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THE ABSTRACTION EFFECT ON LOGIC RULES APPLICATION

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Abstract
The aim of this study is to analyze the relationship between training on abstraction and the comprehension of logic rules. In order to evaluate the possibility of improvement on logic performance we have selected the particular case of the DeMorgan's laws. The dispute between the natural logic approach and the mental models theory is analyzed from the perspective of such abstraction effect. Two experiments are reported. The first one suggests that the presentation of a formal proof promotes a better comprehension of DeMorgan´s laws than the use of visual resources or colloquial examples. The second one offers a stronger test for the same abstraction effect. Some limitations concerned with the syntactic meaning of negation and the differences between constructive and evaluative conditions are discussed. Since the meaning of abstraction for the psychology of reasoning is pointed out as critical some suggestions for further research and possible educational applications are mentioned.

Keywords: *Cognition-reasoning-logic-abstraction*

Reasoning and Logic Rules

We apply logic rules every day without being fully aware of it [1]. We cannot avoid thinking in terms of classes or sets, or to connect elements with abstract operators like implications, negations, conjunctions or disjunctions. Some syllogistic figures are even so spontaneous that their cognitive processing occurs in an automatic way (e.g., the *modus ponens* reasoning rule [2]). This reasoning rule states that given a conditional proposition such as *if p, then q* (or symbolically formulated $p \rightarrow q$), and another proposition that affirms *p*, the necessary conclusion is *q* [3]. This inference can be described as intuitive, and has been well documented in many psychological experiments [4]. The ubiquity of logical components in human thinking has been widely acknowledged by historical and current psychologies of reasoning [5]. The two dominant trends in this field –the syntactic and the semantic approaches to deductive thinking– agree in that logic plays a central role in human cognition [6]. Nevertheless, the cognitive relevance of the formal objects studied by logic for the understanding of human thinking is far from being agreed [5]. The syntactic or rules-driven models propose that logic rules are natural components of the mind [7, 2]. On the other hand, the semantic or meaning-driven theories suggest that the human mind has no formal rules at all, and that we solve logical problems by constructing some kind of mental models that only partially grasp the objects of logic in a fuzzy way [8, 9, 10]. Reasoning is rather a succession of images that are finally tested against counter-examples for the obtained conclusions [11, 12]. During the last decades, the psychology of reasoning has redefined its research agenda to include other issues that emphasize pragmatic restraints like the ecological setting [13] or the similarities between reasoning and decision making [4].

Nevertheless, in this wide and heterogeneous state-of-the-art concerned with the confluence of logic and psychology, the centrality of abstraction is generally recognized and acknowledged by all the theories of human reasoning [10, 2]. The term abstraction is here conventionally defined as the selective reduction of dimensions in the structuring of information about objects and events [14]. In

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this study we suggest that the level of abstraction achieved in reasoning can be experimentally manipulated. Abstraction in this context can be defined as the inferential distance between a concrete experimental material or task and the formality of the logic rules involved in the experiment. Hence, the smaller the distance, the higher the abstraction level of the experimental material. For example, the presentation of a formal proof for a logic rule is more abstract than the illustration of the same rule by means of verbal sentences that apply the same formal structures in concrete daily propositions. Keeping in mind the meaning of abstraction proposed here, we could say that a formal proof is nearer to the domain of logic than any colloquial example made with concrete sentences about daily objects.

In this context, the aim of the study is to contribute to the current discussion about reasoning with focus on the relevance of abstraction for the application of logic rules. The particular case chosen in this study is the pair of logic rules erroneously attributed to the British mathematician Augustus DeMorgan [15] concerned with the negation of conjunctions and disjunctions. Although DeMorgan made many transcendent contributions to mathematics and logic during the nineteenth century, these rules were well known by the medieval logicians [16]. One variety of DeMorgan's laws states two equivalences, one for the negation of conjunctions, and another for the negation of disjunctions [3]. Equations 1 and 2 present the DeMorgan's laws for any propositions p and q [1], using the conventional symbols of negation $\neg$, conjunction $\wedge$, disjunction $\vee$ and equivalence $\Leftrightarrow$.

