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UNPOLARIZED DISTRIBUTIONS WITH TRANSVERSE MOMENTA IN THE CHIRAL QUARK-SOLUTION MODELS: T-ODD STRUCTURE FUNCTIONS IN THE SKYRME-LIKE SOLITONIC NJL MODEL

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Abstract: *We review a formalism for the calculation of the T-odd structure functions of nucleon. The chiral (Skyrme-like) solitonic model of Nambu and Jona- Lasinio (NJL) is used for the nucleon wave function. Corresponding confirmation is discussed. The relevant relations are exposed and corresponding explanations are given. The forms, exhibited below, may be used for further application in calculation of corresponding structure functions. The role of chiral symmetry breaking is underlined.*

Key Words: *Structure functions, polarization, soliton, Skyrme model, NJL model, transverse momentum, T-odd distribution*

I. Introduction

The chiral symmetry plays a crucial role in the single-spin asymmetries observed in the inclusive production of pions in the scattering of transversely polarised protons off unpolarized ones [1]. In particular, it is known [2] that the opposite sign of the asymmetry for positive and negative pions can be related to the underlying chiral symmetry of the model describing the nucleon. This mechanism can be tested by measuring single-spin asymmetries in inelastic hadronic processes $\bar{p}p \to \gamma X$ and $\bar{p}p \to hX$ or in the deep Inelastic Scattering (DIS) of unpolarized leptons on transversely polarised nucleons. It is interesting to discuss a possible origin of single spin asymmetries in the large $p_\perp$ inclusive production of hadrons in the scattering of unpolarized protons or leptons on transversely polarised nucleon. The single transverse spin asymmetries are most easily studied experimentally, as they require only the one polarised particle[3] . These spin effects are T-odd but not related to the CP-violation. They are generated by the term in the cross-section proportional to the polarisation of one of the spin -1/2 particles. On one side we have the quark parton model which successfully describes the scaling behaviour of the structure functions in deep inelastic scattering (DIS) processes. On the other side we have the chiral soliton approach which is motivated by the large N_C expansion of QCD. Also this theory is not explicitly known it can be modelled by assuming that at low energies only the light mesons (pions, kaons) are relevant. When modelling the meson theory one requires the symmetry structure of QCD, In particular besides Poincare invariance we require chiral symmetry and its spontaneous breaking. Baryons emerge as non-perturbative (topological) configurations of the meson

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fields, the so-called solitons. the link between these two pictures can be established by computing structure functions within a chiral soliton model for the nucleon from the hadronic tensor

$$W_{\mu\nu}^{ab}(q)=\frac{1}{4\pi}\int d^4\xi e^{iq\xi}\left\langle N(P)\middle|\left[J_\mu^a(x),J_\mu^b(0)\right]\middle|N(P)\right\rangle \qquad (1)$$

which describes the strong interaction part of the DIS cross-section. In this equation $|N(P)\rangle$ is the nucleon state vector with momentum P and $J_\mu^a(\xi)$ is the hadronic current suitable for the process under consideration. Bellow our consideration is based on the article [4].

2. Chiral soliton model and the nucleon

Here we briefly summarize the basic features of the chiral soliton model in the NJL model and discuss how the states with nucleon quantum numbers are generated.

In the context of the spin structure of the nucleon chiral soliton models are particularly interesting as they provide an explanation for the small magnitude of the quark spin contribution to proton spin, i.e. the vanishingly small matrix element of the singlet axial current. In these models the nucleon is described as a non-perturbative field configuration in some non-linear effective meson theory [5], [6].

The NJL – model Lagrangian [7],[8] contains a quark interaction which is chirally symmetric. Derivatives of the quark fields only appear in form of a free Dirac Lagrangian. Hence the current operator is formally free. Upon bosonization the action may be expressed as

$$A=Tr\ln_\Lambda\left(i\partial\cdot\gamma-mU^{\gamma_5}\right)+\frac{m_0m}{4G}tr\left(U+U^+-2\right) \qquad (2)$$

where we have confined ourselves to the interaction in the pseudoscalar channel. The associated pion fields $\boldsymbol{\pi}$ are contained in the non-linear realization $\exp\left(i\boldsymbol{\pi}\cdot\boldsymbol{\tau}/f_\pi\right)$. In this equation tr denotes discrete flavor trace while Tr also includes the functional trace. The parameters of the model are coupling constant G, the current quark mass m_0 and the UV cut-off Λ. The constituent quark mass m arises as the solution to the Dyson-Schwinger (gap) equation and characterizes the spontaneous breaking of chiral symmetry. A Bethe-Salpeter equation of the pion field can be derived from the Eq.(2) which allows to express the pion mass $m_\pi=135MeV$ and decay constant $f_\pi=93MeV$ in terms of the model parameters. Subsequently an energy functional for non-perturbative but static configurations $U(\boldsymbol{r})$. For the Skyrme-hedgehog ansatz, $U_H(\boldsymbol{r})=\exp\left(i\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}\theta(r)\right)$ the associated one particle Dirac Hamiltonian becomes

$$h=\boldsymbol{\alpha}\cdot\boldsymbol{p}-\beta m\exp\left(i\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}\theta(r)\right),\qquad h\Psi_\mu=\varepsilon_\mu\Psi_\mu \qquad (3)$$

