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The research of the efficiency of the factorial knowledge and methods of designing factor tests

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Abstract

We study the decomposition of knowledge on the set of "micro-knowledge" admitting a single-valued binary test score. Then the task of the test is formulated as a set of micro-knowledge answers that together reveal the level of knowledge of a specific subsection. This allows to regard the process of designing the set of answers to a test question as a problem of investigating multiple-factor dependence and approach it from the standpoint of planning a multiple-factor two-level experiment. It formalizes well both the procedure of designing a test and the procedure of assessing its results making them easy to design for semiskilled teachers. The method is illustrated by an example of constructing and using tests for assessing the level of knowledge in the theory of automatic control.

Key words: *factors of knowledge, experiment planning, two-level multiple-factor experiments, variation of factors, planning matrix.*

1. Status of the issue

In the works [1-6] we proposed a largely new method of tests construction and their implementation with the use of three-factor and two-factor tests. Theoretically, this kind of tests may be based on any number of factors of knowledge. Therefore, we are interested in the generalization of the approach proposed in the cited papers both in terms of the structural organization of multiple-factor tests and in terms of the development of the algorithm assessing the answers to multiple-factor test questions (MF TQ).

2. Structural organization of MF TQ

The essence of compiling multiple-factor test question (MF TQ) involves the selection of elements in the subject studied which are named micro-knowledge by the author [6]. The answers to the questions raised regarding micro-knowledge (MK) should be characterized by a number of properties. First, they must be simple enough to allow the demonstration of learning with one correct answer and the lack of knowledge with one wrong. Second, all of them in total with the correct answers must ensure the full knowledge of the subject. Third, the MK should be as independent as possible, i.e. the possession of one of them does not guarantee the possession of another, and vice versa the ignorance of any MK does not exclude knowing another one. Fourth, the conventional "weight" of each MK in the general knowledge of the subject material should be about the same (comparable). Then the degree of assimilation of the study area of educational material can be characterized by the share of correct answers according to MK of their total amount.

Thus, MK are the elements of knowledge which in the ideal case should have a complete assimilation of the subject studied (or its topic) in the educational process. However, partial assimilation is also possible and estimated.

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As an example of selection of such basic knowledge, we can demonstrate the result of transfer functions (TF) properties analysis and their qualitative and quantitative characteristics. To do this we'll examine a universal form of TF representation in the form of successively structured rational function of the type

$$W(p) = kp^{v} \cdot \frac{\prod_{i=1}^{v1}(T_i p+1)\prod_{j=1}^{ks1}(T_j{}^2 p^2 + 2\xi T_j p+1)}{\prod_{i=1}^{v2}(T_i p+1)\prod_{j=1}^{ks2}(T_j{}^2 p^2 + 2\xi T_j p+1)}, \qquad (1)$$

where k - is a dimensional coefficient transmission link; v determines the order of differentiation ($v > 1$) or astatism ($v < 1$); T_i, T_j - time constants of the elementary units; the degree of damping the second order units.

In the study of the concepts and forms of representation of the dynamic links transfer functions in the "theory of linear control systems" it is advisable to reveal their elementary properties. They are easily and clearly divided into qualitative and quantitative. Qualitative characteristics, which can be formulated as the MK, include the properties of statism (the number of zero roots in the numerator and denominator of the TF is 0), astatism (there are zero roots in the denominator of the TF - sign"-"), the differentiation signal (there are zero roots in the numerator of the TF "+" sign), the fact of physical realizability or unrealizability of the link (the ratio of the numerator and denominator orders of the TF (1)). The knowledge of quantitative characteristics of the TF is manifested in the ability to assess the value of the astatism order (the number of zero roots in the polynomial of the TF denominator) or differentiation (the number of zero roots in the TF numerator), the coefficient value of the transmission, the total order of the link (the degree of the characteristic TF polynomial), etc. Further division of these units of knowledge is either impossible or impractical. Therefore, the answers to the questions about them can be formulated as binary.

Depending on the assigned complexity the MF TQ may include some micro-knowledge associated with the presence of respective properties or quantitative estimates of these factors of knowledge in the TF. For example, the person tested is proposed to estimate correctly the presence or absence of link astatism according to the TF, the total value of the dynamic link order, the equivalent value of its order, etc. Thus, each micro-knowledge can have two options: the right - "Yes" and the wrong - "No", and the test question itself is constructed as a set of micro-knowledge with binary (two-level) answers "yes" and "no" (1 and 0).