$$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q \quad (1)$$

$$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q \quad (2)$$

In logic, a proposition is defined as any sentence that can be considered either true or false [3]. If one of these values cannot be clearly attributed, then such sentence is not a proposition [1]. The term proposition is used here with the same meaning attributed by Tarski [17] to the term sentence. An example for the Equation 1 is the formal equivalence between the following two statements: *a*) It is not true that: summers are happy (p) and winters are sad (q), *b*) summers are not happy ($\neg p$) or winters are not sad ($\neg q$). For Equation 2, an example is the formal equivalence between the following two statements: *c*) It is not true that: Fridays are short days or Mondays are long days, *d*) Fridays are not short days and Mondays are not long days. Statement *a* is equivalent to statement *b*. Statement *c* is equivalent to statement *d*. This equivalence is formal and does not necessary occur in other dimensions like the semantic or the pragmatic [9]. After the early contributions of Wason and Johnson-Laird [18] to the psychology of reasoning, the distinction between structure and content became widely accepted for arguing that rules are structural components and have a formal meaning, but the content is differentiable from the point of view of semantics or pragmatics.

Psychological Experiments With DeMorgan´s Laws

Many previous studies have been conducted to analyze the cognitive processing of negation [19, 20, 2, 18]. For the particular case of DeMorgan´s laws only two previous contributions provide a specific experimental paradigm that makes the study of the relationship between abstraction and the application of logic rules easier. The first contribution referred here [21] evaluates the spontaneuos recognition of DeMorgan´s laws. In that study, the experimental subjects were asked to find equivalent colloquial expressions for certain given sentences, which were presented in capital letters and structured as the first part of Equations 1 and 2. Four possible equivalences were then presented in small letters structured as the second part of Equations 1 and 2. The experimental subjects were supposed to find out which sentence represented by small letters better matched the sentences represented by capital letters. This specific experimental task belongs to two classical paradigms of the psychology of reasoning: the inferential paradigm and the selection paradigm [12]. In the inferential paradigm some sentences are provided and the experimental subjects have to infer a conclusion. This is opposed to the truth table paradigm, which participants are presented with a rule and then they are requested to recognize its correct application [11]. In the selection paradigm participants are presented with a list of possible answers and they have to select the

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option that fits a sound conclusion. The selection paradigm is opposed to the constructive paradigm, in which participants have to fill in the blanks with a conclusion for a given argument.

The experiment conducted in a previous study [21] included a total of ten trials. Five were applications of Equation 1 and the other five applied Equation 2. No experimental manipulations were introduced. The results of this study showed that these laws are difficult to understand, which is consistent with previous findings [22] and with recent contributions [23]. The average of recognitions was barely between two or three out of ten [21]. Not only the correct recognitions were few but also the mistakes showed patterns of difficulty. The mistakes figures were random, especially for Equation 1. Similar results were found in a second experiment that studied the relationship between logic intuition and personality with the same materials and procedures [24]. In this second study, an extension of the first one, the main result suggests that the quality of introspection explains around 43% of the logical intuition of DeMorgan´s laws. This regression model was interpreted as a relevant link between deductive reasoning and personality traits like the tendency to introspection. Similar results were previously obtained by Fumero, Santamaría, and Johnson-Laird [25] for the relationship between personality traits and the recognition of logic rules like *modus tollens* syllogisms ($p \rightarrow q, \neg q \Leftrightarrow \neg p$).

The design for this study followed Macbeth *et al.* [21, 24], and, according to the criteria introduced by Wason and Johnson-Laird [18], can be classified as a *sentential* and *evaluative* variety of negation. The sentential negation refers to propositions in which the negation applies to the whole expression. Its opposite is the constituent negation, which refers to a partial negation inside the proposition [26]. The evaluative condition refers to the nature of the experimental task and is opposed to the constructive condition. The task is evaluative when the experiment provides a list of possible answers and subjects choose one of them. In contrast, the constructive condition offers only a blank space and subjects have to construct their own answers.

