ISSN 1512-1232

A Packet Forwarding using Generion Function of Fibonacci Method

[1]Adio AKINWALE, [2]Daniel BABATUNDE

Federal University of Agriculture, Department of Computer Science, Abeokuta, Nigeria.

[1]aatakinwale@yahoo.com, [2]danieltunde88@gmail.com

Abstract. *In the course of transmitting hundreds of millions of packets per second using a high performance router due to rapid growth of internet and explosively increasing traffic over the network, a situation arises whereby there is a wastage in the communication media adopted for the transmission of the individual packet if all the roots that have been put in place are underutilized. This calls for a fast message packets forwarding using generating function of Fibonacci method, which helps to ascertain the total number of packets that can be transmitted in a given period of time. Packets of varying length were simulated using linear homogenous recurrence relations with constant coefficients of degree two and three. The results of generating function for different routes of Fibonacci method were compared and it was found that more packets could be sent in a given period of time using generating function for Fibonacci method. It was also observed that performance of each route depends on their initial conditions.*

Keywords: *generating function, Fibonacci method, packet forwarding, initial conditions*

1. Introduction

Packet forwarding in simple terms is the forwarding of packets from one node to another from networking point of view, for example, a router or a switch. It is the basic method for sharing information across systems on a network. Packets are transferred between a source and destination interface, usually on two different systems. The interface with the destination internet protocol (IP) address is specified in the packet headers then retrieves the packets from the local network. If the destination address is not on the local network, the packets are then forwarded to the next adjacent network.

During packet forwarding, underutilization or wastage of packet's route may occur due to the number of routes a packet may pass. This arises as a result of lack of prior knowledge of the number of packets that can be sent within a particular time frame. Moreover, at nodes where multiple outgoing links are available, the choice of which, all, or any to use for forwarding a given packet requires a decision making process. This is sometimes bewilderingly complex. Since a forwarding decision must be made for every packet handled by a node, the time required to send a particular number of packets is also of great concern so as to bring about maximization of the available bandwidth when sending packets through the communication media. All these can be achieved using the Fibonacci method to evaluate the number of packets that can be sent at a particular time.

The Fibonacci numbers are defined by the following recurrence:

$$F_0 = 0, \quad F_1 = 1$$

$$f_n = f_{n-1} + f_{n-2}, \text{ for } n \geq 2$$

Thus, each Fibonacci number is the sum of the two previous ones, yielding the sequence:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, …

ISSN 1512-1232

Linear homogenous recurrence relations, which possess characteristic roots can be used for an explicit formula for all the solutions. These roots were employed for modelling message packets that are transmitted from one node to another. The work deals with linear homogenous recurrence relations with constant coefficients of degree two and three. Packets are normally obtained by reading the contents from the local file system which later served into constant coefficients of degrees. The receiver attempts to accept another message packet and writes data received as fast as possible to the file system. This involves that attention must be made to designing the fastest means of sending message packets among nodes.

2. Literature Review

Mohammed J. A. and Mehrdad N. (2003), in their paper proposed an efficient IP packet forwarding based on partitioned lookup table which to a large extent reduces the complexity of the route lookup operation and parallelizes the search process. Focus was made on the uncast (single-source, single-destination) routing. Liao Y. et al., (2010) presented an efficient user mode packet forwarding scheme called "Europa" to facilitate high speed packet forwarding in virtual networks. This forwarding scheme was proposed after identifying the main causes for slow packet forwarding in conventional user mode software router.

Minlan Yu and Jennifer Rexford, (2010) provided practical techniques to apply the basic bloom filter idea to fast packet forwarding in enterprise edge routers. In order to make efficient use of limited fast memory, they optimized the sizes and number of hash functions of the bloom filters. They were able to show that to reach the optimal overall false-positive rate; bloom filters with fewer elements must have fewer false positives than those with more elements. Chazelle B., et al., (2004) also used bloom filters for longest prefix match. They used bloom filters to determine the length of the longest matching prefix for an address, and then perform a direct lookup in a large hash table in slow memory. The goal is to reduce false positives in each bloom filter, thus they choose to use the size of bloom filters proportional to the number of elements in it, and propose mini-bloom filters to deal with various distributions of prefix lengths.

