ISSN 1512-1232

Time-Varying Autoregressive Model Using Multi-Wavelet Basis Functions

G .Ravi Shankar Reddy[1] and Dr. Rameshwar Rao[2]

[1]Dept.of ECE CVR College of Engineering, Hyderabad, India
[2]Ex Vice -Chancellor, JNT University, Hyderabad, India
[1]ravigosula_ece39@yahoo.co.in,[2]rameshwar_rao@hotmail.com

Abstract

A new time-varying autoregressive (TVAR) modeling approach is proposed for non-stationary signal processing and analysis. In the new parametric modeling frame work, the time-dependent coefficients of the TVAR model are represented using a novel multi-wavelet decomposition scheme. The realization of the time-varying AR(TVAR)model here is distinguished from existing time-varying parametric models where the relevant time-dependent coefficients are represented using basis function expansions. In most existing time-varying parametric models, the basis functions used for representing the time-dependent coefficients are global, while the basis functions involved in the new proposed modeling approach are locally defined. The main features of the multi-wavelet approach is that it enables smooth trends to be tracked but also to capture sharp changes in the time-varying process parameters. The associated time-varying coefficients are then estimated by using a Orthogonal least square (OLS) Algorithm. Simulation results show the effectiveness of the proposed method.

Keywords*: TVAR model, Time-dependent coefficients, Multi-wavelet basis, Orthogonal least square(OLS).*

1. Introduction

Many processes are inherently time-varying and cannot effectively be characterized using time invariant models[1].Modeling and analysis of time-varying systems is often a challenging problem. One feature of time-varying systems is that such signals contain non-stationary transient events. One approach To characterize such non-stationary processes is to employ time-varying parametric models for example Time-varying Autoregressive (TVAR)model[2].Approaches for the estimation of time-varying parameters in TVAR model can be broadly classified into two categories[3]: the adaptive recursive algorithm methods and the basis function approximation methods. Adaptive algorithms such as Least mean squares (LMS), recursive least squares(RLS) and Kalman filtering, are applied to estimate the time-varying parameters and are capable of tracking the transient variation providing that the variation is slow and smooth[4]. For the basis function method, time-varying parameters are expanded as a finite sequence of predetermined basis functions; the problem of time-varying estimation can then be reduced to a time invariant parameter estimation problem. The basis function expansion approaches are able to track process parameter changes even those with jumps, provided that appropriate basis functions are used. Many types of basis functions ,such as the Legendre polynomial, Fourier series, Walsh and Haar functions are available and capable of representing TV model coefficients. The choices of basis functions have significant effects on the change speeds and smoothness of the estimated parameters. But there is no uniform selection guideline on how to select the appropriate basis functions from the large-family of available basis functions[1]-[4].

An attractive approach is to expand the time-varying parameters using wavelets as basis functions. Wavelets have distinctive approximation properties and are well suited for approximating general non-stationary signals and thus have been successfully applied to many areas including nonlinear signal processing and parametric identification[1]. The objective of this study is to present a novel TVAR modeling approach, where the time-dependent coefficients are expanded using a finite set of multi-wavelet basis functions [1]-[2]. Based on a multi-wavelet expansion scheme, a new method for time-dependent parameter estimation is then proposed. The term 'multi-wavelet' here has a twofold meaning. Firstly, the TV coefficients of the TVAR model are approximated using several types of wavelet basis functions(i.e. the TV Parameter estimation involves multiple wavelets).Secondly, these wavelet basis functions are combined in a form of multi-resolution wavelet decomposition[3].The advantage of the proposed method, compared with a method involving only a single type of wavlets, is that the multi-wavelet expansion scheme is much more flexible in that it exploits the excellent properties of both non-smooth and smooth wavelet basis functions and thus can effectively track both rapid and slow variations of TV coefficients. In addition, the expansion of TV parameters onto multi-wavelet basis functions is more accurate and effective for dealing with non-stationary signal modeling than traditional power spectral estimation approaches and classical time -invariant parameter models.

The TVAR model with multi-wavelet basis functions is able to track and capture the parameter changes even those with jumps .However, it should be Pointed out that in many applications, not all these candidate wavelet basis functions need to be simultaneously involved in a same time varying coefficient approximation, some wavelet basis functions which play a more important role need be included in the expression, some other wavelet basis functions, however, may only play some little role and can be exclude from the expression. Orthogonal least Squares(OLS)algorithms[6] will be applied to determine which wavelet basis functions should be included in the final approximation expression and which candidate wavelet basis functions should be eliminated From the dictionary[6].

The paper is organized as follows .Section 2 introduces Time varying Autoregressive Model. In section 3, wavelet theory is briefly reviewed to provide the basis of multi-resolution expansions for arbitrary functions. Linear Regression model can be used to determine time varying parameters, and this is introduced in section 4.Simulation examples are provided in section 5, and conclusions are given in section 6.

2.The Time-Varying Auto Regressive Model(TVAR Model)

The p-th order time-varying AR model, TVAR(p),is formulated as below[1]-[2]

$$y(t)=\sum_{i=1}^{p}a_i(t)y(t-i)+e(t)\ , \tag{1}$$

where t is the time instant (or) sampling index of the signal $y(t)$ $e(t)$,is the model residual that can often be treated as a stationary white noise sequence with zero mean and variance $\sigma^2{}_e$,and $a_i(t)$ are the time-varying coefficients. Approaches for the estimation of time-varying parameters can be broadly be classified into two categories: the adaptive recursive algorithm methods and the basis function approximation methods. Adaptive algorithms such as least mean squares (LMS), recursive least squares (RLS) and Kalman filtering, are applied to estimate the time -varying parameters and are capable of tracking the transient variation providing that the variation is slow and smooth. For the basis function method, time-varying parameters are expanded as a finite sequence of predetermined basis functions; the problem of time -varying estimation can then be reduced to a time invariant parameter estimation problem, in the basis function expansion approach the time-varying coefficients $a_i(t)$ using a set of basis functions $\{\pi_m(t):m=1,2,.....L\}$,where $\pi_m(t)$ are scalar functions as below [3].

ISSN 1512-1232

$$a_i(t) = \sum_{m=1}^{L} c_{i,m}\pi_m(t) \quad . \tag{2}$$

Substituting (2) into (1), yields

$$y(t) = \sum_{i=1}^{p}\sum_{m=1}^{L} c_{i,m}\pi_m(t)y(t-i) + e(t) \tag{3}$$

Denote

$$\pi(t) = \left[\pi_1(t), \pi_2(t), ..., \pi_L(t)\right]$$

$$X_i(t) = y(t-i)\pi(t)$$

$$X(t) = \left[X_1(t), X_2(t), ..., X_p(t)\right]$$

$$C_i = \left[C_{i,1}, C_{i,2}, ..., C_{i,M}\right], \qquad C = \left[C_1, C_2, ..., C_p\right]$$

Equation (3) can then be written as

$$y(t) = X(t)C^T + e(t), \tag{4}$$

where the upper script 'T' indicates the transpose of a vector or a matrix. Equation (4) is a standard linear regression model that can be solved using linear least squares algorithms.