The revision of these previous studies suggests that: i) the comprehension of DeMorgan´s laws is difficult [18], ii) the formal meaning of the rules of logic is probably less processed than their semantic or pragmatic dimensions [9, 10, 21, 24]. The first point motivates the experimental intent to generate more understanding about DeMorgan´s laws by cognitive manipulations. The second point suggests that the abstraction level of processing might have a critical relevance for the successful understanding and application of these rules. In this context, the specific aim of this work is to increase the recognition of these laws through the manipulation of the abstraction level during the experimental interventions. The working hypothesis states that more abstract experimental interventions promote better performances in the recognition of DeMorgan´s laws. This hypothesis comes from the idea that logic rules are abstract objects. Although the application of such abstractions occurs in more concrete worlds, an adequate comprehension of logic may require a controlled abstraction effort [2, 6, 7].

Method

Two experiments contributed to study the relationship between the comprehension of DeMorgan´s laws and the level of abstraction in which the cognitive processing is set. The aim of the first experiment was to generate better performances for the recognition of DeMorgan´s laws in test phase through the manipulation of two factors in the study phase. The first factor was the format of the experimental materials. Two conditions were selected for this factor: *i*) the presentation of a formal proof for the theorems in Equations 1 and 2 and, *ii*) the presentation of visual resources. The formal proof was more abstract than its visual representation by means of Venn-Euler diagrams [3]. The second factor was concerned with the use of colloquial daily examples. The two levels defined for this second factor were the use and the lack of use of examples. It is considered here that the lack of illustration by examples is more abstract than its actual presentation [27]. This 2x2 factorial design generates four experimental groups that enable statistical comparisons for interactions between levels of factors and between factors beyond levels distinctions [28]. The fundamental applied purpose of this experiment is to study which intervention promotes better understanding of

DeMorgan´s laws. The main conjecture proposed here states that higher abstraction promotes better logic rules´ comprehension. Although the opposite claim seems reasonable, i.e. that concrete experimental materials facilitate the access to abstract concepts, the main conjecture of the present study is focused on the nature of the logic rules. Such formal structures properly belong to a domain that is abstract *per se*, but also operate in other concrete domains like daily conversations. The central intuition of the present study suggests that cognitive training on abstraction promotes better performances when the same abstract rules are applied in concrete tasks because such rules are abstract by nature [29]. This conjecture does not imply that abstraction is the only valid reasoning promotion method. That educational issue requires other didactic research for specific teaching situations that exceeds the present study. In this context it is suggested that a formal proof is more abstract than the Venn-Euler diagrams. And the lack of examples is considered more abstract than the use of examples.

The second experiment proposes a stronger test for the abstraction hypothesis after the analysis of the results of the first experiment. It focuses only on the extreme positions of a theoretical gradient of abstraction that goes from the most abstract manipulations to the most concrete ones. This second experiment seeks for a bigger effect size through the comparison between the lack of manipulation and the strongest manipulation obtained from the first experiment. All the variables included in the statistical tests of both experiments turned out to be compatible with the normality and homoscedasticity assumptions after Kolmogorov-Smirnov´s and Levene´s tests, respectively.

Experiment 1: Improving Performance in Logic by means of Abstraction

Sample

A sample of 191 university students was randomly recruited at the National University of Entre Ríos, located in the city of Paraná, Argentina. All the recruited students were undergraduates from social sciences careers that exclude logic and mathematics from their study plans. An informative consent form was signed by each subject before taking part in the experiment and the participation remained anonymous. The experimental sessions were conducted in groups of around 50 subjects in a classroom at the same university. 53.3% of the participants (n=102) were female and the average age was 21.3 years (SD=1.2 years). The proportion of male and female students was homogenous in all the experimental groups. Each participant was randomly assigned to one of four experimental groups. The sample size of each group and the corresponding experimental manipulations are described in Table 1.

