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Mathematical and computer modeling of nonlinear processes of elections with two selective subjects

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Abstract

In work the nonlinear mathematical model describing dynamics of voters of pro-government and oppositional parties is offered.

The case when coefficients of attraction of votes of pro-government and oppositional parties are exponential increasing functions from elections to elections is considered. Cauchy's task for nonlinear system of the differential equations with variable coefficients of model is solved by means of the program Matlab environment. Cases as maximum and certain voter turnout on elections, and also the set falsification of voices of opposition party are considered. The following qualitatively various results are received:

- despite superiority of coefficient of attraction of votes of opposition party over pro-governmental, due to administrative impact on voters of opposition party from government institutions, the pro-government party will win the next elections;

- despite superiority of the voters supporting opposition party by the election day due to the best mobilization on elections of the voters, the pro-government party will win the next elections;

- despite superiority of the voters supporting opposition party by the election day at an identical voter turnout on elections, due to a certain falsification of elections, the pro-government party will win the next elections;

- the opposition party, despite the best appearance on elections of voters of pro-government party and a certain falsification of elections, nevertheless will win the next elections.

Keywords: *nonlinear model of elections, the ruling party, the opposition, falsification, voter turnout*

Introduction

During the last decade mathematical and computer modeling has been widely recognized in such disciplines as sociology, political science, and others [1 - 4]. There is an interest in creation of a mathematical model, which would give the opportunity to determine the dynamics of changes in the number of voters of political subjects during the election period. Elections can be divided into two parts: the two-party and multi-party elections.

In works [5 - 7] the mathematical model of political rivalry devoted to the description of fight occurring in imperious elite competing (but not necessarily antagonistic) political forces, for example, power branches is considered. It is supposed that each of the sides has ideas of "number" of the power which this side would like to have itself, and about "number" of the power which she would like to have for the partner.

These papers [8 - 14] present the nonlinear mathematical model of the public or the administrative management (or the macro and micro model). The cases of both constant and variable pressure forces on freethinking people were analyzed. Exact analytical decisions which determine dynamics of a spirit both free-thinking people, and operated (conformists) of people by time are received. During this analyses various governance systems were considered: a liberal, democratic, semi dictatorial and dictatorial.

These works [15 - 20] considered a two- or three-party (one pro-government and two opposition parties) nonlinear mathematical model of elections when coefficients are constant. The assumption was made that the number of voters remain the same between 2 consecutive elections (zero demographic factor of voters). The exact analytical solutions were received. The conditions under which opposition party can win the upcoming elections were established.

In this publication the nonlinear mathematical model with variable coefficients in case of two-party elections which describes dynamics of quantitative change of votes of ruling and oppositional parties is presented. In model three objects are considered:

1. The state and administrative structures that utilize state resources in order to have an influence on the pro opposition voters with the aim to gain their support for the pro-government party.
2. Voters who support opposition party.
3. Voters who support the ruling party.

In the model, there are different indicators of voter turnout on election day, as well as possible cases of falsification by the ruling party.

1. A system of equations and initial conditions.

To describe the dynamics of choosing between two election subjects (pro-government and opposition parties), we propose the following nonlinear mathematical model:

$$\begin{cases} \frac{dN_1(t)}{dt} = (\alpha_1(t)-\alpha_2(t))N_1(t)N_2(t) - f_1(t,N_1(t)) \\ \frac{dN_2(t)}{dt} = (\alpha_2(t)-\alpha_1(t))N_1(t)N_2(t) + f_1(t,N_1(t)) \end{cases} \tag{1.1}$$

$$N_1(0) = N_{10},\ N_2(0) = N_{20},\ N_{10} < N_{20}, \tag{1.2}$$

where $N_1(t), N_2(t)$ the number of voters in support of the opposition and pro-government parties, respectively, at time $t, t \in [0,T]$; $t=0$ --moment the last elections, in consequence of which party won the elections and became the ruling party ($N_{10} < N_{20}$); $t=T$ — time of the next elections (usually T = 4 years or 1460 days);

$\alpha_1(t), \alpha_2(t)$ - coefficients corresponding to the activity to attract the votes of the opposition and the ruling party, respectively, at time t, depending on the program of action, financial and information capabilities of these parties;

$f(t, N_1(t))$ - a positive function of its arguments, characterizing the use of administrative resources devoted to the voters of the opposition party with the aim of attracting to his side, and the preservation of power, which is the goal of any of the authorities. The model (1.1), (1.2) it is assumed that the total number of voters for the election of the sample size is not changed ($N_{10} + N_{20} = a$) in many countries, this change is insignificant compared to the total number of voters. Thus, we believe that the time period between elections the number of deceased voters and number of voters for the first time received the right to vote, equal.

