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NONLINEAR MATHEMATICAL MODEL OF BILATERAL ASSIMILATION

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Abstrac:

In work the new nonlinear mathematical model describing assimilation of the people (population) with some less widespread language by two states with two different widespread languages is offered. In model three subjects are considered: the population and government institutions with the widespread first language, influencing by means of state and administrative resources on the third population with some less widespread language for the purpose of their assimilation; the population and government institutions with the widespread second language, influencing by means of state and administrative resources on the third population with some less widespread language for the purpose of their assimilation; the third population (probably small state education, an autonomy), exposed to bilateral assimilation from two powerful states. In that specific case, a natural zero increase of the population of these three subjects, Cauchy's problem for nonlinear system of the differential equations is solved analytically exactly. Cases of two powerful states assimilating the population of small state education (autonomy), with different number of the population, economic and technological capabilities are considered. The analytical formulas showing in what proportions are received two powerful states assimilate all third population (autonomies). And, conditions under which the first powerful state with the smaller population, than the second powerful state, at the expense of more effective economic and technological capabilities assimilates the most part of the third population (autonomy) are found.

Keywords: *Nonlinear mathematical model; bilateral assimilation; exact analytical solution; result of assimilation.*

Introduction

Mathematical modeling and computing experiment in the last decades gained all-round recognition in science as the new methodology which is roughly developing and widely introduced not only in natural science and technological spheres, but also in economy, sociology, political science and other public disciplines [1 - 4].

In [5 - 7] the mathematical model of political rivalry devoted to the description of fight occurring in imperious elite competing (but not necessarily antagonistic) political forces, for example, power branches is considered. It is supposed that each of the parties has ideas of "number" of the power which this party would like to have itself, and about "number" of the power which she would like to have for the partner.

Works [8 - 12] are devoted to creation of mathematical model of such social process what administrative (state) management is. The last can be carried out as at macro-level (for example, the state) and at micro-level (for example, an educational or research institution, industrial or financial facility, etc.).

In the real work the new nonlinear mathematical model describing assimilation of some poorly widespread language (the people speaking this language) by two other widespread languages is offered. In model three objects are considered:

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1. The population and government institutions with widespread first language (for example, English), influencing by means of state and administrative resources on the population of the third state for the purpose of their assimilation;

2.The population and government institutions with widespread second language (for example, French, Russian, Spanish), influencing by means of state and administrative resources on the population of the third state for the purpose of their assimilation;

3. Population of the third state which is exposed to bilateral assimilation from two powerful states or the coalitions.

1 . System of the equations and initial conditions

For the description of dynamics of the population of three associations (the state, to the coalition of the states with the same state language), speaking different languages on prevalence and opportunities of the respective states, we offer the following nonlinear mathematical model:

$$\begin{cases} \frac{dy(t)}{dt} = \beta_1 y(t) + \alpha_1 y(t)z(t) \\ \frac{dx(t)}{dt} = \beta_2 x(t) + \alpha_2 x(t)z(t) \\ \frac{dz(t)}{dt} = \beta_3 z(t) - \alpha_1 y(t)z(t) - \alpha_2 x(t)z(t) \end{cases} \quad (1.1)$$

$$y(0) = y_0, x(0) = x_0, z(0) = z_0, \quad (1.2)$$

$y(t)$ – number of people at present time t, talking in the first widespread language (for example, English, official language of the UN);

$x(t)$ – number of people at present time t, talking in the second widespread language (for example, French, Russian, Spanish, official languages of the UN);

$z(t)$ – number of people at present time t, talking in the language which is exposed from two widespread languages of assimilation;

α_1, α_2 – respectively coefficients of distribution (assimilation) of the first and second languages (assimilating impact on the people talking on third, not widespread language);

$\beta_1, \beta_2, \beta_3$ - respectively coefficients of natural change (increase, reduction or constancy) populations of the people talking in the first, second and third languages.

We will consider a special case:

$$\beta_1 = \beta_2 = \beta_3 = 0 \quad (1.3)$$

i.e. quantities of a natural increase are equal to zero (quite real situation).

