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MATHEMATICAL MODELING OF NONLINEAR PROCESS OF ASSIMILATION TAKING INTO ACCOUNT DEMOGRAPHIC FACTOR

Chilachava Temur, Chakaberia Maia
Sokhumi State University, St. Anna Politkovskaya No. 9

Abstract

In work mathematical modeling of nonlinear process of assimilation taking into account demographic factor is offered. In considered model taking into account demographic factor natural decrease in the population of the assimilating states and a natural increase of the population which has undergone bilateral assimilation is supposed. At some ratios between coefficients of natural change of the population of the assimilating states, and also assimilation coefficients, for nonlinear system of three differential equations are received the two first integral. Cases of two powerful states assimilating the population of small state formation (autonomy), with different number of the population, both with identical and with various economic and technological capabilities are considered. It is shown that in the first case the problem is actually reduced to nonlinear system of two differential equations describing the classical model "predator - the victim", thus, naturally a role of the victim plays the population which has undergone assimilation, and a predator role the population of one of the assimilating states. The population of the second assimilating state in the first case changes in proportion (the coefficient of proportionality is equal to the relation of the population of assimilators in an initial time point) to the population of the first assimilator. In the second case the problem is actually reduced to nonlinear system of two differential equations describing type model "a predator – the victim", with the closed integrated curves on the phase plane. In both cases there is no full assimilation of the population to less widespread language. Intervals of change of number of the population of all three objects of model are found. The considered mathematical models which in some approach can model real situations, with the real assimilating countries and the state formations (an autonomy or formation with the unrecognized status), undergone to bilateral assimilation, show that for them the only possibility to avoid from assimilation is the natural demographic increase in population and hope for natural decrease in the population of the assimilating states.

Keywords. *Nonlinear process; bilateral assimilation; demographic factor; first integrals; intervals of change of number of the population.*

Introduction

Mathematical modeling and computing experiment in the last decades gained all-round recognition in science as the new methodology which is roughly developing and widely introduced not only in natural science and technological spheres, but also in economy, sociology, political science and other public disciplines [1 - 3].

In [4 - 6] the mathematical model of political rivalry devoted to the description of fight occurring in imperious elite competing (but not necessarily antagonistic) political forces, for example, power branches is considered. It is supposed that each of the sides has ideas of "number" of the power which this side would like to have itself, and about "number" of the power which she would like to have for the partner.

Works [7 - 11] are devoted to creation of mathematical model of such social process what administrative (state) management is. The last can be carried out as at macro-level (for example,

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the state) and at micro-level (for example, an educational or research institution, industrial or financial facility, etc.).

In work [12] computer research of a trajectory of development of three ethnos living in one territory is conducted.

In work [13] we consider the nonlinear mathematical model of bilateral assimilation without demographic factor. It was shown that the most part of the population talking in the third language is assimilated by that widespread language which speaks bigger number of people (linear assimilation). Also it was shown that in case of zero demographic factor of all three subjects, the population with less widespread language completely assimilates the states with two various widespread languages, and the result of assimilation (redistribution of the assimilated population) is connected with initial quantities, technological and economic capabilities of the assimilating states.

In the real work we consider mathematical modeling of nonlinear process of assimilation taking into account demographic factor. In model three objects are considered:

1. The population and government institutions with widespread first language, influencing by means of state and administrative resources on the population of the third state for the purpose of their assimilation;

2.The population and government institutions with widespread second language, influencing by means of state and administrative resources on the population of the third state for the purpose of their assimilation;

3. Population of the third state which is exposed to bilateral assimilation from two powerful states or the coalitions.

1 . System of the equations and initial conditions

For the description of dynamics of the population of three associations (the state, to the coalition of the states with the same state language), speaking different languages on prevalence and opportunities of the respective states, we offer the following nonlinear mathematical model:

$$\begin{cases} \dfrac{dx(t)}{dt} = \beta_1 x(t) + \alpha_1 x(t) z(t) \\ \\ \dfrac{dy(t)}{dt} = \beta_2 y(t) + \alpha_2 y(t) z(t) \\ \\ \dfrac{dz(t)}{dt} = \beta_3 z(t) - \alpha_1 x(t) z(t) - \alpha_2 y(t) z(t) \end{cases} \tag{1.1}$$

$$x(0) = x_0, y(0) = y_0, z(0) = z_0, \tag{1.2}$$

$x(t)-$ number of people at present time t, talking in the first widespread language;