Let $\hat{C}$ be the estimate of C, $\hat{a}_i(t)$ be t he estimate of $a_i(t)$ and $\hat{\sigma}_e^2$ be the estimate of σ_e^2 .The time-dependent spectral function relative to the TVAR model (1) is then given by [3]

$$H(f,t) = \frac{\sigma^2{}_e}{\left|1 + \sum_{i=1}^{p} a_i(t) e^{-j2\pi f / f_s}\right|^2} \quad , \tag{5}$$

where f_s is the sampling frequency. The spectral function (5) is continuous with respect to the frequency f and thus can be used to produce spectral estimates at any desired frequencies up to the Nyquist frequency $f_s/2$ However, the frequency resolution is primarily not infinite, but is determined by the underlying model order and the associated parameters.

Two basic issues are encountered when the basis function expansion and regression approach is applied to general time-varying parametric modeling problems, namely, how to choose the basis functions and how to select the significant ones from the pool of the basis functions. For the first issue, while there are a number of choices and alternatives, for example, Fourier bases, Walsh, Haar functions, wavelets, discrete prolate spheroidal sequences, time polynomial, Chebyshev polynomials, Legendre polynomials, there is no a guideline on how to choose the appropriate ones from these available basis functions for a specific modeling problem[7]. In fact, each family of basis functions possess its own unique tractability and accuracy, for example, polynomial and Fourier basis functions work well for most smoothly and slowly varying coefficients; Walsh and Haar functions, however, perform well for time-varying coefficients that have sharp variations (or)piecewise changes[3].

The second issue involves regression selection and model refinement. For a high dimensional parametric regression modeling problem, the initial full regression model, produced by a basis function expansion approach, often involves a great number of regressors (or) model terms, whatever types of basis functions are employed. In most cases the initial full regression model may be redundant (or) ill-posed, meaning that many of the candidate regressors in the initial full regression equation are linearly dependent on the others and therefore can be removed from the model and the resultant parsimonious model with just a relatively small number of regressors can often produce satisfactory results[6]

In order to alleviate the dilemma that the choice of basis functions has to be highly dependent on a priori information on the signals to be studied, and also to make the modeling algorithm more flexible and able to track both fast and slowly varying trends, we propose a new TVAR modeling approach using a multi-wavelet basis function expansion scheme, where properties of different types of wavelets are exploited and combined in a form of multi resolution decompositions.

3. The Multi-Wavelet Basis Functions

From wavelet theory a square integrable scalar function $f(x)\varepsilon L^2(R)$ can be arbitrarily approximated using the multiresolution wavelet decomposition below [5]

$$f(x) = \sum_k \alpha_{j_{0,k}} \varphi_{j_{0,k}}(x) + \sum_{j \geq j_0} \sum_k \beta_{j,k} \psi_{j,k}(x) \tag{6}$$

where the wavelet family $\psi_{j,k}(x) = 2^{j/2} \psi_{j,k}(2^j x - k)$ and $\varphi_{j,k}(x) = 2^{j/2} \varphi_{j,k}(2^j x - k)$, with $j,k\varepsilon Z$ (Z is a set consisting of whole integers) are the dilated and translated versions of the mother wavelet ψ and the associated scale function φ, $\alpha_{j_0,k}$ and $\beta_{j,k}$ are the wavelet decomposition coefficients, j_0 is an arbitrary integer representing the coarsest resolution (or) scale level. Also from the properties of multi resolution analysis theory, any square integrable function $f(x)$ Can be arbitrarily approximated using the basic scale functions $\varphi_{j,k}(x) = 2^{j/2} \varphi_{j,k}(2^j x - k)$, by setting the resolution scale level to be sufficiently large, that is, there exists an integer J , such that

$$f(x) = \sum_k \alpha_{J,k} \varphi_{J,k}(x) \ . \tag{7}$$

3.1. B-spline wavelets

B-splines as piece-wise polynomial functions with functions with good local properties, were originally introduced by Chui and Wang[5] as wavelet and scaling functions in multi-resolution expansions.

The first order cardinal B-spline is very the well-known Haar function defined as [13]

$$N_1(x) = \chi_{[0,1)}(x) = 1, x\varepsilon[0,1) \tag{8}$$

The B-spline function of *mthorder* is defined by the following recursive formula [9]:

$$N_m(x) = \frac{x}{m-1} N_{m-1}(x) + \frac{m-x}{m-1} N_{m-1}(x-1); m \geq 2 \tag{9}$$

Setting $N_m(x)$ as the scaling function, that is, $\varphi(x) = N_m(x)$, then the scaling function can be expressed in terms of the scaling function $N_m(x)$ as follows

ISSN 1512-1232

$$\varphi(x)=\sum_{k=0}^{m} c_k N_m(2x-k) \tag{10}$$

with the coefficients given by

$$c_k=\frac{1}{2^{m-1}}\binom{m}{k}, \qquad k=0,1,........,3m-2 \quad . \tag{11}$$

Clearly, the support of the mth order B-spline scaling function is $\operatorname{supp}\varphi=\operatorname{supp}N_m=[0,m]$

B-spline scaling function is symmetric in the support. The most commonly used B-spline wavelets are the linear (m=2) and cubic (m=4) cases, both of which can be expressed explicitly. The second, third, fourth and fifth order cardinal B-splines $B_2(x), B_3(x), B_4(x)$ and $B_5(x)$ are given in Table 1.

Table 1. Cardinal B-splines of order from 1 to 5

	$B_1(x)$	$B_2(x)$	$2B_3(x)$	$6B_4(x)$	$24B_5(x)$
$0\le x<1$	1	x	x^2	x^3	x^4
$1\le x<2$	0	$2-x$	$-2x^2+6x-3$	$-3x^3+12x^2-12x+4$	$-4x^4+20x^3-30x^2$ $+20x-5$
$2\le x<3$	0	0	$(x-3)^2$	$3x^3-24x^2+60x-44$	$6x^4-60x^3+210x^2$ $-300x+155$
$3\le x<4$	0	0	0	$-x^3+12x^2-48x+64$	$-4x^4+60x^3-330x^2$ $+780x-655$
$4\le x<5$	0	0	0	0	$x^4-20x^3+150x^2$ $-500x+500$
elsewhere	0	0	0	0	0

One attractive feature of cardinal B-splines is that these functions are completely supported, and this property enables the operation of the multi resolution decomposition (6) to be much more convenient. For example, the mth order B-spline is defined on [0, m], thus, the scale and shift indices j and k for the family of the functions

$$\varphi_{j,k}(x)=2^{j/2}B_m(2^j x-k), 0\le 2^j x-k\le m\,. \tag{12}$$

Assume that the function $f(x)$ that is to be approximated with decompositions (6) or (7) is defined within[0,1],then for any given scale index(resolution level) j, based on $x\varepsilon[0,1]$,and $0\le 2^j x-k\le m$ the effective values for the shift index, k are restricted to the collection $\{k:-m\le k\le 2^j-1\}$ with the $B_m(m)=0$ The first and second order B-splines $B_1(x) and B_2(x)$ are non-smooth piecewise functions, which would perform well for signals with sharp transients and burst-like spikes, B-splines of higher order would work well on smoothly changing signals. Motivated by this consideration, this study proposes using multi-wavelet basis functions for TVAR model. An example of the new multi-wavelet based algorithm is given in the following.