Materials and procedure

All the participants completed a study phase and a test phase. During the study phase different experimental interventions were implemented. An experimental task that reliably measures the individual performance in logic for DeMorgan´s laws was administered to all the participants during the test phase, right after completing the study phase. This experimental task was applied and psychometrically analyzed in previous studies [21, 24]. The instructions were to find an equivalent expression in small letters, among four given possibilities, to other expression in capital letters, as mentioned above. The complete evaluation includes ten trials. Five are applications of the law expressed in Equation 1, and the other five apply the law of Equation 2. The experiment was carried out in a paper and pencil format. An English translation of the original version used in Spanish for the experimental task is shown in Appendix 1. To perform the 2x2 factorial manipulations four experimental groups were defined. The first group received a formal proof of DeMorgan´s laws without colloquial examples. The second group received the same formal proof with examples. The third group received Venn-Euler diagrams instead of a formal proof and no examples. And the fourth one received Venn-Euler diagrams and examples. The proof and the diagrams were considered as levels of the format factor. The presence and the absence of examples were considered as levels of the example factor.

To make it more functional, the proofs introduced as format manipulations during the study phase took the form of mathematical explanations about the necessary equivalence between both sides of

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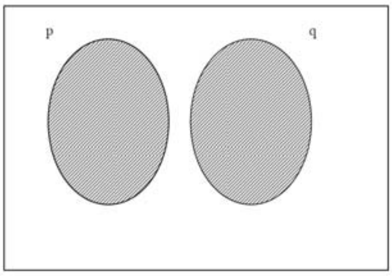
Equations 1 and 2. One proof was presented for Equation 1 and another for Equation 2. The set theory strategy was used for both proofs [3]. The use of symbols for the correspondence of propositions was determined i.e., for disjunction with union it was used $\cup$, for conjunction with intersection it was used $\cap$, and for negation with complementation it was used a line above sets [1]. Equations 3 and 4 introduce the proofs presented in experimental groups 1 and 2. This abstract proof strategy took an arbitrary element *x* and showed that the inclusion of this element in the first set implies its inclusion in the last one. The inclusion of this element in a set was expressed by the symbol $\in$ and its negation by $\notin$.

$$x \in \overline{P \cap Q} \Leftrightarrow x \notin (P \cap Q) \Leftrightarrow x \notin P \vee x \notin Q \Leftrightarrow x \in \overline{P} \vee x \in \overline{Q} \Leftrightarrow x \in (\overline{P} \cup \overline{Q}) \quad (3)$$

$$x \in \overline{P \cup Q} \Leftrightarrow x \notin (P \cup Q) \Leftrightarrow x \notin P \wedge x \notin Q \Leftrightarrow x \in \overline{P} \wedge x \in \overline{Q} \Leftrightarrow x \in (\overline{P} \cap \overline{Q}) \quad (4)$$

The study phase in groups 1 and 2 lasted about ten minutes. The experimenter presented the proof of each equation as follows. First, he wrote Equations 3 and 4 on a whiteboard and invited the participants to pay attention to the sets and elements. Then he wrote the first part of Equation 3 ($x \in \overline{P \cap Q}$) and indicated that an arbitrary element *x* belongs to the complement of the intersection of P and Q. After that the experimenter explained that the latter statement is equivalent to say that the same element *x* does not belong to the intersection of P and Q, and presented the second part of Equation 3 (i.e. $x \notin (P \cap Q)$). Then he moved forward to the next step and continued the same way until the last step. At this moment the experimenter remarked that the element *x* belonging to the complement of an intersection in the first step is the same element belonging to the union of complements in the last step of Equation 3. The same strategy was applied to describe the inferential shift in Equation 4 for the other DeMorgan´s law. The Venn-Euler diagrams presented as an alternative format manipulation with less abstraction processing were introduced as a sequence of each step of Equations 3 and 4. And this experimental manipulation lasted more or less the same time. For this visual level of the format factor introduced in groups 3 and 4, no proof was given. Figure 1 shows a fragment of the set diagrams sequence corresponding to Equation 4. The shadowed sets *p* and *q* represent a union which corresponds to the disjunction operator in logic. If an arbitrary element *x* does not belong to the shadowed area in Figure 1 (second step in Equation 4), this is equivalent to say that this element is not in *p* and not in *q* (third step on Equation 4).

