2, 3]. The long-term control problem of the system is dealt in the paper [4]. Here we suggest mathematical model for the long-term (annual) regime optimization, which is based on on the method of Markov decision process (see [5, 6]).

2. The process of regulation of HPP with reservoir as Markov decision process.

Let's consider scheduling dynamic problem with one year horizon. One year horizon is naturally imposed since the water flow is cyclic periodic. Year is divided into the equal time intervals with appropriate enumeration $t =, 2, \ldots, T$. Generally the length of the interval is either one month or half of the month. Assume that the volume of the water flow in the reservoir with

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volume Q during the time period t equals x^t . $x^t \left(t=1,2,\ldots,T\right)$ are independent integer-valued random variables with known distributions:

$$P(x^t = k) = P^t(k),\ k = \overline{1,K},\quad \sum_{k=1}^{K} P^t(k) = 1\ \ \forall t = \overline{1,T}. \qquad (1)$$

Apparently we assume that the unit of the water flow (volume) is chosen. Traditionally the work of HPP is described as follows (see. [1, 2]): The plant posses supplied power, that is the maximum of water output Y. Thus at every stage the water expenditure is bounded. Define the latter by y^t at stage t. Then the vector $(y^1, y^2, \ldots, y^T)$ is the control policy, i.e. the strategy. If z^t denotes the volume of water at the beginning of stage t in the reservoir of volume Q, then the regulation process can be represented as follows:

$$z^{t+1} = \begin{cases} Q, & z^t + x^t - y^t \geq Q, \\ z^t + x^t - y^t, & z^t + x^t - y^t \leq Q \end{cases} \qquad (2)$$

$$y^t \leq \min\left(z^t + x^t, Q, Y\right),\quad u^t = [z^t + x^t - y^t - Q]^+,$$

where u^t denotes the volume of wasted water and $a^+ = \frac{|a|+a}{2}$. It's clear that the random variable z^{t+1} can be probabilistically defined completely if the value of z^t is known and it does not depend on previous periods (by assumption z^t are independent random variables). Hence, for the reservoir we have the Markov process $\left\{z^t\right\}$ with possible values $0,1,\ldots,Q$. Appling (2) we construct the matrix $\left\{P_{ij}(y^t)\right\}$, $i, j = 0,1,2,\ldots,Q$ of transition probabilities from the state $z^t = i$ to the state $z^{t+1} = j$. For every t the matrix has the following form (Table 1.):

Table 1.

Z^{t+1} \ Z^t	0	1	2	...	k	...	$Q-y-1$	$Q-y$	...	$Q-1$	Q
0	$\hat{P}^t(y)$	$P^t(y+1)$	$P^t(y+2)$	...	$P^t(y+k)$	...	$P^t(Q-1)$	$P^t(Q)$	...	$P^t(Q+y-1)$	$1-\hat{P}^t(Q+y-1)$
1	$\hat{P}^t(y-1)$	$P^t(y)$	$P^t(y+1)$	...	$P^t(y+k-1)$	...	$P^t(Q-2)$	$P^t(Q-1)$	...	$P^t(Q+y-2)$	$1-\hat{P}^t(Q+y-2)$
2	$\hat{P}^t(y-2)$	$P^t(y-1)$	$P^t(y)$	...	$P^t(y+k-2)$	...	$P^t(Q-3)$	$P^t(Q-2)$	...	$P^t(Q+y-3)$	$1-\hat{P}^t(Q+y-3)$
...	...	...	...	...	...	...	...	...	...	...	...
k	$\hat{P}^t(y-k)$	$P^t(y-k+1)$	$P^t(y-k+2)$	...	$P^t(y)$	...	$P^t(Q-k-1)$	$P^t(Q-k)$	...	$P^t(Q+y-k-1)$	$1-\hat{P}^t(Q+y-k-1)$
...	...	...	...	...	...	...	...	...	...	...	...
y	$P^t(0)$	$P^t(1)$	$P^t(2)$	...	$P^t(k)$	...	$P^t(Q-y-1)$	$P^t(Q-y)$	...	$P^t(Q-1)$	$1-\hat{P}^t(Q-1)$
$y+1$	0	$P^t(0)$	$P^t(1)$	...	$P^t(k-1)$	...	$P^t(Q-y-2)$	$P^t(Q-y-1)$	...	$P^t(Q-2)$	$1-\hat{P}^t(Q-2)$
...	...	...	...	...	...	...	...	...	...	...	...
$y+k$	0	0	0	...	$P^t(0)$	...	$P^t(Q-y-k-1)$	$P^t(Q-y-k)$	...	$P^t(Q-k-1)$	$1-\hat{P}^t(Q-k-1)$
...	...	...	...	...	...	...	...	...	...	...	...
$Q-1$	0	0	0	...	0	...	$P^t(0)$	$P^t(1)$	...	$P^t(y)$	$1-\hat{P}^t(y)$

Q	0	0	0	...	0	...	0	$P^t(0)$	...	$P^t(y-1)$	$1-\hat{P}^t(y-1)$

Here, for the sake of simplicity the index t is omitted for the parameter y and

$$\hat{P}^t(k) = P(z^t \le k) = \sum_{j=0}^{k} P^t(j).$$

To the transition matrix, at each stage t corresponds the mathematical expectation of losses, which depend on the quantities z^t, x^t and y^t. The losses at each stage consist of two summands – the losses of deficiency and of unutilized water. The deficiency losses are related to the costs that cover the deficit by means of purchase of expensive energy, to the drop in reputation, the penalty from the consumers etc. These costs can be described separately as the functions of above cited factors. Here, for the simplicity we restrict ourselves by the linear relations. If d denotes the losses, corresponding to the unit deficit and c denotes the losses related to the waste of unit water, then expected losses are

$$l^t(z^t) = \left[R^t - N(y^t)\right]^+ \cdot d + \sum_{k=1}^{K}\left[z^t + x^t - y^t - Q\right]^+ \cdot P^t(x_k)\cdot c, \qquad (3)$$

where R^t is the consumed electric power (cost) at stage t, $N(y^t)$ denotes the quantity of electric power (*kWh*) generated by the y^t quantity of water (m^3). The following formula is valid – $N = (9,8\cdot y\cdot H\cdot\eta)/3600$, where H is the pressure (height difference) at the power plant (in *meters*), η – efficiency of the power plant (turbine). In altitudinal reservoirs H is considered as constant, while in the rest of cases it depends on the average volume of water in the reservoir at the given stage.