ISSN 1512-1232

Take the B-splines of order from 1 to 5 as an example, and consider the decomposition (7).Let $\Gamma_m = \left\{k : -m \le k \le 2^J - 1\right\}$ with $m = 1, 2, ..., 5;$ let, $\varphi_k^{(m)}(x) = 2^{J/2} B_m(2^J x - k)$ with $k\varepsilon\Gamma_m$.The time-varying coefficients $a_i(t)$ in (1) can then be approximated using a combination of functions from the families $\left\{\varphi_k^{(m)} : m = 1, 2, ..., 5; k\varepsilon\,\Gamma_m\right\}$.For example one such combination can be chosen as [1]

$$a_i(t) = \sum_{k\varepsilon\Gamma_q} C_{i,k}^{(q)} \varphi_k^{(q)}\left(\frac{t}{N}\right) + \sum_{k\varepsilon\Gamma_r} C_{i,k}^{(r)} \varphi_k^{(r)}\left(\frac{t}{N}\right) + \sum_{k\varepsilon\Gamma_s} C_{i,k}^{(s)} \varphi_k^{(s)}\left(\frac{t}{N}\right). \tag{13}$$

Where $1 \le q < r < s \le 5$, t=1,2,.....N, and N is number of observations of the signal. Simulation results with a large number of experiments have shown that for most time varying problems, the choice of $q = 4, r = 4, s = 5$ work well to recover the time-varying coefficients. If, however, there is strong evidence that the time-dependent coefficients have sharp changes, then the inclusion of the first and second order B-splines would work well. The decomposition (13) can easily be converted into the form of (2), where the collection, is $\left\{\pi_m(t) : m = 1, 2, ..., L\right\}$ Replaced by the union of the three families [1]-[3]:

$$\left\{\varphi_k^{(q)}(t) : k\varepsilon\,\Gamma_q\right\}, \left\{\varphi_k^{(r)}(t) : k\varepsilon\,\Gamma_r\right\}, \left\{\varphi_k^{(s)}(t) : k\varepsilon\,\Gamma_s\right\}.$$

4. Linear Regression Model

TVAR Model with multi wavelet basis functions can be represented in terms of standard linear regression model. Substitute (13) in (1) leads the following equation

$$C_1^3 = [C_{1,-3}^{(3)}, C_{1,-2,}^{(3)} .... C_{1,7}^{(3)}]$$

$$\begin{aligned} y(t) &= \sum_{i=1}^{p} \sum_{k\varepsilon\Gamma_q} C_{i,k}^{(q)} \varphi_k^{(q)}\left(\frac{t}{N}\right) y(t-i) + \sum_{i=1}^{p} \sum_{k\varepsilon\Gamma_r} C_{i,k}^{(r)} \varphi_k^{(r)}\left(\frac{t}{N}\right) y(t-i) \\ &+ \sum_{i=1}^{p} \sum_{k\varepsilon\Gamma_s} C_{i,k}^{(s)} \varphi_k^{(s)}\left(\frac{t}{N}\right) y(t-i) + e(t) \end{aligned} \tag{14}$$

TVAR Model order p=2 and q=3, r=4, s=5,(third, fourth and fifth order cardinal *B*-splines) works well for most of the mono-component signals. If there is strong evidence that the time-dependent coefficients have sharp changes, then the inclusion of the first and second order B-splines would work well. Scale index (resolution level) J=3.

For q=3 $k\varepsilon\Gamma_3, \Gamma_3 = \left\{k : -3 \le k \le 7\right\}$; for r=4; $k\varepsilon\Gamma_4, \Gamma_4 = \left\{k : -4 \le k \le 7\right\}$; For s=5, $k\varepsilon\Gamma_5, \Gamma_5 = \left\{k : -5 \le k \le 7\right\}$.

$$\begin{aligned} y(t) &= \sum_{i=1}^{2} \sum_{k=-3}^{7} C_{i,k}^{(3)} \varphi_k^{(k)}\left(\frac{t}{N}\right) y(t-i) + \sum_{i=1}^{2} \sum_{k=-4}^{7} C_{i,k}^{(4)} \varphi_k^{(k)}\left(\frac{t}{N}\right) y(t-i) \\ &+ \sum_{i=1}^{2} \sum_{k=-5}^{7} C_{i,k}^{(5)} \varphi_k^{(k)}\left(\frac{t}{N}\right) y(t-i) + e(t) \end{aligned} \tag{15}$$

Denote

$$A(t) = [A_1^{(3)}(t), A_1^{(4)}(t), A_1^{(5)}(t), A_2^{(3)}(t), A_2^{(4)}(t), A_2^{(5)}(t)]$$

$$A_1^{(3)}(t) = [\varphi_{-3}^{(3)}\left(\frac{t}{N}\right), \varphi_{-2}^{(3)}\left(\frac{t}{N}\right), ..., \varphi_7^{(3)}\left(\frac{t}{N}\right) \otimes y(t-1)$$

$$A_1^{(4)}(t) = [\varphi_{-4}^{(4)}\left(\frac{t}{N}\right), \varphi_{-3}^{(4)}\left(\frac{t}{N}\right), ..., \varphi_7^{(4)}\left(\frac{t}{N}\right) \otimes y(t-1)$$

$$A_1^{(5)}(t) = [\varphi_{-5}^{(5)}\left(\frac{t}{N}\right), \varphi_{-4}^{(5)}\left(\frac{t}{N}\right), ..., \varphi_7^{(5)}\left(\frac{t}{N}\right) \otimes y(t-1)$$

$$A_2^{(3)}(t) = [\varphi_{-3}^{(3)}\left(\frac{t}{N}\right), \varphi_{-2}^{(3)}\left(\frac{t}{N}\right), ..., \varphi_7^{(3)}\left(\frac{t}{N}\right) \otimes y(t-2)$$

$$A_2^{(4)}(t) = [\varphi_{-4}^{(4)}\left(\frac{t}{N}\right), \varphi_{-3}^{(4)}\left(\frac{t}{N}\right), ..., \varphi_7^{(4)}\left(\frac{t}{N}\right) \otimes y(t-2)$$

$$A_2^{(5)}(t) = [\varphi_{-5}^{(5)}\left(\frac{t}{N}\right), \varphi_{-4}^{(5)}\left(\frac{t}{N}\right), ..., \varphi_7^{(5)}\left(\frac{t}{N}\right) \otimes y(t-2)$$