Figure 1
Venn-Euler Diagram Representing the Second and Third Steps in Equation 4



Hypotheses
The experimental hypothesis H1 predicts a higher mean of correct answers for the experimental group 1 than for the other groups. The experimental hypothesis H2 states that the proof format generates a better performance than the visual format. The hypothesis H3 states that the lack of examples generates better performances than the use of them. Hypothesis H1 is an interaction conjecture that combines levels of both factors: proof presentation and no examples. Hypothesis H2 and H3 are factorial conjectures that operate inside each factor. Only if the interaction does not occur, then the factor conjectures are reasonable to be tested [28]. In this way, H1, H2, and H3 predict better performances for more abstract interventions. Thus, it is assumed that in H1 proof plus no examples is the most abstract intervention in this experiment, in H2 proofs are more abstract than diagrams, and finally that in H3 the lack of examples is more abstract than the use of them.

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Results and discussion

The main results of the factorial ANOVA are presented in Table 1 and Figure 2. H1 (F=2.878, p=0.091, η_p^2= 0.015) and H3 (F=0.038, p=0.538, η_p^2= 0.002) were rejected. H2 was compatible with the evidence (F=5.927, p=0.016, η_p^2= 0.031) with a small effect size for a partial eta squared bigger than 0.01 and smaller than 0.06.

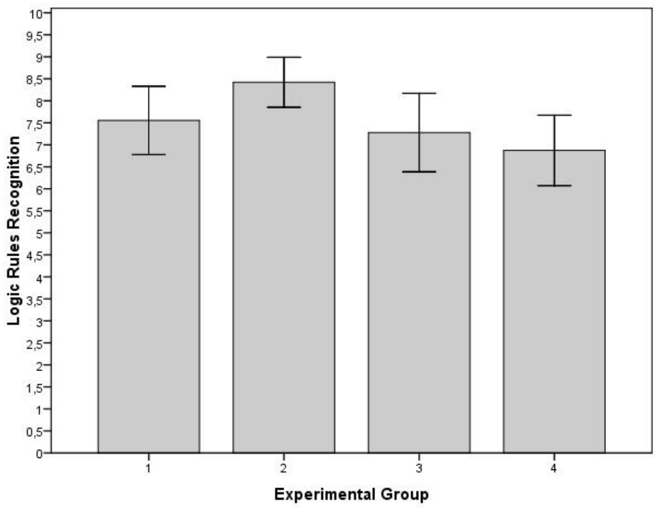
Table 1

Description of the Experimental Groups and their Performance in Logic

Experimental Group	Sample Size	Manipulation		Mean	Standard Deviation
		Format	Example		
1	47	proof	no	7.55	2.64
2	50	proof	yes	8.42	1.84
3	47	visual	no	7.28	3.03
4	47	visual	yes	6.87	2.72

Figure 2

Recognition of DeMorgan´s Laws



Note: Error bars represent a 95% confidence interval for the mean

No interaction effect was found for the combination of format and example factors. Although in Figure 2 the highest performance corresponded to the experimental group 2, the factorial ANOVA did not show significance for this combination of proof plus examples. Thoroughly, a t test for the difference between groups 1 and 2 showed no significant differences (t=-1.883, p=0.063, Cohen´s d=0.384). Nevertheless, further tests with higher statistical power ideally near to 1-β=0.80 would probably generate significance for the observed effect size of Cohen´s d. If we accept this assumption, the results of Experiment 1 are consistent with the abstraction hypothesis. An abstraction effect might be postulated after this evidence. The use of formal proofs during the study phase generated better performances in the recognition of DeMorgan´s laws during the test phase. The non-significant results obtained for the example factor suggests that the use of colloquial illustrations for these rules does not improve comprehension. It seems that examples are complementary but not structural for the cognitive processing of DeMorgan´s laws. This result was obtained for the case of selection tasks. The experimental paradigm employed here is the inferential one. When an alternative paradigm was applied in previous studies (i.e. the truth tables paradigm),

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the best performances were obtained for the experimental training on thematic schemata [27]. This probably means that the abstraction effect occurs in inferential paradigms but not in truth table paradigms. Further research is also needed to compare with the constructive condition of negation referred by Klima [26], in which subjects are asked to construct their own expressions instead of selecting one from a given list. Hence, a limitation of the present study is that only the sentential and evaluative conditions were analized inside the inferential paradigm.