In case of (1.3) systems of the differential equations (1.1) first integral, taking into account entry conditions (1.2) has an appearance:

$$y(t) + x(t) + z(t) = b = y_0 + x_0 + z_0 \ . \quad (1.4)$$

In case of (1.3) from the first and second equations (1.1) it is easy to receive the second first integral of system (1.1)

$$y(t) = \frac{y_0}{x_0^a} x^a(t), \qquad a = \frac{\alpha_1}{\alpha_2} \ . \quad (1.5)$$

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From the second equation (1.1), taking into account the first integrals (1.4), (1.5) we will receive Cauchy's following problem:

$$\frac{dx(t)}{dt}=\alpha_2 x(t)\left[b-x(t)-\frac{y_0}{x_0^a}x^a(t)\right],\quad x(0)=x_0. \tag{1.6}$$

We will consider some special cases.

1.1. $a=1, \alpha_1=\alpha_2=\alpha$.

Then the exact solution of Cauchy's problem (1.6) has an appearance

$$x(t)=\frac{x_0 b e^{b\alpha t}}{(x_0+y_0)e^{b\alpha t}+z_0}\quad . \tag{1.7}$$

Respectively from (1.4), (1.5), (1.7) we will receive

$$y(t)=\frac{y_0 b e^{b\alpha t}}{(x_0+y_0)e^{b\alpha t}+z_0}\ , \tag{1.8}$$

$$z(t)=\frac{z_0 b}{(x_0+y_0)e^{b\alpha t}+z_0}\ , \tag{1.9}$$

From (1.7), (1.8). (1.9) it is easy to receive an asymptotics

$$\begin{aligned}&\lim_{t\to\infty} x(t)=\frac{x_0 b}{x_0+y_0}\ ,\\ &\lim_{t\to\infty} y(t)=\frac{y_0 b}{x_0+y_0}\ ,\\ &\lim_{t\to\infty} z(t)=0\ .\end{aligned} \tag{1.10}$$

Respectively the first and second parties assimilate

$$\Delta y=\frac{y_0 z_0}{x_0+y_0}\ ,\quad \Delta x=\frac{x_0 z_0}{x_0+y_0}\ , \tag{1.11}$$

parts of the population z_0, i.e. the most part of the population talking in the third language is assimilated by that widespread language which speaks bigger number of people (linear assimilation).

1.2. $a=2,\ \alpha_1=2\alpha_2$.

Then the exact solution of Cauchy's problem (1.6) has an appearance

$$\frac{1}{2b}\ln\left|\frac{x^2(t)z_0}{x_0^2\left(sx^2(t)+x(t)-b\right)}\right|-$$
$$-\frac{1}{2b\sqrt{1+4bs}}\ln\left|\frac{\left(2sx(t)+1-\sqrt{1+4bs}\right)\left(2y_0+x_0+x_0\sqrt{1+4bs}\right)}{\left(2sx(t)+1+\sqrt{1+4bs}\right)\left(2y_0+x_0-x_0\sqrt{1+4bs}\right)}\right|=\alpha_2 t,\quad s=\frac{y_0}{x_0^2}. \tag{1.12}$$

We investigate a $x(t)$ function asymptotics on infinity $(t\to\infty)$.

Lemma 1. For function $x(t)$ the following asymptotics on infinity $(t\to\infty)$ is just

$$x(t)\to\frac{-1+\sqrt{1+4bs}}{2s},\quad t\to\infty\quad . \tag{1.13}$$

Proof. In equality (1.12) having passed to a limit at $t\to\infty$, we will receive

$$\lim_{t\to\infty} x(t)=\frac{-1+\sqrt{1+4bs}}{2s}=x_*.$$

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Then it agrees (1.5), (1.13) we will receive

$$\lim_{t\to\infty} y(t) = s{x_*}^2 = y_* \ , \tag{1.14}$$

It is clear, that equalities are just

$$\lim_{t\to\infty} z(t) = 0\,, \tag{1.15}$$

$$x_* + y_* = b\,.$$

Lemma 2. If the inequality $y_0 \ge x_0$ takes place, then the inequality $y_* > x_*$ is just

Proof. From (1.13), (1.14) it is easy to receive

$$\frac{(x_0 + y_0 + z_0)y_0}{x_0^2} > 2,$$

which is carried out at $y_0 \ge x_0$.