Bloom filters have also been used by Broder A. and Mitzenmacher, M. (2005) and Rhea S. C. and Kubiatowicz J. (2002). They applied bloom filters to probabilistic algorithms for locating resources. The general idea was to use one bloom filter to store the list of resources that can be accessed through each neighboring node.

Jianfeng W., et al., (2005), provided high-rate, reliable and energy efficient wireless communications in mobile ad hoc networks. They presented a new cross-layer approach based on ad hoc on-demand routing and 802.11 Medium Access Control (MAC) to utilize the local path diversity. Two basic issues were addressed. The first issue is to find out the most cost efficient virtual path, in which hop has a primary forwarding node and several alternative forwarding nodes. The second issue is how to enhance the 802.11 MAC to provide opportunistic MAC layer under the guidance of routing preference.

Akinwale Adio, (2012), provided a fast way of forwarding packets of varying lengths which were generated and simulated using linear homogenous recurrence relations with constant coefficients of degree two and three. All previous works on packet forwarding do not estimate the number of packets that can be sent per time, except for this work. Hence this work is to maximize the number of packets that can be forwarded in predetermined time.

3. Generating Function for Fibonacci method with two roots

ISSN 1512-1232

The recurrence relations $f_n = f_{n-1} + f_{n-2}$ is a linear homogenous recurrence relation of degree two. It also satisfies the initial conditions of $f_0 = 0$ and $f_1 = 1$. The roots of the characteristic equation $r^2 - r - 1 = 0$ are

$$r_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$r_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

$$r_1 = \frac{-(-1) + \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$r_1 = \frac{1 + \sqrt{5}}{2}$$

$$r_2 = \frac{-(-1) - \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)}$$

$$r_2 = \frac{1 - \sqrt{5}}{2}$$

It follows that Fibonacci numbers are given by

$$f_n = a_1(\frac{1+\sqrt{5}}{2})^n + a_2(\frac{1-\sqrt{5}}{2})^n \qquad \text{(i)}$$

for some constants α_1 and α_2. The initial conditions $f_0 = 0$ and $f_1 = 1$ can be used to find the constants.

We have

$$f_0 = a_1 + a_2 = 0 \quad (ii)$$

$$\therefore \qquad \alpha_1 = -\alpha_2$$

From equation (i),

$$f_1 = a_1(\frac{1+\sqrt{5}}{2}) + a_2(\frac{1-\sqrt{5}}{2}) = 1 \qquad (iii)$$

By substituting the value of $\alpha_1 = -\alpha_2$ into equation (*ii*), we have

$$= -\alpha_2\left(\frac{1+\sqrt{5}}{2}\right) + \alpha_2\left(\frac{1-\sqrt{5}}{2}\right) = 1$$

$$= -\alpha_2(1+\sqrt{5}) + \alpha_2(1-\sqrt{5}) = 2$$

$$= -\alpha_2 - \alpha_2\sqrt{5} + \alpha_2(1-\sqrt{5}) = 2$$

$$= -2\alpha_2\sqrt{5} = 2 \Rightarrow -\alpha_2 = \frac{2}{2\sqrt{5}} \Rightarrow \alpha_2 = -\frac{1}{\sqrt{5}}$$

Using equation (*ii*) and by substituting the value of $\alpha_2 = -\frac{1}{\sqrt{5}}$ into it, we have

$$\alpha_1 + \alpha_2 = 0$$

ISSN 1512-1232

$$\alpha_1 + \left(-\frac{1}{\sqrt{5}}\right) = 0 \Rightarrow \alpha_1 - \frac{1}{\sqrt{5}} = 0 \Rightarrow \alpha_1 = \frac{1}{\sqrt{5}}$$

Thus, $\alpha_1 = \frac{1}{\sqrt{5}}$ and $\alpha_2 = -\frac{1}{\sqrt{5}}$

Consequently, the Fibonacci numbers with two roots are given by

$$f_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$

This proof of $f_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$ concludes that different message packets can be transmitted in any *n* times using two routes.