The format factor that obtained significance in this experiment showed a small effect size (η_p^2= 0.031). It is suggested here that this abstraction effect needs more evaluation because the factor design mixes the example factor inside the statistical comparisons. Although the example factor resulted non-significant, it adds some variability to the comparison inside the format factor. From this point of view, it seems relevant to propose a stronger and clearer test for the abstraction conjecture. Thus, a second experiment was conducted with that aim.

Experiment 2: A Stronger Test for the Abstraction Effect Hypothesis

Only two groups were compared in this experiment. A control group which received no treatment at all, and an experimental group which received the most abstract material and procedures according to Experiment 1. To prevent any promotion of abstraction, there was no manipulation in the control group. The study phase was completely omitted for this group of subjects. On the other hand, the experimental group received the same proof presentation as the group 1 in Experiment 1. This manipulation is the most abstract in a theoretical gradient of abstraction that goes from the concrete application of examples on one side, to the pure formality of a mathematical proof on the other side. The purpose of this comparison between the absence of abstraction and the highest abstraction is to generate a statistical significant difference between performances. This strategy can be considered as a stronger test for the central conjecture proposed in the present study. Technically, the specific aim of Experiment 2 is to produce a bigger effect size than the small partial eta squared obtained in Experiment 1 for the abstraction effect conjecture. It is also proposed that in Experiment 1 the effects were confused because of the factor design applied. In this Experiment 2, instead, the interventions are clear-cut and not combined. While the control group remains in a natural state, the experimental group receives the strongest and clearest intervention of the abstract proof strategy.

Sample

A new sample of 66 students was randomly recruited from the same target population as in Experiment 1. The students that had participated in Experiment 1 did not take part in this experiment. The selected participants were randomly assigned to the control group or to the experimental group. Similar proportions of gender and age average as for Experiment 1 were chosen for the Experiment 2.

Materials and procedure

The same materials applied in Experiment 1 were used in Experiment 2. The control group received no treatment at all. Whereas the experimental group received a formal proof non-technically explained in the study phase. This intervention took about five minutes. In the test phase all the participants received the same task used in Experiment 1.

Hypothesis

The experimental hypothesis predicts a big effect size for the comparisons between the absence of treatment and the most abstract treatment for the recognition of DeMorgan´s laws. An effect size bigger than 0.80 Cohen´s *d* is suggested.

Results and discussion

The experimental group (mean=8.61, SD=2.57) obtained a better performance than the control group (mean=2.53, SD=2.62). The difference resulted significant (t=9.27, p<0.001) and the effect size was big (Cohen´s d=2.34). These findings suggest that the cognitive processing of DeMorgan´s laws for the sentential and evaluative conditions [26] inside the inferential paradigm does improve when the study phase is set in an abstract level. A relevant theoretical gradient of abstraction levels might be postulated after these findings. The proximity to concrete materials and procedures during the study phase probably generates the worst performances during the test phase. Inversely, the proximity to the extreme abstract side of the gradient during the study phase produces the best performances during the test phase. A limitation in this experiment is that only the sentential and evaluative conditions were studied. It is recommended for further studies to test the abstraction conjecture in constituent and constructive conditions according to the distinction proposed by Klima [26] and emphasized in classical studies of reasoning by Wason and Johnson-Laird [18]. It is also necessary to test the abstraction effect for the truth table paradigm according to the current psychology of reasoning.

Discussion

The relationship between logic rules axiomatically constructed and its application in plausible human contexts can be considered as one of the central issues for the psychology of reasoning [2, 4]. Another way to put forward this problem is to ask for the meaning of abstraction in human reasoning. Both theoretical trends in the psychology of thinking, i.e. the *syntactic* and the *semantic*, agree with the relevance of abstraction but disagree with the meaning of abstraction [5]. The present contribution suggests that the experimental materials and procedures have a critical relevance for the improvement in comprehension of abstract rules. DeMorgan´s laws seem to be better understood when the setting of the experimental tasks is previously adjusted to its proper level of abstraction. The presentation of a formal proof during the study phase resets the cognitive processes in a more adjusted environment that improves comprehension during the test phase. It is suggested here that mental models of the experimental subjects become more accurate and explicit when abstraction is experimentally controlled through materials and procedures. The deductive inferences that move from abstraction to concrete applications become more efficient when the rules have been previously processed in its natural level of abstraction. If the rules are abstract, their adequate understanding needs to occur in an abstract scenarios. In the concrete situations where theses rules are feasible and consistent, a coherent application requires previous abstract comprehension. A possible corollary for this abstraction conjecture suggests that many results in reasoning experiments might be different when the abstraction level is controlled during the study phase. Since this is concerned with environmental issues, the findings of the present contribution can be considered consistent with the ecological approach to the study of human cognition [13].