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The distinct level(ν), which is bound in the background of U_H, is referred to as the valence quark state. Its explicit occupation guarantees unit baryon number. The chiral angle $\theta(r)$ of the soliton is determined by self-consistently minimizing the energy functional. This soliton configuration does not yet carry nucleon quantum numbers. To generate them the (unknown) time dependent field configuration is approximated by elevating the zero modes to time dependent collective coordinates $U(\boldsymbol{r},t)=A(t)U_H(\boldsymbol{r})A^+(t)$, $A(t)\in SU(2)$ Upon canonical quantization the angular velocities $\boldsymbol{\Omega}=-2itr\left(\boldsymbol{\tau}A^+A\right)$, are replaced by the spin operator $\boldsymbol{J}$ via $\boldsymbol{\Omega}=\boldsymbol{J}/\alpha^2$ with α^2 being the moment of inertia while the nucleon states $|N\rangle$ emerge as Wigner D-functions. To compute nucleon properties the action (2) is expanded in powers of $\boldsymbol{\Omega}$ corresponding to an expansion in $1/N_C$ [6]. In particular the valence quark wave-function $\Psi_\nu(\boldsymbol{x})$ acquires a linear correction

$$\Psi_\nu(\boldsymbol{x},t)=e^{-i\varepsilon_\nu t}A(t)\left\{\psi_\nu(\boldsymbol{x})+\sum_{\mu\neq\nu}\psi_\mu(\boldsymbol{x})\frac{\langle\mu|\tau\cdot\Omega|\nu\rangle}{2\left(\varepsilon_\nu-\varepsilon_\mu\right)}\right\}=e^{-i\varepsilon_\nu t}A(t)\psi_\nu(\boldsymbol{x}) \qquad (4)$$

Here $\psi_\nu(\boldsymbol{x})$ refers to the spatial part of the body-fixed valence quark wave-function with rotational corrections included.

3. The valence quark approximation for the structure functions

The starting point for constructing the unpolarized structure functions is the symmetric part of hadronic tensor in a form suitable for localized fields

$$W^{lm}_{\{\mu\nu\}}(q)=\varsigma\int\frac{d^4k}{(2\pi)^4}S_{\mu\rho\nu\sigma}k^\rho\,\mathrm{sgn}(k_0)\,\delta\left(k^2\right)\int_{-\infty}^{+\infty}e^{i(k_0+q_0)t}\times$$

$$\times\int d^3x_1d^3x_2\exp\left\{-i(\boldsymbol{k}+\boldsymbol{q})\cdot(\boldsymbol{x}_1-\boldsymbol{x}_2)\right\}$$
$$\langle N|\left\{\hat{\bar{\Psi}}(\boldsymbol{x}_1,t)\tau_l\tau_m\gamma^\sigma\hat{\Psi}(\boldsymbol{x}_2,0)-\hat{\bar{\Psi}}(\boldsymbol{x}_2,0)\tau_m\tau_l\gamma^\sigma\hat{\Psi}(\boldsymbol{x}_1,t)\right\}|N\rangle \qquad (5)$$

The quark spinors are functionals of the soliton. Here $S_{\mu\rho\nu\sigma}=g_{\mu\rho}g_{\nu\sigma}+g_{\mu\sigma}g_{\nu\rho}-g_{\mu\nu}g_{\rho\sigma}$, and ς is the numerical coefficient, depended on the current's type. The matrix element between nucleon states is taken in the space of the collective coordinates.

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4. Chiral odd structure function $h_T(x)$

The chiral odd structure functions are computed as Fourier transformations of nuclear matrix elements of bilocal quark operators on the light –cone. In the nucleon rest frame (RF) the contribution of the forward moving intermediate quark to the chiral odd structure functions may be expressed as [9]

$$h_T^{(+)}(x) = N_C \frac{2M\sqrt{2}}{8\pi\sqrt{}} \int d\xi^- \exp\left(-i\xi^- \frac{Mx}{\sqrt{2}}\right)\times$$
$$\times\int d^3x_0 \left\langle \boldsymbol{S}_\perp \middle| \Psi_+^+(\xi - x_0)\gamma_\perp\gamma_5 Q^2 \Psi_+(-x_0) \middle| \boldsymbol{S}_\perp \right\rangle_{\xi^+=\xi_\perp=0} \tag{6}$$