$\otimes$ Indicates kroneker delta product

$$C = [C_1^3, C_1^4, C_1^5, C_2^3, C_2^4, C_2^5]$$

$$C_1^3 = [C_{1,-3}^{(3)}, C_{1,-2,}^{(3)} ....C_{1,7}^{(3)}]$$

$$C_1^4 = [C_{1,-4}^{(4)}, C_{1,-3,}^{(4)} ....C_{1,7}^{(4)}]$$

$$C_1^5 = [C_{1,-5,}^{(5)} C_{1,-4}^{(5)}, ...C_{1,7}^{(5)}]$$

$$C_2^3 = [C_{2,-3}^{(3)}, C_{2,-2,}^{(3)} ....C_{2,7}^{(3)}]$$

$$C_2^4 = [C_{2,-4}^{(4)}, C_{2,-3,}^{(4)} ....C_{2,7}^{(4)}]$$

$$C_2^5 = [C_{2,-5}^{(5)}, C_{2,-4,}^{(5)} ....C_{2,7}^{(5)}]$$

Equation (15) can be represented as below

$$Y = H\theta + e \tag{16}$$

Where

$$Y^T = [y(1), y(2)..........y(N)],$$
$$H^T = [A(1), A(2)..........A(N)],$$
$$\theta = [C^T],$$
$$e^T = [e(1), e(2), ..., e(N)].$$

Here H is a regression vector and θ is coefficient vector the initial full regression equation (15) may involve a great number of free parameters; the associated regressors may be highly correlated, and the ordinary least squares algorithm may fail to produce reliable results for such ill-posed problems. These problems, however, can easily be overcome by performing an effective model refinement procedure where significant model terms (or) regressors can be selected one by one [6].The well-known Orthogonal least squares(OLS)type of algorithms have been proven to be very effective to deal with multiple dynamical regression problems, which involve a great number of candidate model terms (or) regressors that may be highly correlated. In this present study, the OLS

ISSN 1512-1232

algorithm given in [6] is used to solve the regression equation (16).This includes a model refinement procedure involving the selection of significant regressors and model parameter estimation. The resultant estimates will then be used to recover the time-varying coefficients $a_i(t)$ using (13) in the TVAR model (1).

5. Time-Dependent Spectrum Estimation

The time-varying frequency can be extracted from the TVAR parameters. $a_i(t)$ Since the nonstationary signal is modeled as the output of the TVAR process, with a zero-mean white noise input, $e(t)$ the time varying power spectral density of the nonstationary signal is given by [3]

$$H(f,t) = \frac{\sigma^2{}_e}{\left|1+\sum_{i=1}^{p} a_i(t) e^{-j2\pi f/f_s}\right|^2} \quad , \tag{17}$$

where f_s is the sampling frequency. The spectral function (17) is continuous with respect to the frequency f and thus can be used to produce spectral estimates at any desired frequency up to the Nyquist frequency $f_s/2$.

Simulation Examples

To verify the performance of the multi-wavelet basis functions approach, three examples will be studied.

1) Signaltest1: Consider a TVAR model of order 2 below

$$y_1(t) = a_1(t)y_1(t-1) + a_2(t)y_1(t-2) + e(t) \tag{18}$$

where $e(t)$ is zero-mean Gaussian white noise. The TV parameters in (18) are given by:

$$a_1(t) = \begin{cases} 0.32\cos(1.5-\cos(\frac{4\pi t}{N}+\pi)), 1 \le t \le \frac{N}{4} \\ 0.32\cos(3-\cos(\frac{4\pi t}{N}+\pi/2)), \frac{N}{4}+1 \le t \le 3N/4 \\ 0.32\cos(1.5-\cos(\frac{4\pi t}{N}+\pi)), 3N/4+1 \le t \le N \end{cases}$$

$$a_2(t) = 0.4\cos(\frac{4\pi t}{N}), 1 \le t \le N \ , \tag{19}$$

Where the length of data N is 512.The variance of the noise $e(t)$ was chosen to be 0.04, and this made the signal-to-noise ratio to be around 13dB. A second order TVAR model was estimated to describe the time-varying signal $y_1(t)$. The third, fourth and fifth order B-splines, as shown by (15) where the scale index (resolution level) J was chosen to be 3,were employed to approximate the time-varying parameters $a_i(t)$ with $i = 1,2$ and $n = 1, 2, ..., 512$ .An OLS algorithm was then applied to estimate and refine the model including significant regressor selection and model parameter estimation. The true and estimates of the two time-varying coefficients $a_1(t)$ and $a_2(t)$ are shown in Figure 1.and Figure 2.

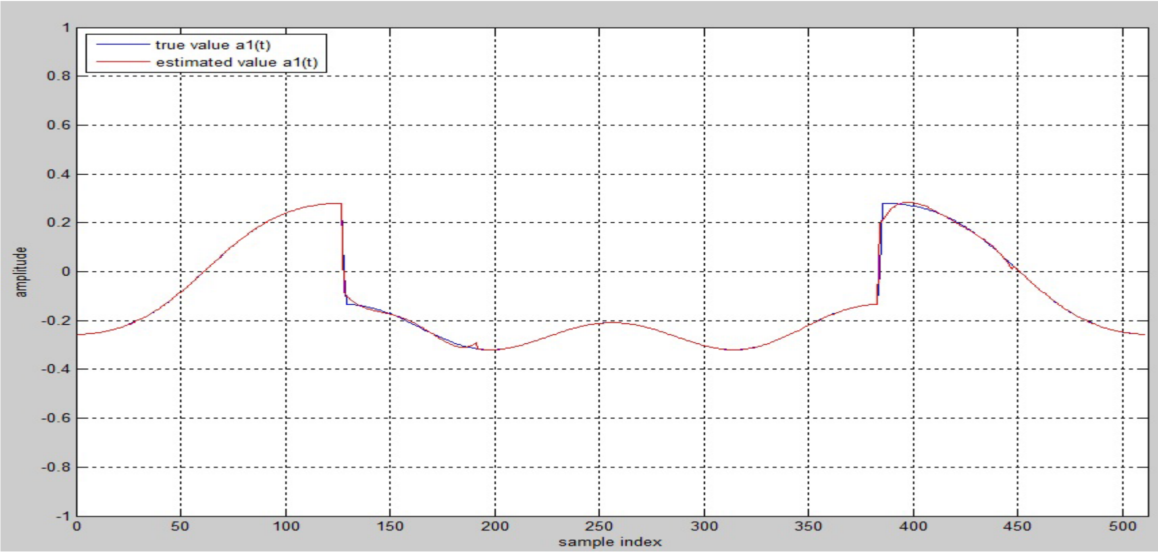

Figure 1 The true and estimates of the time- varying coefficients $a_1(t)$ for the signaltest1