The use of visual resources and illustrative examples to promote the comprehension of DeMorgan´s laws might be important for different phases of the cognitive processing. One limitation of the present contribution is that only one phase was implemented. A sequence of increasing abstraction interventions would probably lead to a better comprehension of DeMorgan´s laws. For example, synchronic and diachronic conditions may produce different results.

Another important issue that remains unanswered after the present study is the possible presence or absence of formal rules in human cognitive architecture. Though the results obtained here favour a more syntactic interpretation, the core philosophical problem of rules versus images remains distant from this psychological experiment. The evidence collected here makes no contribution to that philosophical debate.

Other limitation of the present study concerns the lack of experimental control of the truth value of the propositions. The truth or falseness associated to the atomic or non-compound propositions included in the experimental tasks was no controlled by design. Although it is well known in the reasoning literature that content affects reasoning [18], the experimental task of the present study was syntactically, but not semantically, constructed. This weakness, however, did not affect the occurrence of the abstraction effect. Nevertheless, further research is needed to evaluate the same phenomenon under semantically controlled conditions.

A possible account for the evidence collected in the present study is concerned with a rule of thumb that associates the negation of conjunctions to a disjunction, and the negation of disjunctions to a conjunction. The experimental subjects might simply remember that the denial of *and* is *or* and the denial of *or* is *and* after following the step-by-step presentation of DeMorgan´s laws proofs. The

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analytical processing of these materials during the study phase may facilitate the construction of such fast rules. The introduction of examples and the visual presentation for DeMorgan´s laws probably helps comprehension, but may not produce the direct isolation of clear-cut heuristics like the proposed rule of thumb. This rules of thumb conjecture can be considered compatible with the current theoretical trend in the reasoning literature that treats deduction and decision making as branches of the same cognitive architecture [4, 5].

The abstraction effect conjectured in the present study might have didactic implications for logic and mathematics education. The use of more abstract materials seems to lead to a better understanding of DeMorgan´s laws. This result is consistent with previous experiments [29] that found that an abstraction strategy (i.e., alternatives generation with false premises in conditional reasoning) improved abstract reasoning in 9- to 19-year-olds. Because of the counter-intuitive aspects of this result, some further research might be recommended. It is reasonable to consider that different materials may be optimal for different teaching moments. Maybe the introductory phases need more concrete materials. The abstraction effect would probably help to achieve a deeper comprehension when the logic rules have already been partially understood through visual resources.

Conclusions

Two evidence-based conclusions can be proposed. The first one suggests that a formal proof presentation is the best training strategy for the improvement in the comprehension of DeMorgan´s laws. This effect occurs when an inferential paradigm with evaluative tasks is applied. The second conclusion suggests that this proof intervention seems to be more efficient than the use of visual diagrams or the use of colloquial examples. These empirical conclusions support an abstraction effect hypothesis. This hypothesis states that the level of abstraction is a critical variable for the study of reasoning. Because human thinking includes dynamic inferential processes that iteratively move through different levels of abstraction, it is important to consider the distance between the formality of logic rules evaluated in some reasoning experiments and the concreteness of applied contexts.

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Appendix 1
Experimental Task on Logic Performance for DeMorgan´s Laws

Instructions: ten sentences in capital letters are presented below. After reading each one, please select one of the four possible equivalences listed in small letters. One and only one of these four alternatives is the correct one, and the other three are not equivalences. Please note that the equivalence sought is formal. That is, two sentences are equivalent when they have the same abstract meaning. Consider also that sentences with disjunctive connectors (when the word "or" appears) mean that both alternatives can happen at the same time. Or at least one is true. That is, the disjunctions are inclusive rather than exclusive. Please give an answer for each one of the ten exercises.