Note that ξ refers to a four vector which in light-cone coordinates reads $(\xi^+, \xi^-, \boldsymbol{\xi}_\perp)$. this coordinate enters the light-cone variables via $\xi^\pm = (t \pm z)/\sqrt{2}$. The notation $\boldsymbol{S}_\perp$ is synonymous for the spin being perpendicular to the coordinate z.The "good" and "bad" light-cone components of the quark wave functions are the projections $\Psi_\pm = P_\pm \Psi$, with $P_\pm = \frac{1}{2}\gamma^\mp\gamma^\pm$ being the corresponding projections operators. Above $Q = diag(2/2, -1/3)$ is the quark charge fraction's matrix and the zero momentum nuclear states are given by, $\left|\boldsymbol{p} = 0, \boldsymbol{S}\right\rangle = \left\{(2\pi)^3 2M\right\}^{1/2} \left|\boldsymbol{S}\right\rangle$. Introducing Fourier transforms for the spatial part of the valence quark wave functions

$$\psi\left(\boldsymbol{\xi}_\perp, \xi_3 = -\frac{\xi^-}{\sqrt{2}}\right) = \int \frac{d^2p_\perp dp_3}{2\pi^2} \exp\left[i\left(\frac{p_3\xi^-}{\sqrt{2}} - \boldsymbol{p}_\perp \cdot \boldsymbol{\xi}_\perp\right)\right]\psi\left(\boldsymbol{p}_\perp, p_3\right) \tag{7}$$

yields

$$h_T^{(+)}(x) = N_C \frac{M}{\sqrt{2}\pi^2} \int d\xi^- p^2 dp d(\cos\vartheta) d\phi \exp\left(\frac{-i\xi^-\left(Mx - \varepsilon_\nu + p\cos\vartheta\right)}{\sqrt{2}}\right)\times$$
$$\left\langle \boldsymbol{S}_\perp \middle| \tilde{\psi}_+^+(\boldsymbol{p})\gamma^\perp\gamma^5 Q^2 \tilde{\psi}_+(\boldsymbol{p}) \middle| \boldsymbol{S}_\perp \right\rangle \tag{8}$$

The square of the charge operator is redefined (because of quantization) to

$$Q^2 = \frac{5}{18} I + \frac{1}{6} D_{3i}\tau_i \tag{9}$$

Where $D_{ij} = \frac{1}{2} tr\left(\tau_i A(t)\tau_j A^+(t)\right)$ denotes the adjoint representation of the collective rotation, defined above. After performing integration over ξ^- and ϑ,it results in the forward moving quark contribution to the transverse chiral odd nucleon structure function

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$$h_T^{(+)} = N_C \frac{2M}{\pi} \int_{p_{\min}}^{\infty} pdpd\phi \left\langle S_\perp \left| \tilde{\psi}_+^+ (p) \gamma_\perp \gamma_5 Q^2 \tilde{\psi}_+ (p) \right| S_\perp \right\rangle_{\cos\vartheta = \frac{\varepsilon - Mx}{p}} \quad (10)$$

and $p_{\min} = \left| Mx - \varepsilon_\nu \right|$

The calculation of the longitudinal function is analogous and gives

$$h_L^{(+)}(x) = N_C \frac{2M}{\pi} \int_{p^{\mp}{}_{\min}}^{\infty} pdpd\phi \left\langle \boldsymbol{S}_z \left| \tilde{\psi}_+^+ (\boldsymbol{p}) \gamma_0 \gamma_5 Q^2 \tilde{\psi}_- (\boldsymbol{p}) - \tilde{\psi}_-^+ (\boldsymbol{p}) \gamma_0 \gamma_5 \tilde{\psi}_+ (\boldsymbol{p}) \right| \boldsymbol{S}_z \right\rangle \Big|_{\cos\vartheta_p^{\mp} = \frac{\varepsilon - Mz}{p}} \quad (11)$$

where $p_{\min}^{\pm} = \left| Mx \pm \varepsilon_\nu \right|$ and $\cos\vartheta_p^{\pm} = \left(Mx \pm \varepsilon_\nu \right) / p$, while $\tilde{\psi}(p_\pm) = \tilde{\psi}\left(p, \cos\vartheta_p^{\pm}, \phi\right)$. The contributions of backward moving quarks are easily obtained from $h_{T,L}^{(+)}(-x)$ by reversing the appropriate signs in above equations. Finally we can summarize our results by decomposition of the proton structure functions into their isospin components as follows

$$\begin{aligned} h_T(x) &= h_{T,+}^{I=0}(x) + h_{T,+}^{I=1}(x) + \left(h_{T,-}^{I=0}(x) + h_{T,-}^{I=1}(x) \right) \\ h_L(x) &= h_{\mathrm{L},+}^{I=0}(x) + h_{\mathrm{L},+}^{I=1}(x) + \left(h_{\mathrm{L},-}^{I=0}(x) + h_{\mathrm{L},-}^{I=1}(x) \right) \end{aligned} \quad (12)$$

The isoscalar piece (I=0) originates from unit matrix in the decomposition while the isovector part (I=1) follows from the terms involving the collective coordinates. The explicit expressions for these structure functions in terms of the static quark wave functions are collected in Appendix of [4].
These equations represent the starting point of further calculation in the chiral models for deriving the numerical results.