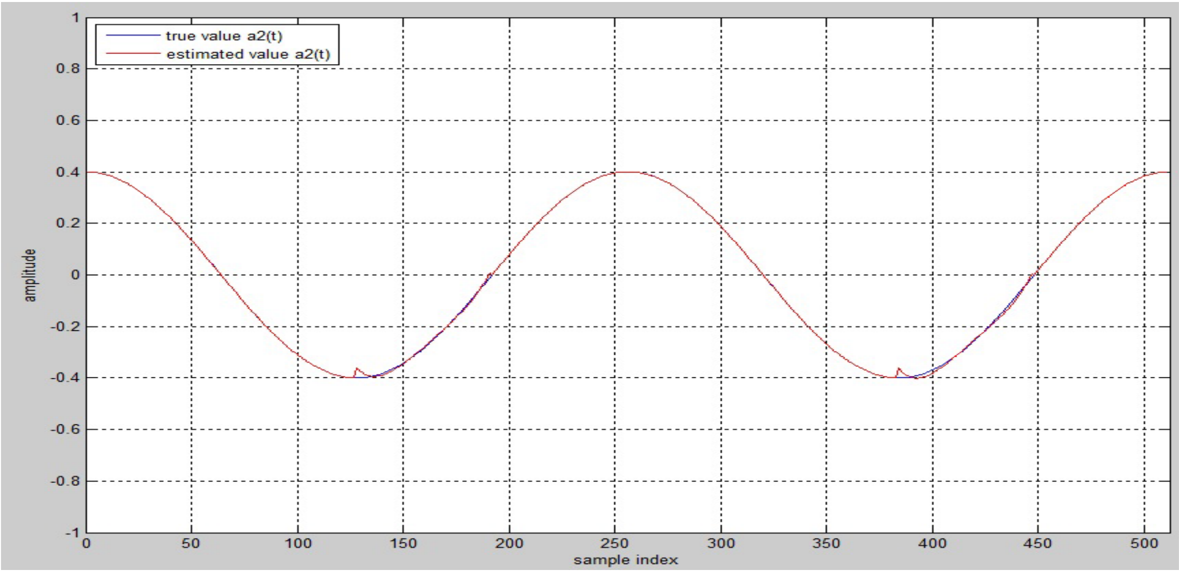

Figure 2 The true and estimates of the time- varying coefficients $a_2(t)$ for the signaltest1

2) Signaltest2 a sinusoid with normalized frequency nonlinearly varying in a periodic manner over N=256 samples, starting from f_0 and oscillating between $f_{\max} = 0.4$ and $f_{\min} = 0.1$ with a sweep rate of $\mu_f = \frac{3.2}{N}$

$$y_2(n) = \cos\left\{2\pi\left[f_0 n - \frac{\mu}{2\pi\mu_f}\cos(2\pi\mu_f n)\right]\right\}, 1 \le n \le 256, \tag{20}$$

Where

$$\mu = \left|\frac{f_{\max} - f_{\min}}{2}\right|.$$

A second order TVAR model was estimated to describe the time-varying signal $y_2(n)$. The third, fourth and fifth order B-splines, as shown by(15) where the scale index(resolution level) J was chosen to be 4,were employed to approximate the time-varying parameters $a_i(t)$ with $i = 1, 2$

and $n = 1, 2, ..., 256$ An OLS algorithm was then applied to estimate and refine the model including significant regressor selection and model parameter estimation. The estimates of the two time-varying coefficients $a_1(t)$ and $a_2(t)$ are shown in Figure 3.The topographical map of the time-dependent spectrum estimated from the TVAR model is shown in Figure4, and the 2-D image of the time-dependent spectrum produced from the 3-D topographical map is shown in Figure 5.In this example the main features of the multi-wavelet approach is that it enables smooth trends to be tracked in the time-varying process parameter.

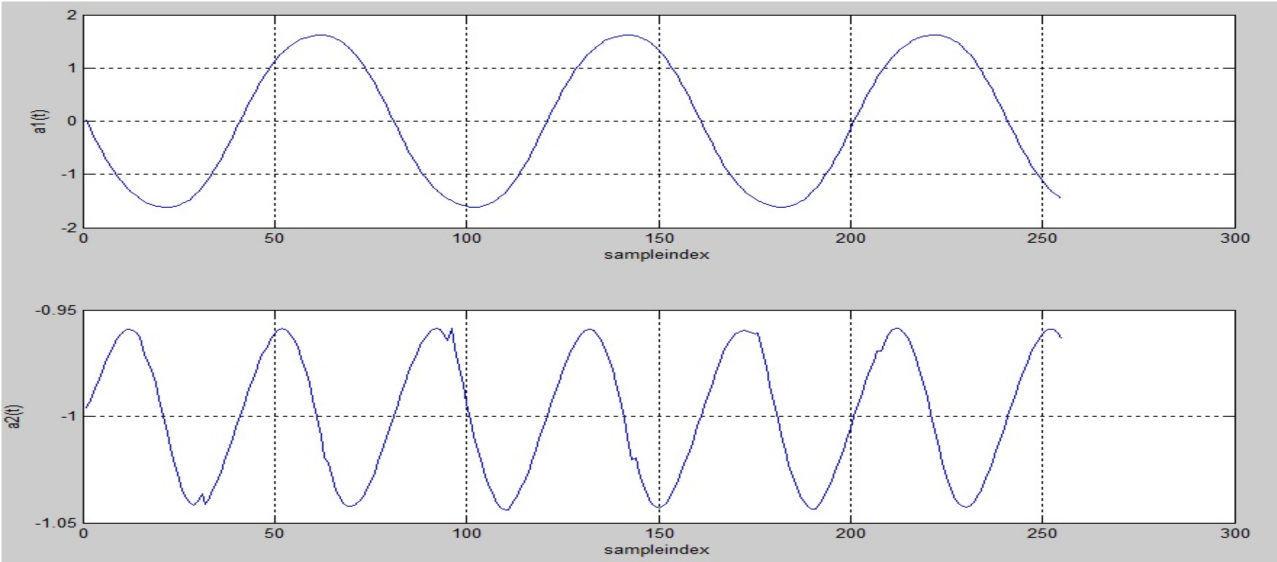

Figure.3 The estimates of the two time -varying coefficients $a_1(t)$ and $a_2(t)$ for the signal test2.