1. IT IS NOT TRUE THAT: WINTERS ARE SAD AND SUMMERS ARE HAPPY.
Circle the option (a, b, c, or d) you think it is equivalent to the statement above. Follow the same instruction for each of the capital letter statements below.
  a. Winters are not sad and summers are not happy.
  b. Winters are not sad or summers are not happy.
  c. If winters are not sad, then summers are not happy.
  d. Winters are not sad, or summers are not happy, but not both.

2. IT IS NOT TRUE THAT: THE FUTURE IS TODAY AND THE PAST IS OBLIVION.
  a. If the future is not today, then the past is not oblivion.
  b. Or the future is not today, or the past is not oblivion, but not both.
  c. The future is not today and the past is not oblivion.
  d. The future is not today or the past is not oblivion.

3. IT IS NOT TRUE THAT: THE BAD ONES ALWAYS WIN AND LIFE IS A DREAM.
  a. The bad ones do not always win or life is not a dream.
  b. The bad ones do not always win and life is not a dream.
  c. Or the bad ones do not always win, or life is not a dream, but not both.
  d. If the bad ones do not always win, then life is a dream.

4. IT IS NOT TRUE THAT: NATURE IS WISE AND MAN IS A RATIONAL ANIMAL.
  a. Nature is not wise, or man is not a rational animal, but not both.
  b. If nature is not wise, then man is not a rational animal.
  c. Nature is not wise or man is not a rational animal.
  d. Nature is not wise and man is not a rational animal.

5. IT IS NOT TRUE THAT: TO BE IGNORANT GIVES MUCH TROUBLE AND THE BEST IS ENEMY OF THE GOOD.
  a. Or being ignorant does not give much trouble, or the best is not enemy of the good, but not both.
  b. Being ignorant does not give much trouble and the best is not enemy of the good.
  c. Being ignorant does not give much trouble or the best is not enemy of the good.
  d. If being ignorant does not give much trouble, then the best is not enemy of the good.

6. IT IS NOT TRUE THAT: FRIDAYS ARE SHORT DAYS AND MONDAYS ARE LONG DAYS.
  a. If Fridays are not short days, then Mondays are not long days.
  b. Or Fridays are not short days, or Mondays are not long days, but both cannot be true.
  c. Fridays are not short days or Mondays are not long days.
  d. Fridays are not short days and Mondays are not long days.

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7. IT IS NOT TRUE THAT: MAN VISITED THE MOON OR THE MOON HAS A GOOD WEATHER.

a. Man did not visit the moon or the moon does not have a good weather.
b. Man did not visit the moon and the moon does not have a good weather.
c. If man did not visit the moon, then the moon does not have a good weather.
d. Or man did not visit the moon, or the moon does not have a good weather, but not both.

8. IT IS NOT TRUE THAT: WITCHES DO EXIST OR VAMPIRES DO EXIST.

a. Or witches do not exist, or vampires do not exist, but both things cannot be true.
b. If witches do not exist, then vampires do not exist.
c. Witches do not exist and vampires do not exist.
d. Witches do not exist or vampires do not exist.

9. IT IS NOT TRUE THAT: NAPOLEON WAS RIGHT-HANDED OR JOAN OF ARC WAS LEFT-HANDED.

a. Napoleon was not right-handed and Joan of Arc was not left-handed.
b. Napoleon was not right-handed or Joan of Arc was not left-handed.
c. Or Napoleon was not right-handed, or Joan of Arc was not left-handed, but both things cannot be true.
d. If Napoleon was not right-handed, then Joan of Arc was not left-handed.

10. IT IS NOT TRUE THAT: EVERYTHING CHANGES OR NOTHING CHANGES.

a. If it is not true that everything changes, then it is not true that nothing changes.
b. It is not true that everything changes and it is not true that nothing changes.
c. Or it is not true that everything changes, or it is not true that nothing changes, but both things cannot be true.
d. It is not true that everything changes or it is true that nothing changes.

Note: Correct answers according to DeMorgan´s laws are in bold letters.