$y(t)-$ number of people at present time t, talking in the second widespread language;

$z(t)-$ number of people at present time t, talking in the language which is exposed from two widespread languages of assimilation;

$\alpha_1, \alpha_2 -$ respectively coefficients of distribution (assimilation) of the first and second languages (assimilating impact on the people talking on third, not widespread language);

$\beta_1, \beta_2, \beta_3$ - respectively coefficients of natural change (increase, reduction or constancy) populations of the people talking in the first, second and third languages.

ISSN 1512-1232

2. **Case of the states with identical assimilating opportunities and coefficients of natural change of the population and various initial number of the population.**

We will assume that coefficients of natural change of the population of the assimilating states in (1.1) are negative, and the population which has undergone bilateral assimilation – is positive (quite real situation).

Thus, it is supposed that the following inequalities take place

$$\beta_1 < 0, \quad \beta_2 < 0, \quad \beta_3 > 0 \ . \tag{2.1}$$

We will assume that equalities are right:

$$\frac{\beta_1}{\beta_2} = \frac{\alpha_1}{\alpha_2} = \delta > 0 \ . \tag{2.2}$$

In case of (2.2) from the first and second equations (1.1) it is easy to receive the first integral of system (1.1)

$$x(t) = \frac{x_0}{y_0^{\delta}} y^{\delta}(t) \ . \tag{2.3}$$

We will consider two special cases:

1. $\delta = 1$ **2.** $\delta = 2$

In the first case ($\delta = 1$) we have two powerful states with identical assimilating opportunities and coefficients of natural change of the population (at the initial moment probably with different number of the population), and in the second case ($\delta = 2$) – with various assimilating opportunities and coefficients of natural change of the population.

In a case $\delta = 1$ entering designation

$$\frac{x_0}{y_0} = \gamma \ . \tag{2.4}$$

From (1.1), (1.2), (2.1) – (2.4) we will receive Cauchy's problem for nonlinear system of two differential equations

$$\begin{cases} \dfrac{dy(t)}{dt} = \beta_2 y(t) + \alpha_2 y(t) z(t) \\ \\ \dfrac{dz(t)}{dt} = \beta_3 z(t) - (\alpha_1\gamma + \alpha_2) y(t) z(t) \end{cases} \tag{2.5}$$

$$y(0) = y_0, \qquad z(0) = z_0$$

As in (2.5) have inequality places

$$\beta_3 > 0, \quad \alpha_1\gamma + \alpha_2 > 0, \quad \beta_2 < 0, \quad \alpha_2 > 0 \tag{2.6}$$

we received system of the Lotka - Volterra equations for the classical “predator- victim” model.

Thus, naturally the role of the victim plays the population which has undergone assimilation, and a predator role the population of one of the assimilating states.

The population of the second assimilating state agrees (2.3), (2.4) changes in proportion (the coefficient of proportionality is equal to the relation of the population of assimilating sides in an initial time-point) to the population of the first assimilating state.

$$x(t) = \gamma y(t) \ . \tag{2.7}$$

From (2.5) it is possible to receive the second first integral of system of the equations (1.1)

$$\alpha_2(z - z_0) + (\alpha_1\gamma + \alpha_2)(y - y_0) = (-\beta_2)\ln\frac{z}{z_0} + \beta_3 \ln\frac{y}{y_0} \ . \tag{2.8}$$

ISSN 1512-1232

As it is known (2.8) at various initial data y_0, z_0 represent family of the closed integrated curves round some point $M\left[\frac{-\beta_2}{\alpha_2}, \frac{\beta_3}{\alpha_1\gamma+\alpha_2}\right]$, located in the first quadrant the Ozy plane of variables $z(t), y(t)$.