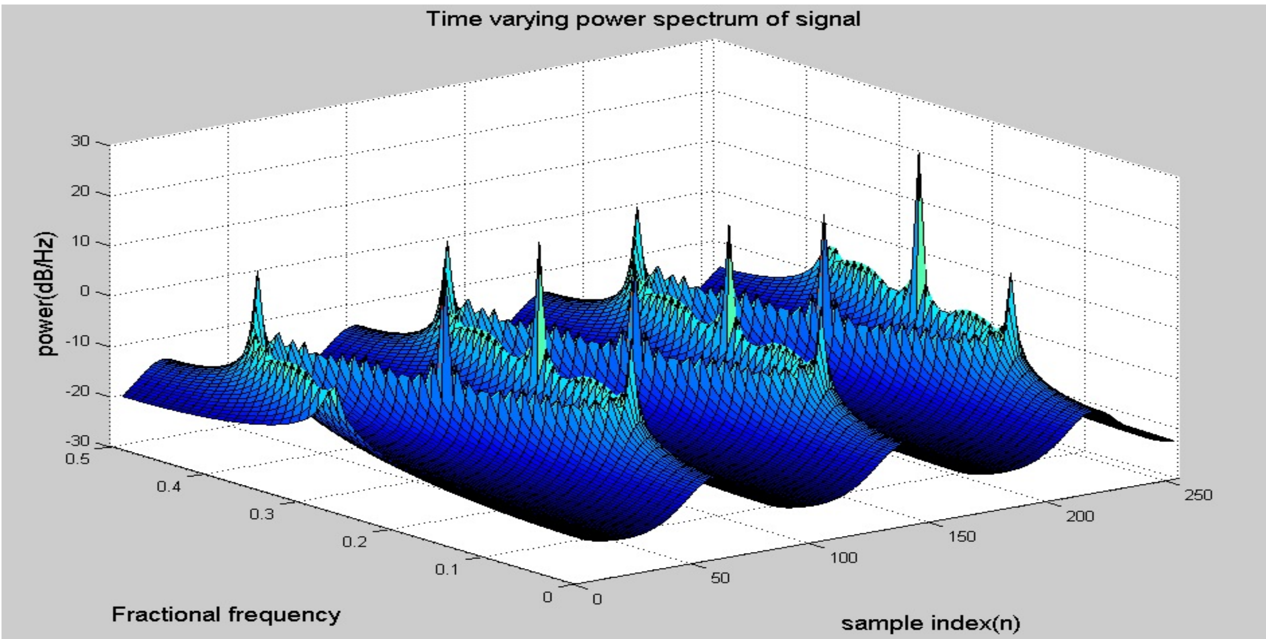

Figure.4 The 3-D topographical map of the time-dependent spectrum estimated from the TVAR model for the signal test2

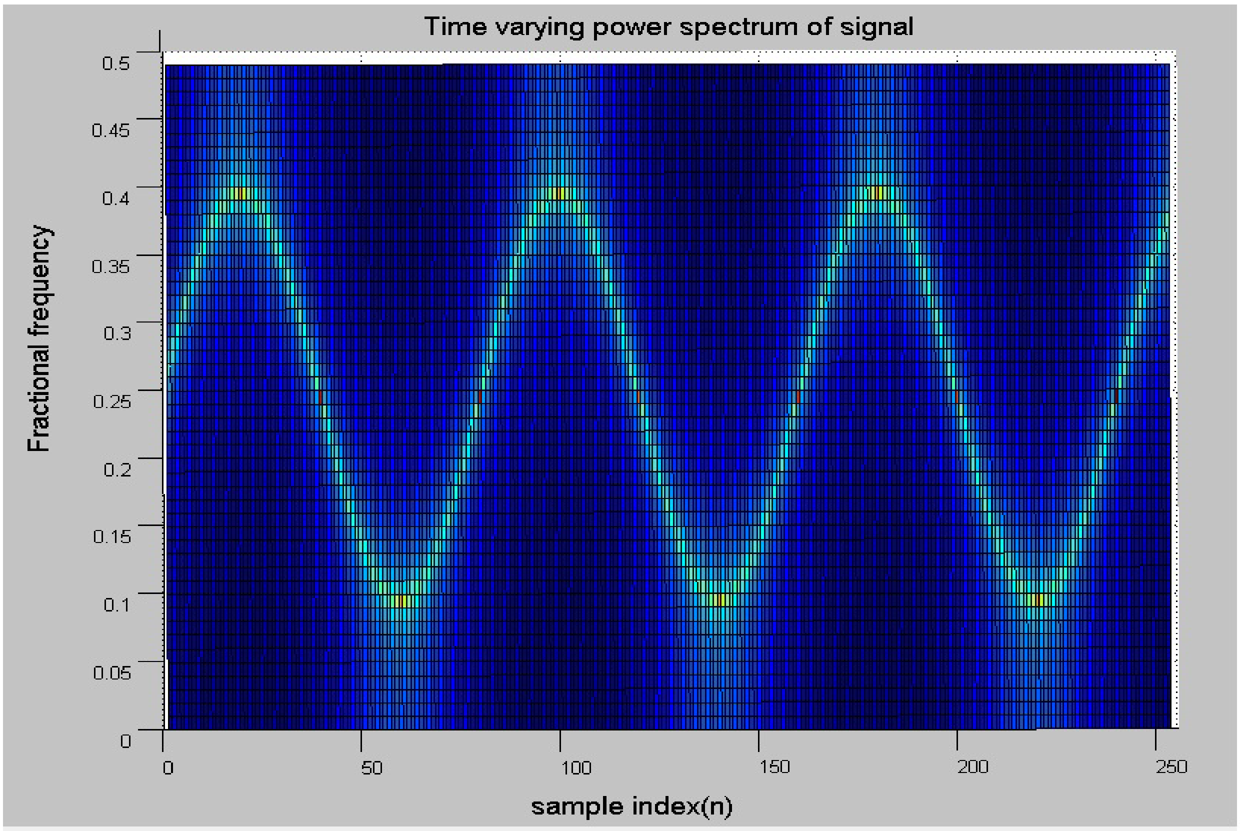

Figure.5 The 2-D image of the time-dependent spectrum produced from the 3-D topographical map shown in Figure4.

3) Signaltest3 a sinusoid with frequency jump. The frequency remains constant up to $f_0 = 0.1$.for first 127 samples and then it jumps to $f_N = 0.4$ at the 128th sample and remains constant over the next 128 samples

$$y_3(n) = \begin{cases} \cos(2\pi f_0 n), 1 \le n \le 127 \\ \cos\left[2\pi(f_0 + \Delta f)n\right], 128 \le n \le 256 \end{cases} \tag{21}$$

Where

$$\Delta f = f_N - f_0 .$$

A second order TVAR model was estimated to describe the time-varying signal. $y_3(n)$ The first, second and third order B-splines, as shown by(15)where the scale index(resolution level) J was chosen to be 3,were employed to approximate the time-varying parameters $a_i(t)$ with $i = 1, 2$ and $n = 1, 2, ..., 512$ An OLS algorithm was then applied to estimate and refine the model including significant regressor selection and model parameter estimation. The estimates of the two time-varying coefficients $a_1(t)$ and $a_2(t)$ are shown in Figure 6.The topographical map of the time-dependent spectrum estimated from the TVAR model is shown in Figure 7,and the 2-D image of the time-dependent spectrum produced from the 3-D topographical map is shown in Figure 8.In this example, the main features of the multi-wavelet approach is that it enables to capture sharp changes in the time-varying process parameter.