In this regard, the process of constructing the MF TQ can be considered using the analogy of planning the study of multiple-factor dependence in the theory of experimental planning (TEP). Then a possible set of answers to a test question becomes similar to the matrix of planning a two-level multiple-factor experiment, if the answer is positive - "Yes" it must be replaced with +1, and a negative answer - "No" with -1. Thus, by varying answers to micro-knowledge on two levels one can obtain a set of rows-answers to a full test question which implements all of their possible combinations by analogy with a full factorial experiment.

The test material built on the basis of the defined principles and the similarity to the methods of TEP will be called a "factorial test" (FT). Micro-knowledge used for constructing questions for the FT will be called "factors of knowledge" (FK).

3. Assessment of the factorial testing results

The second major problem after working out the structure of test construction is to assess its the results. With respect to the FT it should be noted that when the FK are used the test structure must include the answers to test questions which contain clearly defined right or wrong answers. This fact implies that the most natural implementation of the first stage of assessment related to the assimilation of factors of knowledge is supposed to use binary scale. According to the method

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developed in [1,5,6] it differs from the normalized assessment of the factors in TEP (+1, -1) replacing the lower level for the lack of knowledge with a more natural zero as shown in Table 1.

Table 1. The structure of answers according to the factors of knowledge

Answers to the factors of knowledge				Assessment of the answers
x_1	x_2	…	x_n	
Right answer $\boldsymbol{tr_1}$	Right answer $\boldsymbol{tr_2}$	…	Right answer $\boldsymbol{tr_n}$	1
Wrong answer $\boldsymbol{er_1}$	Wrong answer$\boldsymbol{er_2}$	…	Wrong answer $\boldsymbol{er_n}$	0

4. Rating scale of partially correct answers to the MF TQ

If we consider all the factors of knowledge to be equivalent (which is not always true), it is possible to get $n+1$ options of assessing e_j answers to the examined TQ which are calculated as weighted shares of right (single) assessments of these factors in the answer (see Table 2)

$$e_j = \frac{n-j}{n} \in [0,1], \tag{2}$$

As a result a possible assessment of the answer to a TQ from FT as a whole is distributed in the range $[0,1]$. In other words, it is calculated as an average of the set of assessments of the FK contained in the question.

Table 2. Assessment of answer combinations by the factors of knowledge of a random TQ

Options of the assessment combinations of the factors of knowledge	x_1	1	0	0	…	0	…	0
	x_2	1	1	0	…	0	…	0
	x_3	1	1	1	…	0	…	0
	…	…	…	…	…	…	…	…
	x_i	1	1	1	…	0	…	0
	x_{i+1}	1	1	1	…	1	…	0
	…	…	…	…	…	…	…	…
	x_n	1	1	1	…	1	…	0
Assessments of the options		1	$(n-1)/n$	$(n-2)/n$	…	$(n-j)/n$	…	0

The answer to the question of a multiple-factor test, the assessment of which is calculated by the formula (2), satisfying the inequality

$$0 < e_j < 1, \tag{3}$$

will be called "partially correct", and e_j will be considered as a share measure of knowledge for a question of the FT. The share measure of knowledge for a question is easily converted into numerical score by scaling.

The number of answers with different fractional assessments isn't the same. In the full set of options only absolutely right and absolutely wrong answers are found once. The frequency *m* of the remaining options depends on the values *n* and *j*. It is defined by the formula

$$\forall j = \overline{0,n} \rightarrow m_j = \frac{n!}{j! \cdot (n-j)!}, \quad (4)$$

similar to the Newton's binomial formula.

Thus, the least likely (by simple guessing) are absolutely right and absolutely wrong answers to the TQ. Their probability is

$$p_1 = p_0 = \frac{1}{n}, \quad (5)$$

i.e. it decreases hyperbolically with increasing number of factors of knowledge involved for testing.

Any "partially correct" answers (PCA), the frequency of which is determined by the formula (2), have the probability

$$p_j > p_{1,0}, \quad (6)$$

which is always greater than the probability of absolute answers. The principle of distributing this probability has a single maximum at even *n*, and two identical maximums at odd *n*.

Naturally, the following condition is fulfilled

$$\sum_{j=0}^{n} \frac{n!}{j! \cdot (n-j)!} = 1. \quad (7)$$

5. Conclusion

The proposed model of assessing the results of a factor test is quite logical, simple and rather adequately reflects the level of knowledge of a tested person. This has been repeatedly confirmed by experimentally performed procedures of testing the students studying the theory of automated control.

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