At the same time if the inequality is executed

$$(y_0 + 2x_0)(y_0 - x_0) + z_0 y_0 \le 0\,,$$

$$x_0 - y_0 \ge \frac{z_0}{1 + \frac{2x_0}{y_0}}\,,$$

then $x_* \ge y_*$.

We will enter designations:

$$\Delta x = x_* - x_0 = \frac{x_0(\sqrt{x_0^2 + 4by_0} - 2y_0 - x_0)}{2y_0} \ , \tag{1.16}$$

$$\Delta y = y_* - y_0 = \frac{-x_0\sqrt{x_0^2 + 4by_0} + 2by_0 + x_0^2 - 2y_0^2}{2y_0}\,. \tag{1.17}$$

From ratios (1.16), (1.17) taking into account assumptions of mathematical model

$$z_0 << y_0 \ , \qquad z_0 << x_0\,, \tag{1.18}$$

according to formulas of asymptotic decomposition in the small parameter (leaving the main member of decomposition [13]), we will receive

$$\Delta x \approx \frac{x_0}{x_0 + 2y_0} z_0 \ , \qquad \Delta y \approx \frac{2y_0}{x_0 + 2y_0} z_0 \ . \tag{1.19}$$

Formulas of asymptotic decomposition (1.19) show that in spite of the fact that the first party has bigger coefficient of assimilation, than the second party $(\alpha_1 = 2\alpha_2)$ under a condition

$$x_0 > 2y_0$$

the second party will be able to assimilate the most part of the population z_0, which is talking in the third language, undergone to bilateral assimilation.

In a case $x_0 < 2y_0$ the strength first more assimilates the most part of the third population.

1.3. $a = 3,\ \alpha_1 = 3\alpha_2$.

We will enter designation

$$f(x) \equiv b - x - px^3 \ , \tag{1.20}$$

$$f(0) = b > 0, \quad f(b) = -pb^3 < 0, \quad p = \frac{y_0}{x_0^3}.$$

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As $f(x)$ function is continuous on a $[0,b]$ segment, that according to the theorem of Bolzano- Cauchy exists a point $x_{**} \in (0,b)$, such that

$$f(x_{**}) = 0,$$

$$x_{**} = \sqrt[3]{\frac{b}{2p} + \sqrt{Q}} - \sqrt[3]{\frac{-b}{2p} + \sqrt{Q}}, \quad Q = \left(\frac{1}{3p}\right)^3 + \left(\frac{b}{2p}\right)^2 \quad . \tag{1.21}$$

Then the exact solution of Cauchy's problem (1.6) has an appearance

$$-\frac{1}{b}\ln x + B\ln|x - x_{**}| + I_2 = -\alpha_2 t - C_1 \ , \tag{1.22}$$

$$B = \frac{1}{x_{**}\ (2px_{**}^2 + \frac{b}{x_{**}})},$$

$$C_1 = \frac{1}{b}\ln x_0 - B\ln|x_0 - x_{**}| - \frac{C}{2p}\ln\left|\left(x_0 + \frac{x_{**}}{2}\right)^2 + q^2\right| - \frac{\left(D_1 - \frac{Cx_{**}}{2}\right)}{p}\frac{1}{q}arctg\frac{\left(x_0 + \frac{x_{**}}{2}\right)}{q},$$

$$I_2 = \frac{C}{2p}\ln\left|\left(x + \frac{x_{**}}{2}\right)^2 + q^2\right| + \frac{\left(D_1 - \frac{Cx_{**}}{2}\right)}{p}\frac{1}{q}arctg\frac{\left(x + \frac{x_{**}}{2}\right)}{q},$$

$$\frac{b}{px_{**}} - \frac{x_{**}^2}{4} = \frac{3b + x_{**}}{4px_{**}} = q^2 > 0,$$

where C, D_1 constants easily are from system of the linear algebraic equations

$$\begin{cases} \frac{1}{b}(x_{**} - 1)\left(p + px_{**} + \frac{b}{x_{**}}\right) + B\left(p + px_{**} + \frac{b}{x_{**}}\right) + (1 - x_{**})(C + D_1) = 1 \\ \frac{1}{b}(x_{**} - 2)\left(4p + 2px_{**} + \frac{b}{x_{**}}\right) + 2B\left(4p + 2px_{**} + \frac{b}{x_{**}}\right) + \\ + 2(2 - x_{**})(2C + D_1) = 1 \end{cases}$$

We investigate a $x(t)$ function asymptotics on infinity $(t \to \infty)$.