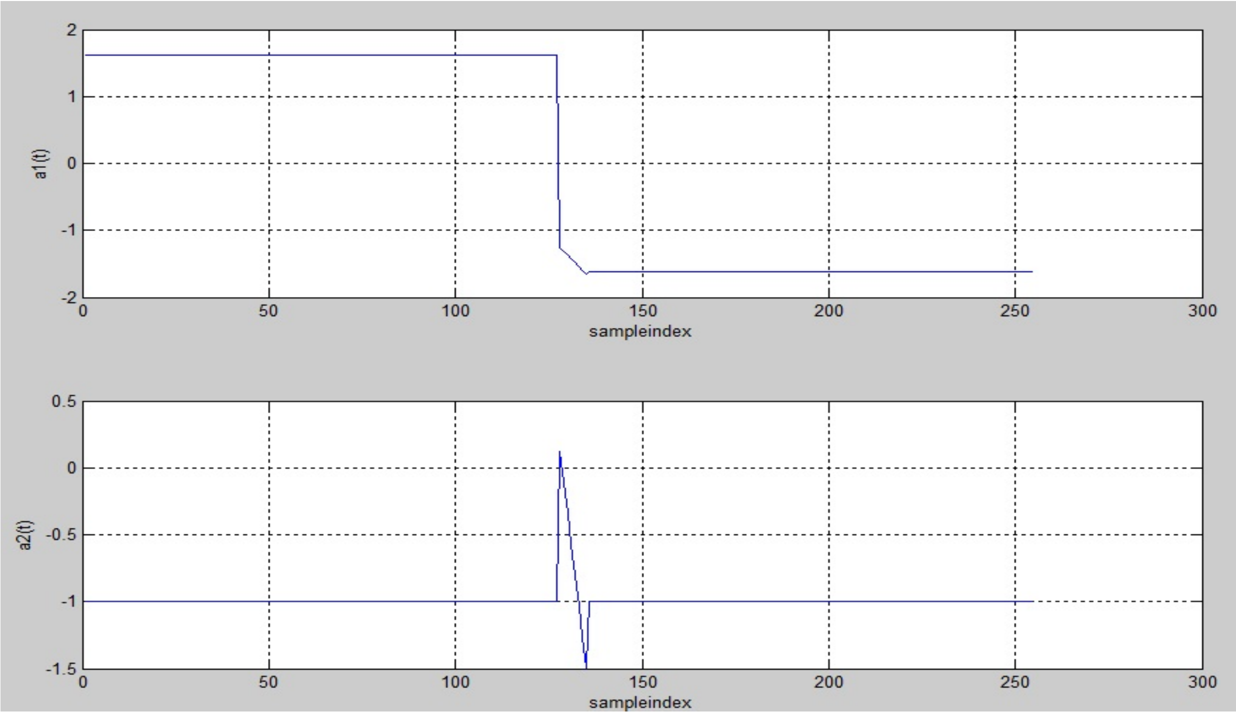

Figure 6 The estimates of the two time-varying coefficients $a_1(t)$ and $a_2(t)$ for the signal test 3

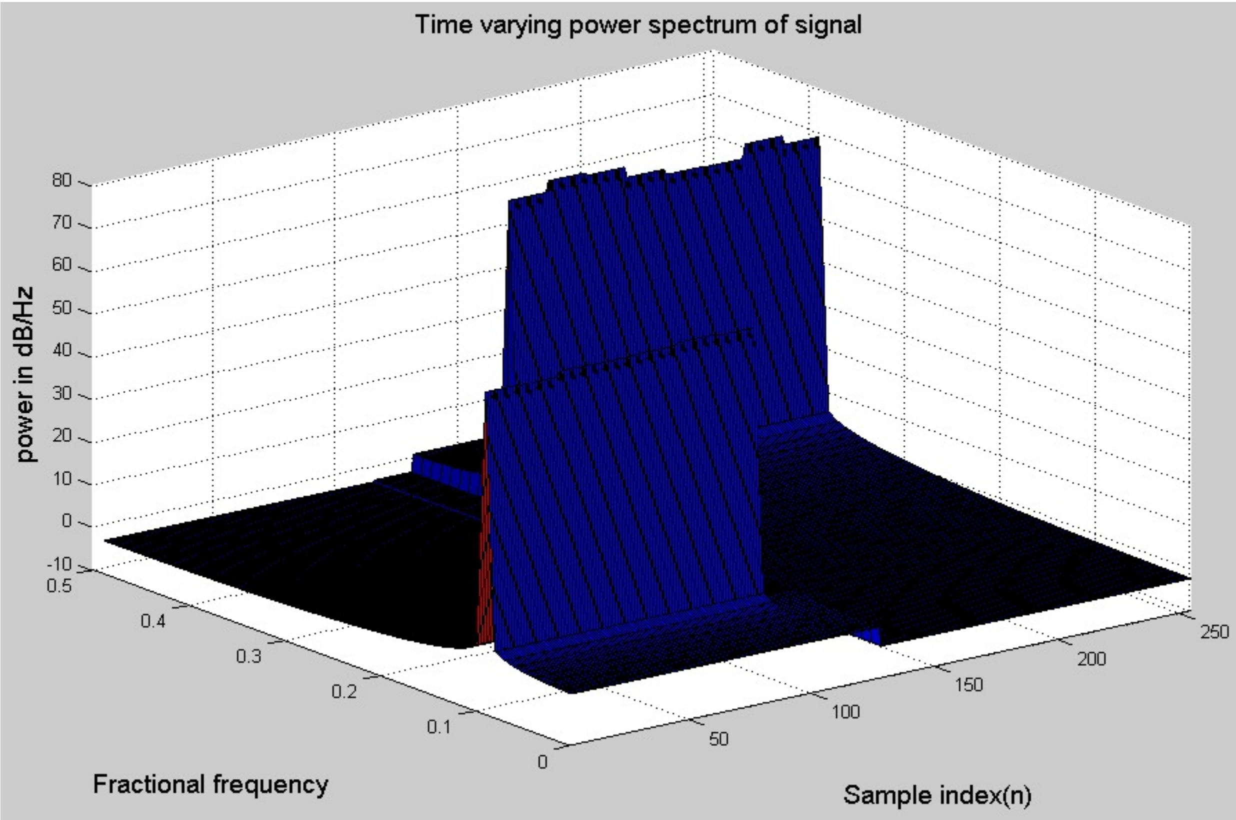

Figure 7 The 3-D topographical map of the time-dependent spectrum estimated from the TVAR model for the signal test3