Thus, at $t \geq 0$ function

$$z(t) \in [z_{\min}, z_{\max}],$$

$0 < z_{\min}, z_{\max}$ are defined from the transcendental equation

$$\alpha_2(z - z_0) + (\alpha_1\gamma + \alpha_2)(\frac{\beta_3}{\alpha_1\gamma+\alpha_2} - y_0) = (-\beta_2)\ln\frac{z}{z_0} + \beta_3 \ln\frac{\beta_3}{(\alpha_1\gamma+\alpha_2)y_0} \tag{2.9}$$

We will enter designation

$$y^* \equiv \frac{\beta_3}{\alpha_1\gamma+\alpha_2} > 0, \qquad z^* \equiv \frac{-\beta_2}{\alpha_2} > 0 \tag{2.10}$$

$$A(y_0) \equiv (\alpha_1\gamma+\alpha_2)(y^* - y_0) - \beta_3 \ln\frac{y^*}{y_0}$$

As ratios are fair

$$A(y_0) \in C(0,\infty), \qquad A(y^*) = 0, \qquad \lim_{y_0 \to 0_+} A(y_0) = -\infty, \ \lim_{y_0 \to +\infty} A(y_0) = -\infty \tag{2.11}$$

$$\frac{dA}{dy_0} = \frac{(\alpha_1\gamma+\alpha_2)(y^* - y_0)}{y_0} \qquad \frac{d^2A}{dy_0^{\,2}} = -\frac{(\alpha_1\gamma+\alpha_2)y^*}{y_0^{\,2}} < 0$$

$$\frac{dA}{dy_0}(y^*) = 0, \qquad \frac{d^2A}{dy_0^{\,2}}(y^*) = -\frac{(\alpha_1\gamma+\alpha_2)}{y^*} < 0$$

that is right an inequality

$$A(y_0) \leq 0, \ y_0 \in (0,\infty), \tag{2.12}$$

$$\max_{y_0 \in (0,\infty)} A(y_0) = A(y^*) = 0$$

Then (2.9), according to (2.10) will correspond in the following look

$$\alpha_2(z - z_0) + A = (-\beta_2)\ln\frac{z}{z_0} \quad . \tag{2.13}$$

We will enter designation

$$f(z) \equiv \alpha_2(z - z_0) + A - (-\beta_2)\ln\frac{z}{z_0} \quad . \tag{2.14}$$

It is easy to show that for $f(z)$ function are fair

$$f(z) \in C(0,\infty), \qquad f(z_0) = A \leq 0, \qquad \lim_{z \to 0_+} f(z) = +\infty, \ \lim_{z \to +\infty} f(z) = +\infty \tag{2.15}$$

$$f'(z) = \frac{\alpha_2\left[z - \frac{(-\beta_2)}{\alpha_2}\right]}{z}$$

i.e. for the equation

$$f(z) = 0 \tag{2.16}$$

at $A = 0$ - has two decisions $z = z_0 = z_{\min,1}$, $z = z_{\max,1}$, and in a case $A < 0$ - has two decisions $z = z_{\min,2} < z_0$, $z = z_{\max,2} > z_0$.

The first decision doesn't make physical sense as performance is necessary

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$z_{\min} \le z_0,$

Thus, we will finally receive

$$A = 0 \qquad 0 < z_0 = z_{\min,1} < z_{\max,1}, \qquad A < 0 \qquad 0 < z_{\min,2} < z_0 < z_{\max,2} \tag{2.17}$$

Similarly, at $t \ge 0$ function

$y(t) \in [y_{\min}, y_{\max}],$

$0 < y_{\min}, y_{\max}$ are defined from the transcendental equation

$$\alpha_2(z^* - z_0) + (\alpha_1\gamma + \alpha_2)(y - y_0) = (-\beta_2)\ln\frac{z^*}{z_0} + \beta_3 \ln\frac{y}{y_0} \tag{2.18}$$