Thus, by defining the generating function for Fibonacci numbers as the formal power series whose coefficients are the Fibonacci numbers, we have

$$F(x) = \sum_{n=0}^{\infty} F_n x^n$$

$$= 0 + x + x^2 + 2x^3 + 3x^4 + 5x^5 + 8x^6 + 13x^7 + 21x^8 + \cdots$$

Therefore, base on the generating function for Fibonacci number, we can set the recurrence relation for the two routes $f_n = f_{n-1} + f_{n-2}$ to become

$$f_n = i_1 f_{n-1} + i_2 f_{n-2},$$

where i_1 and i_2 are the initial values in microseconds.

Examples

For the transmission of packets over a communication channel using two routes, if the packet forwarding of one route requires one microsecond and the transmission of the other packet in another route is two microseconds, the Fibonacci recurrence relation for the two routes is given by:

$$f_n = f_{n-1} + f_{n-2}$$

$$f_0 = 1, f_1 = 1$$

$$f_2 = f_{(2-1)} + f_{(2-2)} = f_{(1)} + f_{(0)} = 1 + 1 = 2\ packets$$

$$f_3 = f_{(3-1)} + f_{(3-2)} = f_{(2)} + f_{(1)} = 2 + 1 = 3\ packets$$

$$f_4 = f_{(4-1)} + f_{(4-2)} = f_{(3)} + f_{(2)} = 3 + 2 = 5\ packets$$

$$f_5 = f_{(5-1)} + f_{(5-2)} = f_{(4)} + f_{(3)} = 5 + 3 = 8\ packets$$

$$f_6 = f_{(6-1)} + f_{(6-2)} = f_{(5)} + f_{(4)} = 8 + 5 = 13\ packets$$

$$f_7 = f_{(7-1)} + f_{(7-2)} = f_{(6)} + f_{(5)} = 13 + 8 = 21\ packets$$

$$f_8 = f_{(8-1)} + f_{(8-2)} = f_{(7)} + f_{(6)} = 21 + 13 = 34\ packets$$

$$f_9 = f_{(9-1)} + f_{(9-2)} = f_{(8)} + f_{(7)} = 34 + 21 = 55\ packets$$

$$f_{10} = f_{(10-1)} + f_{(10-2)} = f_{(9)} + f_{(8)} = 55 + 34 = 89\ packets$$

This implies that after ten (10) microseconds, the total number of packets that can be sent is 89.

However, using the generating function recurrence relation, we have

ISSN 1512-1232

$$f_n = i_1 f_{n-1} + i_2 f_{n-2}$$

$$f_0 = 1, \; f_1 = 1, \; i_1 = 1, i_2 = 2$$

$$f_2 = 1f_{(2-1)} + 2f_{(2-2)} = 1f_{(1)} + 2f_{(0)} = (1 \, x \, 1) + (2 \, x \, 1) = 3 \, packets$$

$$f_3 = 1f_{(3-1)} + 2f_{(3-2)} = 1f_{(2)} + 2f_{(1)} = (1 \, x \, 3) + (2 \, x \, 1) = 5 \, packets$$

$$f_4 = 1f_{(4-1)} + 2f_{(4-2)} = 1f_{(3)} + 2f_{(2)} = (1 \, x \, 5) + (2 \, x \, 3) = 11 \, packets$$

$$f_5 = 1f_{(5-1)} + 2f_{(5-2)} = 1f_{(4)} + 2f_{(3)} = (1 \, x \, 11) + (2 \, x \, 5) = 21 \, packets$$

$$f_6 = 1f_{(6-1)} + 2f_{(6-2)} = 1f_{(5)} + 2f_{(4)} = (1 \, x \, 21) + (2 \, x \, 11) = 43 \, packets$$

$$f_7 = 1f_{(7-1)} + 2f_{(7-2)} = 1f_{(6)} + 2f_{(5)} = (1 \, x \, 43) + (2 \, x \, 21) = 85 \, packets$$