The T-odd effects are higher twist, appearing at order 1/Q [4]. Including transverse momenta of quarks, there are leading order effects. One can have fragmentation of transversely polarised quarks into unpolarised spin zero hadrons or production of transversely polarised hadrons in the fragmentation of unpolarised quarks [1]. For distribution functions, it has been conjectured that T-odd quantities also might appear without violating time-reversal invariance. This might be due to soft initial state interaction or, as suggested recently [2], be a consequence of chiral symmetry breaking.

To study time-reversal for the single-quark state we write the Dirac equation. It reads

$$\left[k_\mu \gamma^\mu - g\left(\sigma + i\gamma_5 \boldsymbol{\tau}^T \cdot \boldsymbol{\pi} \right) \right] u(k) = 0 \quad (13)$$

We seek now the time reversed solution of the same equation, corresponding to the substitution $\boldsymbol{k} \to -\boldsymbol{k}$. Using the standard procedure one obtains the equation

$$\left[k_\mu \gamma^\mu - g\left(\sigma - i\gamma_5 \boldsymbol{\tau}^T \cdot \boldsymbol{\pi} \right) \right] \gamma_5 C u^*\left(\tilde{k} \right) = 0 \quad (14)$$

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where $\tilde{k} = (k_0, -\boldsymbol{k})$ and $C = i\gamma_0\gamma_2$. If the pion field is absent, the time-reversed solution is given by $\gamma_5 C u^*(\tilde{k})$. On the other hand, the term containing the pion has been modified by the previous transformation. To compensate, one needs to introduce an isospin rotation. Since $(-i\tau_2)(-\tau^T)(i\tau_2) = \tau_2$, the time- reversed solution reads as $(-i\tau_2)\gamma_5 C u^*(\tilde{k})$ and therefore under time-reversal the isospin of single quark is reversed. Actually, quark states of fixed isospin are not eigenstates of chiral Hamiltonian. A good example is provided by the hedgehog form for the mean pion field models.

In summary, quark states of specific spin and flavour are not eigenstates of the Hamiltonian. It is indeed possible to build states of definite spin and flavour in chiral Lagrangians, but they correspond to a mixing of quarks and pions.

5. Conclusions

We have developed the method of the chiral soliton calculation of the leading twist parts of the transverse and longitudinal chiral odd structure functions of the nucleon. The most important feature of the present quark based model is that it is chirally invariant and that the symmetry is dynamically broken. After bosonization the NJL model becomes an effective meson theory in which baryon emerge as self-consistent soliton solutions exactly the way as expected from large N_C considerations in QCD [6]. Chiral soliton models are particularly interesting in the context of the nucleon spin structure as these models nicely explain the small contribution of the quarks total nucleon spin.

In the NJL chiral soliton model there are two contributions to nucleon properties. First, there is the contribution of the distinct valence quark level. This is the lowest level in the quark spectrum and bound in the background of the chiral soliton. Second, there is the part which is associated to the polarization (by the soliton) of the vacuum. For many static nucleon properties the latter contribution is quite small, in particular for those which are related to the axial (spin) properties of the nucleon. This is a strong indication that the vacuum contribution to the chiral odd structure functions is negligible as well. Inclusion of $1/N_C$ corrections together with a consistently regularized treatment of the vacuum polarization is technically rather involved and beyond the scope of the present paper.

We plan consider the same problems in the framework of QCD.

References

1. D.Adams et al., Phys. Lett. (1991) , B264, 462
2. M. Anselmino et al., hep-ph/9703303.
3. O.A.Teryaev. Czechoslovak Journal of Physics. (2003), Vol.53 Suppl.A
4. L.Gamberg, H.reinhardt, H. Weigel. Phys. Rev. (1998), D58 054014.
5. T.H.R.Skyrme. Proc. R. Soc. (1961), 127, 260.
6. G. Adkins, C. Nappi, E. Witten, Nucl. Phys. (1983), B228 552.
7. Y. Nambu, G.Jona- Lasinio, Phys. Rev. (1961) , 122, 345; (1961) 124, 139.
8. H. Weigel, L. Gamberg, H. Reinhardt. Phys. Rev. (1997), D55 6910.
9. R.Jaffe. Phys. Rev. (1975) , D11 1953.
10. D.Boer, P. Mulders. Phys. Rev. (1998) , D57 83.

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