We will enter designation

$$B(z_0) \equiv \alpha_2(z^* - z_0) - (-\beta_2)\ln\frac{z^*}{z_0} \tag{2.19}$$

As ratios are fair

$$B(z_0) \in C(0,\infty), \quad B(z^*) = 0, \quad \lim_{z_0 \to 0_+} B(z_0) = -\infty, \quad \lim_{z_0 \to +\infty} B(z_0) = -\infty \tag{2.20}$$

$$\frac{dB}{dz_0} = -\frac{\alpha_2(z_0 - z^*)}{z_0} \qquad \frac{d^2B}{dz_0^{\ 2}} = -\frac{\alpha_2 z^*}{z_0^{\ 2}} < 0$$

$$\frac{dB}{dz_0}(z^*) = 0, \qquad \frac{d^2B}{dz_0^{\ 2}}(z^*) = -\frac{\alpha_2}{z^*} < 0$$

that is right an inequality

$$B(z_0) \le 0,\ z_0 \in (0,\infty), \qquad \max_{z_0 \in (0,\infty)} B(z_0) = B(z^*) = 0 \tag{2.21}$$

Then (2.18), according to (2.19) will correspond in the following look

$$(\alpha_1\gamma + \alpha_2)(y - y_0) + B = \beta_3 \ln\frac{y}{y_0} \ . \tag{2.22}$$

We will enter designation

$$g(y) \equiv (\alpha_1\gamma + \alpha_2)(y - y_0) + B - \beta_3 \ln\frac{y}{y_0} \tag{2.23}$$

It is easy to show that for $g(y)$ function are fair

$$g(y) \in C(0,\infty), \quad g(y_0) = B \le 0, \quad \lim_{y \to 0_+} g(y) = +\infty, \quad \lim_{y \to +\infty} g(y) = +\infty \tag{2.24}$$

$$\frac{dg}{dy} = \frac{(\alpha_1\gamma + \alpha_2)(y - y^*)}{y}$$

Thus equation

$$g(y) = 0 \tag{2.25}$$

at $B = 0$ - has two decisions $y = y_0 = y_{\min,1},\ y = y_{\max,1},$ and in a case $B < 0$ - has two decisions $y = y_{\min,2} < y_0,\ y = y_{\max,2} > y_0.$

Thus, we will finally receive

$$B = 0 \qquad 0 < y_0 = y_{\min,1} < y_{\max,1} \qquad B < 0 \qquad 0 < y_{\min,2} < y_0 < y_{\max,2} \tag{2.26}$$

Then for $x(t)$ function according to (3.7) will take place

$$B = 0 \qquad 0 < x_0 = x_{\min,1} < x_{\max,1} \tag{2.27}$$

ISSN 1512-1232

$$B<0 \qquad 0<x_{\min,2}<x_0<x_{\max,2}$$

Thus, ratios are naturally right

$$x_{\min,i}=\gamma y_{\min,i}, \qquad x_{\max,i}=\gamma y_{\max,i} \qquad i=1,2 \tag{2.28}$$

3. **Case of the states with various assimilating opportunities and coefficients of natural change of the population.**

In a case $\delta=2$ entering designation

$$\frac{x_0}{y_0^{\,2}}=p \quad . \tag{3.1}$$

From (1.1), (1.2), (2.1) – (2.3), (2.10), (3.1) we will receive Cauchy's problem for nonlinear system of two differential equations

$$\begin{cases} \dfrac{dy(t)}{dt}=\beta_2 y(t)+\alpha_2 y(t)z(t)=\alpha_2\left[z(t)-z^*\right]y(t) \\ \\ \dfrac{dz(t)}{dt}=(\beta_3-\alpha_1 py^2-\alpha_2 y)z(t)=-\alpha_1 p(y-y_1)(y-y_2)z(t) \end{cases} \tag{3.2}$$

$$y(0)=y_0, \qquad z(0)=z_0$$

where

$$y_1=\frac{-\alpha_2+\sqrt{\alpha_2^2+4\alpha_1 p\beta_3}}{2\alpha_1 p}>0, \qquad y_2=\frac{-\alpha_2-\sqrt{\alpha_2^2+4\alpha_1 p\beta_3}}{2\alpha_1 p}<0 \tag{3.3}$$