$$f_8 = 1f_{(8-1)} + 2f_{(8-2)} = 1f_{(7)} + 2f_{(6)} = (1 \, x \, 85) + (2 \, x \, 43) = 171 \, packets$$

$$f_9 = 1f_{(9-1)} + 2f_{(9-2)} = 1f_{(8)} + 2f_{(7)} = (1 \, x \, 171) + (2 \, x \, 85) = 341 \, packets$$

$$f_{10} = 1f_{(10-1)} + 2f_{(10-2)} = 1f_{(9)} + 2f_{(8)} = (1 \, x \, 341) + (2 \, x \, 171) = 683 \, packets$$

This implies that after ten (10) microseconds, the total number of packets sent is 683.

Also, given that the packet forwarding of one route requires two (2) microseconds and the other packet in another route is three (3) microseconds, the number of packets that can be sent within ten (10) microseconds is evaluated below:

The Fibonacci recurrence relation is given by:

$f_n = f_{n-2} + f_{n-3}$, due to the transmission time of the two routes (i.e. when n $\geq$ 3), with initial conditions as follows:

$$f_0 = 0, \;\; f_1 = 1, \; f_2 = 1$$

$$f_3 = f_{(3-2)} + f_{(3-3)} = f_{(1)} + f_{(0)} = 1 + 0 = 1 \, packet$$

$$f_4 = f_{(4-2)} + f_{(4-3)} = f_{(2)} + f_{(1)} = 1 + 1 = 2 \, packets$$

$$f_5 = f_{(5-2)} + f_{(5-3)} = f_{(3)} + f_{(2)} = 1 + 1 = 2 \, packets$$

$$f_6 = f_{(6-2)} + f_{(6-3)} = f_{(4)} + f_{(3)} = 2 + 1 = 3 \, packets$$

$$f_7 = f_{(7-2)} + f_{(7-3)} = f_{(5)} + f_{(4)} = 2 + 2 = 4 \, packets$$

$$f_8 = f_{(8-2)} + f_{(8-3)} = f_{(6)} + f_{(5)} = 3 + 2 = 5 \, packets$$

$$f_9 = f_{(9-2)} + f_{(9-3)} = f_{(7)} + f_{(6)} = 4 + 3 = 7 \, packets$$

$$f_{10} = f_{(10-2)} + f_{(10-3)} = f_{(8)} + f_{(7)} = 5 + 4 = 9 \, packets$$

From the above, it can be inferred that the total number of packets that can be sent in ten (10) microseconds is 9.

However, using the generating recurrence relation which is given by

$$f_n = i_1 f_{n-2} + i_2 f_{n-3},$$

where i_1 = 2 and i_2 = 3, we have

$$f_0 = 0, \; f_1 = 1, \; f_2 = 1$$

$$f_3 = 2f_{(3-2)} + 3f_{(3-3)} = 2f_{(1)} + 3f_{(0)} = (2 \, x \, 1) + (3 \, x \, 0) = 2 \, packets$$

$$f_4 = 2f_{(4-2)} + 3f_{(4-3)} = 2f_{(2)} + 3f_{(1)} = (2 \, x \, 1) + (3 \, x \, 1) = 5 \, packets$$

$$f_5 = 2f_{(5-2)} + 3f_{(5-3)} = 2f_{(3)} + 3f_{(2)} = (2 \, x \, 2) + (3 \, x \, 1) = 7 \, packets$$

ISSN 1512-1232

$$f_6 = 2f_{(6-2)} + 3f_{(6-3)} = 2f_{(4)} + 3f_{(3)} = (2 \, x \, 5) + (3 \, x \, 2) = 16 \, packets$$
$$f_7 = 2f_{(7-2)} + 3f_{(7-3)} = 2f_{(5)} + 3f_{(4)} = (2 \, x \, 7) + (3 \, x \, 5) = 29 \, packets$$
$$f_8 = 2f_{(8-2)} + 3f_{(8-3)} = 2f_{(6)} + 3f_{(5)} = (2 \, x \, 16) + (3 \, x \, 7) = 53 \, packets$$
$$f_9 = 2f_{(9-2)} + 3f_{(9-3)} = 2f_{(7)} + 3f_{(6)} = (2 \, x \, 29) + (3 \, x \, 16) = 106 \, packets$$
$$f_{10} = 2f_{(10-2)} + 3f_{(10-3)} = 2f_{(8)} + 3f_{(7)} = (2 \, x \, 53) + (3 \, x \, 29) = 193 \, packets$$

From the above, it can be inferred that the total number of packets that can be sent in ten (10) microseconds is 193.