We apply the functional equation method of dynamic programming. Denote $f_t(i)$ the expected losses when following the optimal policy from the stage t to stage T (inclusively), where the volume of water in the reservoir at the beginning of the stage t equals i. Due to the optimality principle

$$f_i(i) = \min_{y^t}\left\{l^t(z^t) + \sum_{j=0}^{Q} P_{ij}(y^t)\cdot f_{t+1}(j)\right\}, \quad t = T-1, T-2, \ldots, 1. \qquad (4)$$

Applying the standard procedure for each initial state i, we obtain the optimal policy $\overline{y_1}, \overline{y_2}, \ldots, \overline{y_T}$ and the corresponding summary losses $f_i(i)$. $f_T(i)$ is easily calculated by means of $y^t = z^t + x^t$.

3. Control of Energy System with several HPPs with reservoirs.

Assume the system consists of M HPPs, each one with technical parameters – $H_\mu, \eta_\mu, K_\mu, Y_\mu, Q_\mu$ $(\mu = \overline{1,M})$. In the corresponding relations (1), (2) and the transition matrices (M matrices at each stage) we have the following notations – $x_\mu^t, y_\mu^t, z_\mu^t, u_\mu^t, P_\mu^t(k), P_{\mu ij}^t$. It's more convenient to deal with the matrices of equal order. For this sake, in the transition matrices we consider $\overline{Q} = \max(Q_\mu)$ and $\overline{Y} = \max(Y_\mu)$ instead of the quantities Q_μ and Y_μ and assume that the transition probabilities corresponding to the impossible states equal zero.

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At every stage we choose the vector $Y^t = (y_1^t, y_2^t, \ldots, y_M^t)$. This vector transfers the system from the state $Z^t = (z_1^t, z_2^t, \ldots, z_M^t)$ to new state $-Z^{t+1}$. Corresponding transition probability is $\prod_{\mu=1}^{M} P_{z_\mu^t \cdot z_\mu^{t-1}}$. At the stage t the system suffers losses, which depend on the vectors Z^t and Y^t. Analogously as above the following holds

$$l^t(Z^t, Y^t) = \left[R^t - \sum_{\mu=1}^{M} N(y_\mu^t) \right]^+ \cdot d + \sum_{\mu=1}^{M} \sum_{x_\mu=1}^{K} \left[z_\mu^t + x_\mu^t - y_\mu^t - Q_\mu \right]^+ \cdot P_\mu^t(x_\mu^t) \cdot c \qquad (5)$$

The sequence of the vectors $Y^t, (Y^1, Y^2, \ldots, Y^T)$ – the strategy, should be chosen to minimize expected annual summary losses. Here, as above we write the functional equations based on the principle of dynamic programming. Denote $F_t(Z^t)$ the expected losses when following the optimal strategy from the stage t to stage T (inclusively), where at stage t the system is in the state Z^t. For $t = T-1, T-2, \ldots, 1$ (5) implies

$$\begin{gathered} F_t(Z^t) = \min_{Y^t} \left\{ l^t(Z^t, Y^t) + \prod_{\mu=1}^{M} P_{z_\mu^t \cdot z_\mu^{t+1}} \cdot F_{t+1}(Z^{t+1}) \right\}, \quad t = T-1, T-2, \ldots, 1. \\ F_T(Z^t) = \min_{Y^t} l^T(Z^t, Y^t). \end{gathered} \qquad (6)$$

$F_t(Z^t)$ is easily calculated when $Y^T = Z^T + x^T$.

4. Remarks.

a) In the presented model, like in cited works, hydro energy, in fact is regarded free of charge. This assumption is justified for a single structure or a single country when the model is closed. In the regional models, where the buy-sell process of energy is essential factor, the prices are taken into account and the criteria should be the maximization of the profit. If necessary the model may include the constraint on the balance of capacities.

b) The problem of dimension occurs to be very hard in the method of dynamic programming. In case of energy system with three HPPs the problem can be practically resolved. In case of more HPPs (e.g. there are 6 HPPs with considerable reservoirs in Georgia) we have presented the following suggestion (see [7]): at every stage instead of the vector $(y_1^t, y_2^t, \ldots, y_M^t)$ we deal with one parameter N^t – the sum of electric power generated at all HPPs at stage t. N^t should be distributed to the plants proportionally to the expected losses $N_i(u_i^t)$. As above, the losses are represented by the formula $N_i^t = (9{,}8 \cdot u_i^t \cdot H_i \cdot \eta_i)/3600$ *(kWh)* and they depend on the quantities z_i^t, y_i^t and $\overline{x}_i^t$, where $\overline{x}_i^t$ is mathematical expectation of x_i^t. Following this procedure at every stage we obtain the partition $(N_1^t, N_2^t, \ldots, N_M^t)$ and appropriate vector $(y_1^t, y_2^t, \ldots, y_M^t)$. Such an approach allows corresponding simple software and significantly reduces the dimension problem.

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