Lemma 3. For $x(t)$ function the following asymptotics on infinity $(t \to \infty)$ is just

$$x(t) \to x_{**}, \quad t \to \infty \, . \tag{1.23}$$

Proof. In equality (1.22) having passed to a limit at $t \to \infty$, we will receive

$$\lim_{t\to\infty} x(t) = x_{**}.$$

Then it agrees (1.5), (1.23) we will receive

$$\lim_{t\to\infty} y(t) = px_{**}^3 = y_{**}. \tag{1.24}$$

It is clear, that equalities are just

$$\lim_{t\to\infty} z(t) = 0 \ , \tag{1.25}$$

$$x_{**} + y_{**} = b \, .$$

Lemma 4. If the inequality $y_0 \geq x_0$ takes place, then the inequality $y_{**} > x_{**}$ is just.

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Proof. From (1.20), (1.21) it is easy to receive

$$f(0)=b>0,\ f(b/2)\equiv\frac{b(4-pb^2)}{8}<0,\ \frac{(x_0+y_0+z_0)^2y_0}{x_0^3}>4.$$

The last inequality is carried out at $y_0 \geq x_0$. Thus $x_{**} < b/2$.

We will enter designations:

$$\Delta x = x_{**} - x_0 = \sqrt[3]{\frac{b}{2p}+\sqrt{Q}} - \sqrt[3]{\frac{-b}{2p}+\sqrt{Q}} - x_0, \quad (1.26)$$

$$\Delta y = y_{**} - y_0 = b - x_{**} - y_0 = b - \sqrt[3]{\frac{b}{2p}+\sqrt{Q}} + \sqrt[3]{\frac{-b}{2p}+\sqrt{Q}} - y_0. \quad (1.27)$$

Taking into account (1.18) and according to formulas of asymptotic decomposition in the small parameter (leaving the main member of decomposition [13]), from (1.26), (1.27) we will receive

$$\Delta x \approx \frac{1}{6p\sqrt{Q_1}}\left[(\sqrt{Q_1}+\frac{x_0+y_0}{2p})^{1/3}+(\sqrt{Q_1}-\frac{x_0+y_0}{2p})^{1/3}\right]z_0, \quad (1.28)$$

$$Q_1=\left(\frac{1}{3p}\right)^3+\left(\frac{x_0+y_0}{2p}\right)^2$$

$$x_0:\quad x_0+y_0-x-px^3=0,$$

$$\Delta y = z_0 - \Delta x \approx z_0 - \frac{1}{6p\sqrt{Q_1}}\left[(\sqrt{Q_1}+\frac{x_0+y_0}{2p})^{1/3}+(\sqrt{Q_1}-\frac{x_0+y_0}{2p})^{1/3}\right]z_0. \quad (1.29)$$

Formulas of asymptotic decomposition (1.28), (1.29) show that in spite of the fact that the first party has bigger coefficient of assimilation, than the second party $(\alpha_1 = 3\alpha_2)$ under a condition

$$\frac{\omega}{3\sqrt{q}}\left[(\sqrt{q}+\frac{\omega+1}{2})^{1/3}+(\sqrt{q}-\frac{\omega+1}{2})^{1/3}\right]>1,$$

$$\frac{x_0}{y_0}\equiv\omega,\qquad q=\left(\frac{\omega}{3}\right)^3+\left(\frac{\omega+1}{2}\right)^2.$$

the second party will be able to assimilate the most part of the z_0 population, which is talking in the third language, undergone to bilateral assimilation.

In case of inequality performance

$$\frac{\omega}{3\sqrt{q}}\left[(\sqrt{q}+\frac{\omega+1}{2})^{1/3}+(\sqrt{q}-\frac{\omega+1}{2})^{1/3}\right]<1$$

the first more powerful party assimilates the most part of the population being talked in the third language, undergone to bilateral assimilation.

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