4. Generating Function for Fibonacci number with three roots

The recurrence relation $f_n = f_{n-1} + f_{n-2} + f_{n-3}$ is a linear homogenous recurrence relation of degree three. It also satisfies different initial conditions.

$$r^3 - r^2 - r - 1 = 0$$
$$r(r^2 - r - 1) = 1$$

For a cubic equation the roots should be taken as $\alpha - \beta, \ \alpha, \ \alpha + \beta$.

After following the process of Fibonacci number with two roots, the Fibonacci numbers with three roots that can transmit message packets in any *n* times are given as follows:

$$f_n = \left(\frac{1}{3}\right)^n + \left(\frac{7 - i\sqrt{26}}{6i\sqrt{26}}\right)\left(\frac{1 + \sqrt{26}}{3}\right)^n + \left(\frac{7 - i\sqrt{26}}{6i\sqrt{26}}\right)\left(\frac{1 - \sqrt{26}}{3}\right)^n$$

The process specifies generating function for Fibonacci method that can be used to transmit messages in *n* microseconds using two and three roots with different initial conditions.

5. Implementation

The generating function for Fibonacci method with two and three roots were coded in Java programming language which was run on Netbean IDE 7.2 environment. The time of transmitting packets has been converted into microseconds. In determining the number of packets that can be transmitted after *n* microseconds, the whole process was simulated. This system allows the user to input the number of routes to use (either two (2) or three (3)) together with their initial values which regulate the time to send the messages. For example, the user is allowed to enter 1 as first initial value and 2 as the second initial value and the range of the time for the simulation is shown in figure 1. The button inscribed the word "generate" would be pressed to compute the number of packets to be sent in a specified time as illustrated in figure 1. The same process was also done for figure 2 by changing the first initial value to 2 and second initial value to 3. The results were displayed under the number of the packets transmitted at specified time. A simulation process was also carried out for three roots using initial values of 1, 2 and 2. The results were illustrated in figure 3 while figure 4 displayed the results of three roots with initial values of 1, 2 and 3.

6. Results and discussion

The results of packet forwarding using generation function for Fibonacci number for two and three roots with specified times are described in Table 1 and Table 2 respectively. Looking at the table 1 using two roots with initial values of 1 and 2, 89 packets could be sent in 10ms while 1,346,269 packets could be transmitted in 30ms. In table 2 using two roots with the same initial

ISSN 1512-1232

values (1 and 2), 686 packets could be sent in 10ms while 715,827,883 packets could be transmitted in 30ms.

When one of the roots requires 2ms for packet transmission and the packet forwarding of the second root requires 3ms, 12 packets could be transmitted in 10ms while 3,329 packets would be sent in 30ms as shown in table 1. In table 2, 193 packets could be transmitted in 10ms and 129, 696, 365 packets would be sent in 30ms. Applying three different roots of which one root requires 1ms for packet transmission and the other two roots demand 2ms each for packet transmission, 341 packets would be sent in 10ms and 35,7913,941 packets would be sent in 30ms as illustrated in table 1 while in table 2, 2,929 packets would be sent in 10ms and 4.36390E+11 packets would be sent in 30ms.