According to (3.3)

$$y-y_2>0 \qquad \forall t\in[0,\infty),$$

therefore

$$\operatorname{sgn}\frac{dy(t)}{dt}=\operatorname{sgn}\left[z(t)-z^*\right], \qquad \operatorname{sgn}\frac{dz(t)}{dt}=-\operatorname{sgn}\left[y(t)-y_1\right] \tag{3.4}$$

From (3.2) it is possible to receive the second first integral of system of the equations (1.1)

$$\alpha_2(z-z_0)-(-\beta_2)\ln\frac{z}{z_0}=\beta_3\ln\frac{y}{y_0}-\frac{\alpha_1 p}{2}(y^2-y_0^2)-\alpha_2(y-y_0) \quad . \tag{3.5}$$

Ratio (3.5) at various initial data represent family of the closed integrated curves round some point $M\left[z^*,y_1\right]$, located in the first quadrant the Ozy plane of variables z (t), y (t).

Thus, at $t\geq 0$ function

$$z(t)\in\left[z_{\min},z_{\max}\right],$$

$0<z_{\min},z_{\max}$ are defined from the transcendental equation

$$\alpha_2(z-z_0)-(-\beta_2)\ln\frac{z}{z_0}=\beta_3\ln\frac{y_1}{y_0}-\frac{\alpha_1 p}{2}(y_1^{\,2}-y_0^2)-\alpha_2(y_1-y_0) \tag{3.6}$$

We will enter designation

$$A_1\equiv-\beta_3\ln\frac{y_1}{y_0}+\frac{\alpha_1 p}{2}(y_1^{\,2}-y_0^2)+\alpha_2(y_1-y_0) \tag{3.7}$$

Then (2.6), according to (2.7) will correspond in the following look

$$\alpha_2(z-z_0)+A_1=(-\beta_2)\ln\frac{z}{z_0} \tag{3.8}$$

As ratios are fair

$$A_1(y_0)\in C(0,\infty), \qquad A_1(y_1)=0, \qquad \lim_{y_0\to 0_+}A_1(y_0)=-\infty, \quad \lim_{y_0\to+\infty}A_1(y_0)=-\infty \tag{3.9}$$

ISSN 1512-1232

$$\frac{dA_1}{dy_0} = -\frac{\alpha_1 p(y_0 - y_1)(y_0 - y_2)}{y_0} \qquad \frac{d^2 A_1}{dy_0^{\,2}} = -\frac{\beta_3 + \alpha_1 p y_0^2}{y_0^{\,2}} < 0$$

$$\frac{dA_1}{dy_0}(y_1) = 0, \qquad \frac{d^2 A_1}{dy_0^{\,2}}(y_1) = -\frac{\beta_3 + \alpha_1 p y_1^2}{y_1^{\,2}} < 0$$

that is right an inequality

$$A_1(y_0) \le 0,\ y_0 \in (0,\infty), \qquad (3.10)$$
$$\max_{y_0 \in (0,\infty)} A_1(y_0) = A_1(y_1) = 0$$

We will enter designation

$$f_1(z) \equiv \alpha_2(z - z_0) + A_1 - (-\beta_2)\ln\frac{z}{z_0} \qquad (3.11)$$

It is easy to show that for $f_1(z)$ function are fair

$$f_1(z) \in C(0,\infty), \qquad f_1(z_0) = A_1 \le 0, \qquad \lim_{z \to 0_+} f_1(z) = +\infty, \ \lim_{z \to +\infty} f_1\ (z) = +\infty \qquad (3.12)$$
$$\frac{df_1}{dz} = \frac{\alpha_2[z - z^*]}{z}$$

The equation (3.8) taking into account (3.11) will correspond in a look

$$f_1(z) = 0 \qquad (3.13)$$

at $A_1 = 0$ - has two decisions $z = z_{\min,3} = z_0$, $z = z_{\max,3}$, and in a case $A_1 < 0$ - has two decisions $z = z_{\min,4} < z_0$, $z = z_{\max,4} > z_0$.