ISSN 1512-1232

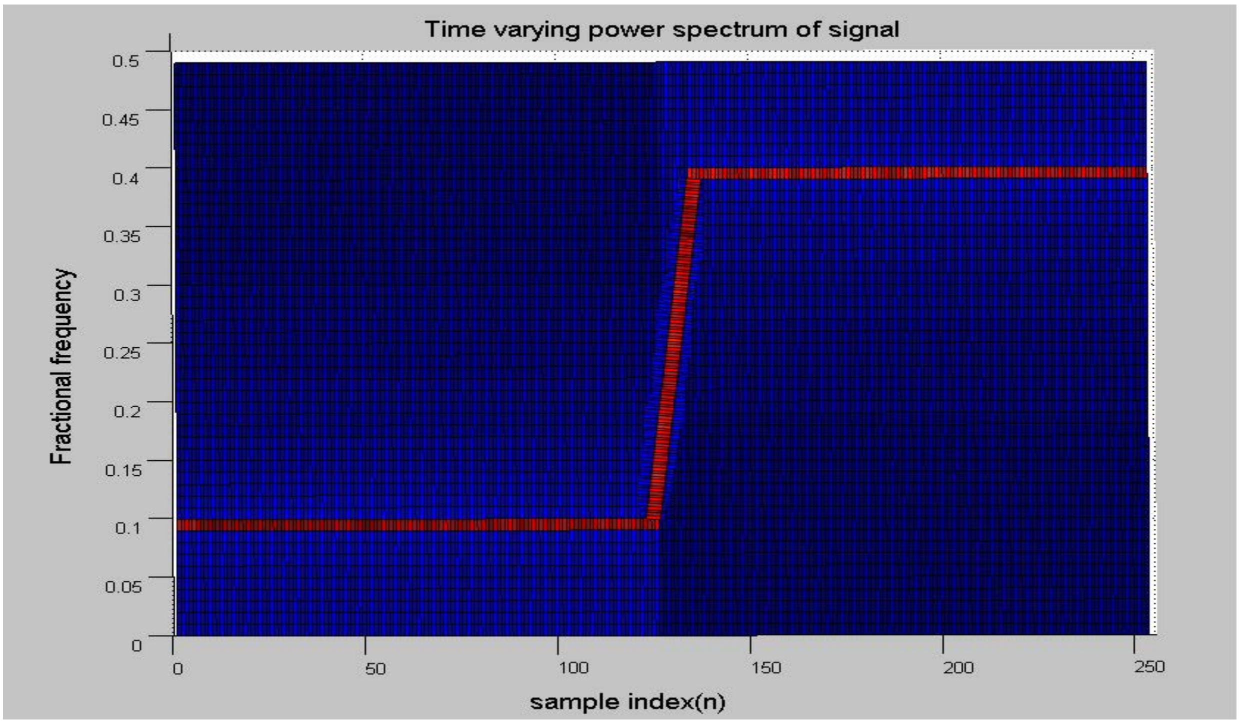

Figure 8 The 2-D image of the time-dependent spectrum produced from the 3-D topographical map shown in Figure7.

6. Conclusions

Time-varying parameters in TVAR model have been estimated using a new multi-wavelet basis function approach with OLS algorithm introduced in this study where the associated time dependent coefficients are expanded using multi-wavelet basis functions. The orthogonal least square (OLS) algorithm is then applied to refine the model parameter estimates of the TVAR model. From the results above, it can be concluded that the main features of the multi-wavelet approach is that it enables smooth trends to be tracked but also to capture sharp changes in the time-varying process parameter. The time-dependent spectrum, calculated from the multi-wavelet based TVAR model, has a capability that not only reveals the global frequency behavior of the signal but also reflects the local variations of the signal along the time course.

References

[1] Yang Li,Hua-liang Wei,and S.A.Billings,"Identification of Time-Varying Systems Using Multi- Wavelet Basis Functions," IEEE Transactions on control systems *Technology, Vol.19, No.3,May 2011.*

[2] *H.L.Wei and S.A.Billings,"Identification of time-varying systems using multiresolution wavelet models," International Journal of System Science,vol.33,no.15,pp.1217-1228,Dec.2002.*

[3] *H.L.Wei ,S.A.Billings, and J.Liu,"Time-varying parametric modeling and time-dependent spectral characterization with application to EEG signal using multiwavelets,"Int.J.Modell., Identificatiol Control,vol.9,no.3,pp.215-224,2010.*

[4] *S.A.Billings and H.L.Wei,"A new class of wavelet networks for nonlinear system Identification,"IEE Trans.Neural Netw.,vol.16,no.4, pp.862-874,Jun.2005.*

[5] *S.Malat,A Wavelet Tour of Signal Processing,secondedition.London,U.K,:Academic Press,1999.*

[6] *Billings,S.A.,Wei,H.L."Sparse model identification using a forward orthogonal regression algorithm aided by mutual information."IEEE Transactions on Neural Networks,Vol.18,No.1,pp.306-310,2007*

ISSN 1512-1232

[7] *Liu,J.,Billings,S.A.,Zhu,Z.Q.and Shen,J."Enhanced frequency analysis using wavelets"International Journal of Control, Vol.75,N0.15.15,pp.1145-1158,2002*

[8] *M.Niedzwiecki,"Identification of time-varying systems with abrupt parameters changes sourse,"Automatica, Vol.30, no.3 pp.447-459, Mar.1994.*

[9] *H.L.Wei and S.A.Billings,"An efficient nonlinear cardinal B-spline model for high tide forecasts at the Venice lagoon," Nonlinear Processes in Geophysics,vol.13,no.5,pp.577-584,2006.*

[10] *M.K.Tsatsanis and G.B.Giannakis,"Time-varying system identification and model validation using wavelets," IEEE Trans.Signal Process.,vol.41,no.12,pp.3512-3523,Dec.1993.*

[11] *Chen S,Wang XX,Harris CJ."NARX-based nonlinear system identification using orthogonal least squares basis hunting." IEEE Transactions on Control Systems Technology ,vol.16,pp78-84,2008*

[12] *Chon KH,Zhao H,Zou R,Ju K."Multiple time-varying dynamic analysis using multiple sets of basis functions," IEEE Transactions on Biomedical Engineering, vol .52,pp956-960,2005*

[13] *Chen SL, Lai HC, Ho Kc."Identification of linear time varying systems by Haar Wavelet," International Journal of Systems Science vol.37,pp619-628,2006.*

[14] *Wei HL,Billings SA"Long term prediction of non-linear time series using multiresolution wavelet models."International Journal of Control vol 79,pp 569-580,2006.*

[15] F.N.*Chowdhury,"Input-output modeling of nonlinear systems with time-varying linear models," IEEE Trans.Autom.Control, vol 45,no.6,pp 1355-1358,Jun 2000.*

Article received: 2015-10-09