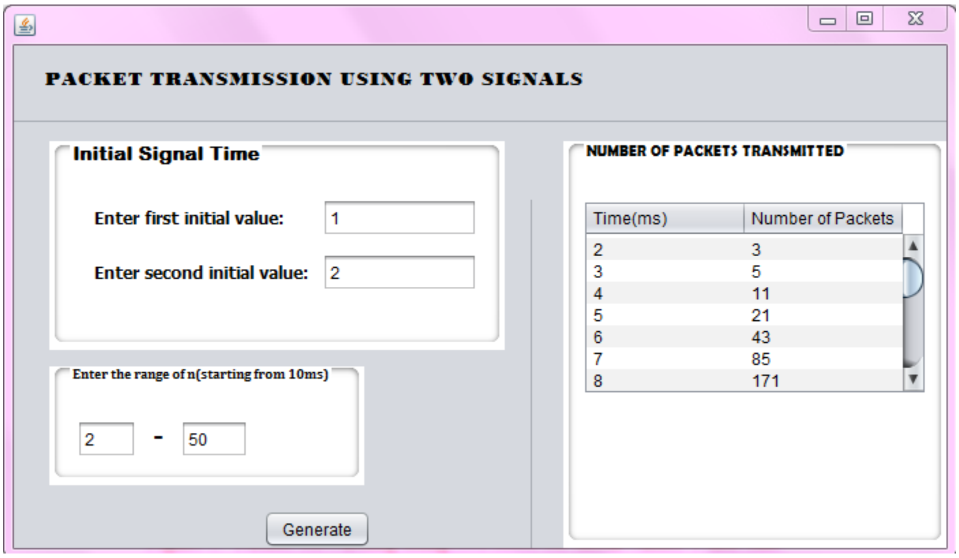

Figure 1: Packet forwarding using two roots for initial values 1 and 2ms

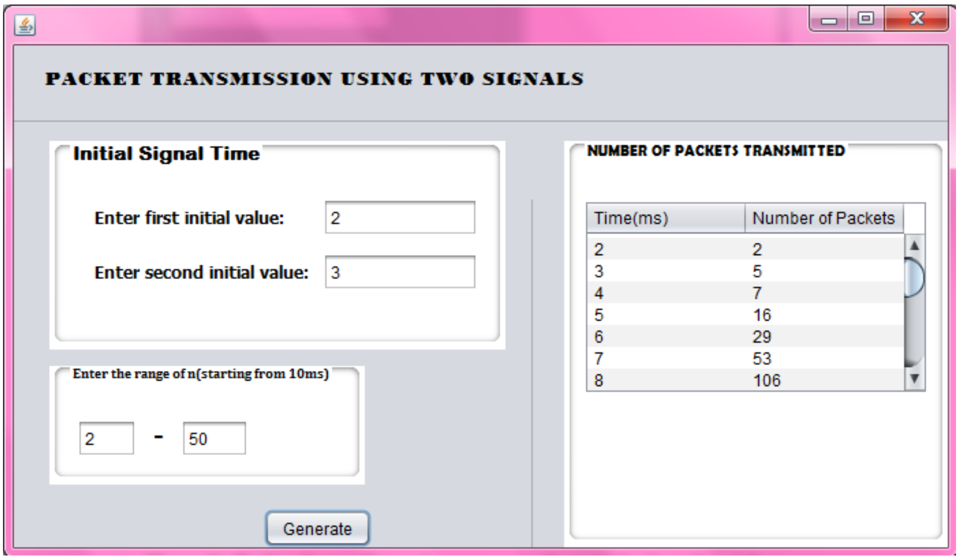

Figure 2: Packet forwarding using two roots for initial values 2 and 3ms

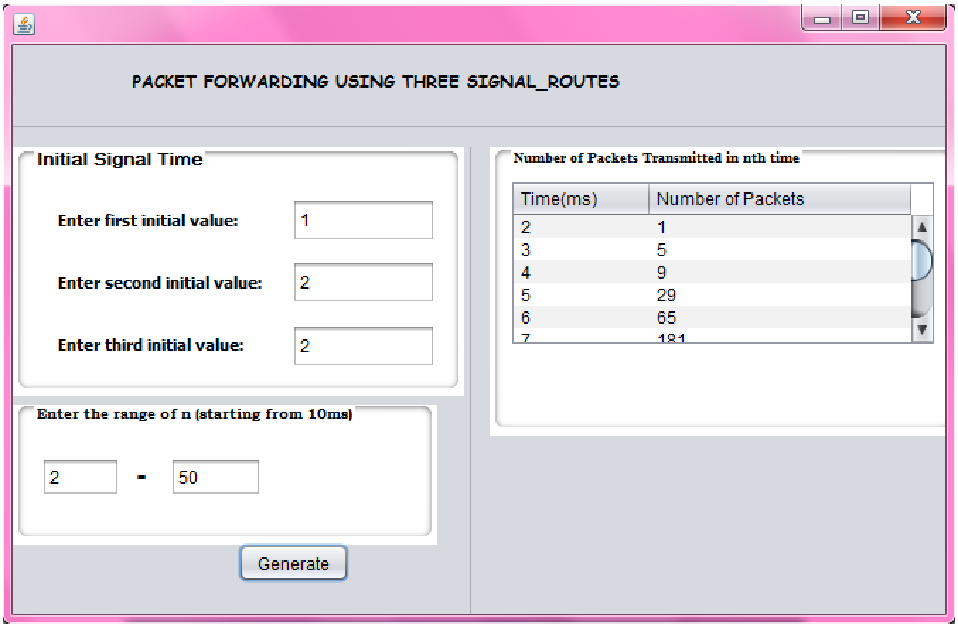

Figure 3: Packet forwarding using three roots for initial values 1, 2 and 2ms

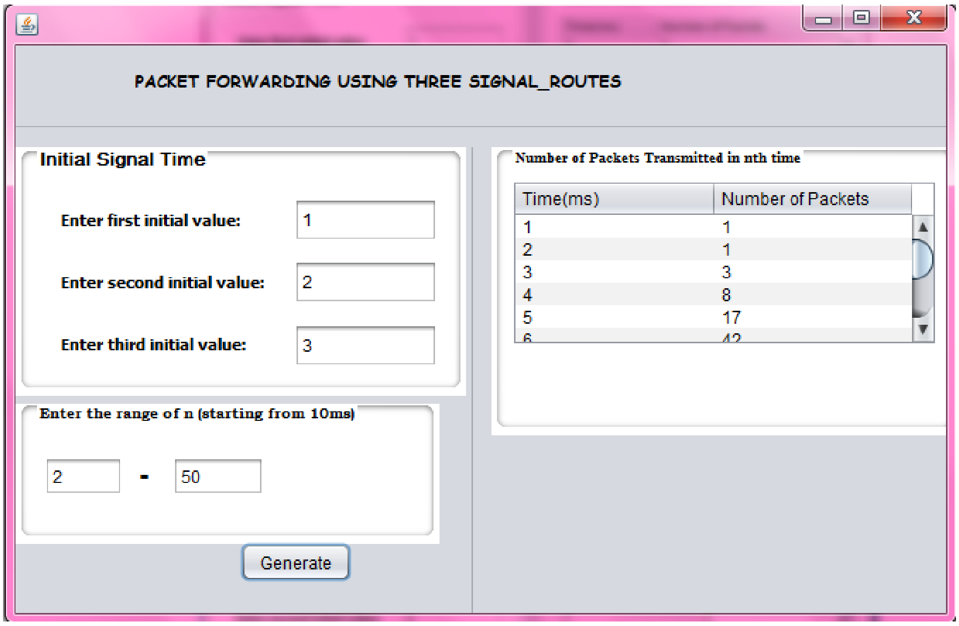

Figure 4: Packet forwarding using three roots for initial values 1, 2 and 3ms

ISSN 1512-1232

Table 1: Results of packet forwarding using Fibonacci method for two and three routes

Time(m)	Number of packets			
	Using two routes		Using three routes	
	1 and 2ms	2 and 3ms	1, 2 and 2ms	1, 2 and 3ms
10	89	12	341	149
20	10946	200	349525	66012
30	1346269	3329	357913941	29249425
40	165580141	55405	36650387592	12960201916
50	20365011074	922111	3.752999E+14	5.74257E+12
60	2.504731E+12	15346786	3.843072E+17	1.383410E+15
70	3.080615E+14	255418101	6.148914E+20	1.127444E+18