Thus, we will finally receive

$$\begin{array}{ll} A_1 = 0 & 0 < z_0 = z_{\min,3} < z_{\max,3} \\ A_1 < 0 & 0 < z_{\min,4} < z_0 < z_{\max,4} \end{array} \qquad (3.14)$$

Similarly, at $t \ge 0$ function

$$y(t) \in [y_{\min}, y_{\max}],$$

$0 < y_{\min}, y_{\max}$ are defined from the transcendental equation

$$\alpha_2(z^* - z_0) - (-\beta_2)\ln\frac{z^*}{z_0} = \beta_3 \ln\frac{y}{y_0} - \frac{\alpha_1 p}{2}(y^2 - y_0^2) - \alpha_2(y - y_0) \quad . \qquad (3.15)$$

Then (3.15), according to (2.19) will correspond in the following look

$$\frac{\alpha_1 p}{2}(y^2 - y_0^2) + \alpha_2(y - y_0) + B = \beta_3 \ln\frac{y}{y_0} \qquad (3.16)$$

We will enter designation

$$g_1(y) \equiv \frac{\alpha_1 p}{2}(y^2 - y_0^2) + \alpha_2(y - y_0) + B - \beta_3 \ln\frac{y}{y_0} \qquad (3.17)$$

It is easy to show that for function $g_1(y)$ are fair

$$g_1(y) \in C(0,\infty), \qquad g_1(y_0) = B \le 0, \qquad \lim_{y \to 0_+} g_1(y) = +\infty, \ \lim_{y \to +\infty} g_1(y) = +\infty \qquad (3.18)$$
$$\frac{dg_1}{dy} = \frac{\alpha_1 p(y - y_1)(y - y_2)}{y}$$

Thus equation

$$g_1(y) = 0 \qquad (3.19)$$

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at $B = 0$ - has two decisions $y = y_0 = y_{\min,3}$, $y = y_{\max,3}$, and in a case $B < 0$ - has two decisions $y = y_{\min,4} < y_0$, $y = y_{\max,4} > y_0$.

Thus, we will finally receive

$$\begin{array}{ll} B = 0 & 0 < y_0 = y_{\min,3} < y_{\max,3} \\ B < 0 & 0 < y_{\min,4} < y_0 < y_{\max,4} \end{array} \quad (3.20)$$

Then for $x(t)$ function according to (2.3), (3.1) will take place

$$\begin{array}{ll} B = 0 & 0 < x_0 = x_{\min,3} < x_{\max,3} \\ B < 0 & 0 < x_{\min,4} < x_0 < x_{\max,4} \end{array} \quad (3.21)$$

Thus, ratios are naturally right

$$x_{\min,i} = p y^2_{\min,i}, \qquad x_{\max,i} = p y^2_{\max,i}, \quad i = 3,4 \quad (3.22)$$

In both cases 2, 3 there is no full assimilation of the population to less widespread language. Intervals of change of number of the population of all three objects of model are found.

Conclusion

In considered model taking into account demographic factor natural decrease in the population of the assimilating states and a natural increase of the population which has undergone bilateral assimilation is supposed. At some ratios between coefficients of natural change of the population of the assimilating states, and also assimilation coefficients, for nonlinear system of three differential equations are received the two first integral. Cases of two powerful states assimilating the population of small state formation (autonomy), with different number of the population, both with identical and with various economic and technological capabilities are considered. It is shown that in the first case the problem is actually reduced to nonlinear system of two differential equations describing the classical model "predator - the victim", thus, naturally a role of the victim plays the population which has undergone assimilation, and a predator role the population of one of the assimilating states. The population of the second assimilating state in the first case changes in proportion (the coefficient of proportionality is equal to the relation of the population of assimilators in an initial time point) to the population of the first assimilator. In the second case the problem is actually reduced to nonlinear system of two differential equations describing type model "a predator – the victim", with the closed integrated curves on the phase plane. In both cases there is no full assimilation of the population to less widespread language. Intervals of change of number of the population of all three objects of model are found.

The considered mathematical models which in some approach can model real situations, with the real assimilating countries and the state formations (an autonomy or formation with the unrecognized status), undergone to bilateral assimilation, show that for them the only possibility to avoid from assimilation is the natural demographic increase in population and hope for natural decrease in the population of the assimilating states.

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