Table 2: Results of packet forwarding using generation function for Fibonacci number for two and three routes

Time(m)	Number of packets			
	Using two routes		Using three routes	
	1 and 2ms	2 and 3ms	1, 2 and 2ms	1, 2 and 3ms
10	686	193	2929	1331
20	699051	219130	35877321	7579198
30	715827883	12696365	4.36390E+11	4.31723E+10
40	7.33008E+11	7.67536E+10	5.30772E+15	2.45916E+14
50	7.50600E+14	4.54221E+13	9.21646E+19	1.40078E+18

By modifying the initial conditions whereby one root requires 1ms, second root is 2ms and third root has initial value of 3ms for packet transmission, 149 packets would be sent in 10ms while 29,249,425 packets would be sent in 30ms as depicted in table 1. In table 2, 1331 packets would be sent in 10ms and 4.31723E+10 packets would be sent in 30ms. The constraint is that each root of a packet is followed by the next root. The results indicated that generating function for Fibonacci number performed better than Fibonacci method without generating function using three roots or two roots with any initial conditions. The results from the generation function for Fibonacci number with three roots with different initial values still performed better in packet forwarding than two roots with any initial values as shown in table 2 and figures 1 to 4.

7. Conclusions

Packet forwarding from the source to its destination could be done efficiently using the generating function of Fibonacci method to compute the number of packets that could be sent in *n*

ISSN 1512-1232

microseconds. It was observed that initial values play a vital role on the number of message packets that could be sent in *n* microseconds. Generating function for Fibonacci method with three routes performed better than with two routes without considering delay time in the network. Using generating function for Fibonacci method could lead to maximization of the number of packets that could be transmitted on the available bandwidth and minimization of the communications channel waste. This is due to the fact that the total number of packets could be determined within a given time range compared to other packet forwarding techniques which do not allow the estimation of the number of packets that can be sent per time. Therefore, with this functionality being added for packet processing, it would enhance the rate at which message packets could be forwarded from one node to another in a network environment.

References

1 [Akinwale, 2012] Akinwale A. T. (2012), A fast message packet forwarding using Fibonacci method, *GESI: Computer Science and Telecommunication,* 2(34), 64-69
2 [Broder, 2005] Broder A. Mitzenmcher M. (2005), Network application of bloom filter, *A survey in internet mathematics*, 1(1), 485-509
3 [Chazelle, 2004] Chazelle B., Kilian J., Rubinfeld R., Tal A., (2004) The bloomier filter: An efficient data structure for static support lookup table, *In proceeding of the fifteenth annula ACM-SIAM symposium on discrete algorithm (SODA)*, 30-39
4 [Gossett, 2009] Gosset E. (2009), *Discrete mathematics with proof*, second edition, John Wiley and Sons, New Jersey
5 [Jianfeng, 2005] Jianfeng W. (2005), Reliable and efficient packet forwarding by utilizing path diversity in wireless ad-hoc network, *Canadian journal of electrical and computer engineering,* Special issues on advances in wireless communication and networking, 29(2), 129-134
6 [Liao, 2010] Liao Y., Yin D., Gao L. (2010), Europa: Efficient user node packet forwarding in neteork, Northwestern Polytech University, China
7 [Minla, 2010] Minla Y., Jennifer R., (2010), Has don’t cash fast forwarding foe enterprise edge routers, Princeton University
8 [Mohammed, 2003] Mohammed J. A. Mehrdad N., (2003), An IP forwarding technique based on partitioned lookup table, *Centre for integrated circuits and systems*, University of Texas, Delta
9 [Rhea, 2002] Rhea S. C., Kubiatowicz J., (2002), Probabilistic location and routing, *In proceeding of IEEE,* INFOCOM, 23-34
10 [Rosen, 2007] Rosen R. K., (2007), *Discrete mathematics and its application,* seventh edition, McGraw-Hill

Article received: 2015-04-29