2. Nonlinear mathematical models with variable coefficients in the case of two-party elections

We consider two cases: when the elections are held, without falsification, and when in the falsification takes place during the elections. In the model, we consider the case of variable coefficients. In particular, we assume that during the period between elections coefficients of involvement of voters are exponential increasing function of time.

$$\alpha_1(t) = \alpha_{10} e^{\delta \frac{t}{T}} \tag{2.1}$$

$$\alpha_2(t) = \alpha_{20} e^{\delta \frac{t}{T}} \tag{2.2}$$

$$\alpha_{10} > 0, \alpha_{20} > 0, \delta_1 > 0, \delta_2 > 0$$

and for administrative resources function $f(t, N_1(t))$ we consider three different cases:

1. $f(t, N_1(t)) = b = const > 0$
2. $f(t, N_1(t)) = b_1 N_1(t), b_1 > 0$
3. $f(t, N_1(t)) = F_0 e^{\delta \frac{t}{T}}$

Thus we receive Cauchy's tasks:

$$\begin{cases} \frac{dN_1(t)}{dt} = (\alpha_1(t) - \alpha_2(t)) N_1(t) N_2(t) - b \\ \frac{dN_2(t)}{dt} = (\alpha_2(t) - \alpha_1(t)) N_1(t) N_2(t) + b \end{cases} \tag{2.3}$$

$$\begin{cases} \frac{dN_1(t)}{dt} = (\alpha_1(t) - \alpha_2(t)) N_1(t) N_2(t) - b_1 N_1(t) \\ \frac{dN_2(t)}{dt} = (\alpha_2(t) - \alpha_1(t)) N_1(t) N_2(t) + b_1 N_1(t) \end{cases} \tag{2.4}$$

$$\begin{cases} \frac{dN_1(t)}{dt} = (\alpha_1(t) - \alpha_2(t)) N_1(t) N_2(t) - F_0 e^{\delta \frac{t}{T}} \\ \frac{dN_2(t)}{dt} = (\alpha_2(t) - \alpha_1(t)) N_1(t) N_2(t) + F_0 e^{\delta \frac{t}{T}} \end{cases} \tag{2.5}$$

Tasks (2.3)− (2.5) analytically aren't solved. For obtaining numerical decisions the package of the applied MATLAB programs was used.

Since nontrivial model will be obtained only if $\alpha_1(t) > \alpha_2(t), t \in [0, T]$, for numerical computations take $\alpha_{10} \in [10^{-10} - 10^{-9}], \alpha_2(t) \in [10^{-10} - 10^{-9}]$.

Numerical decisions are received when is falsification and when it isn't present. The results obtained in both cases may be grouped (sorted) as the several of different models. Let us refer to:

k_1 -- relative value of the supporters of opposition who voted in the election day from all number of the voters supporting opposition party;

k_2 -- relative value of the supporters of ruling party who voted in the election day from all number of the voters supporting ruling party;

k_3 -- relative value of falsification in the election day, that is relative value of number of the damaged (forged) bulletins from all number of the voted oppositionists.

Consider the first case when the falsification during the elections is absent $(k_3 = 0)$:

1) $\boldsymbol{k_1 > k_2}$, i.e. the relative importance of voters on election day opposition supporters by the total number of voters in support of the opposition party is greater than the relative importance of voters on election day supporters of the current government of the total number of voters support the ruling party. $\boldsymbol{N_2(T) > N_1(T)}$ --- number of voters supporting pro-government party at the time of the election more than the number of voters supporting the opposition party. Solving simple inequality, we obtain a $\boldsymbol{k_1^{*} = \min k_1}$, where $\boldsymbol{k_1}$ $\boldsymbol{k_1^{*} * N_1(T) > k_2 * N_2(T)}$. I.e. we define, what

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smallest number of voters from opposition party has to vote to ensure a victory on elections of opposition party.

On this graph (as well as on all the subsequent) points of the corresponding color (the green - color of supporters of opposition, red – supporters of ruling party) correspond to real number of the voted voters from this or that party.

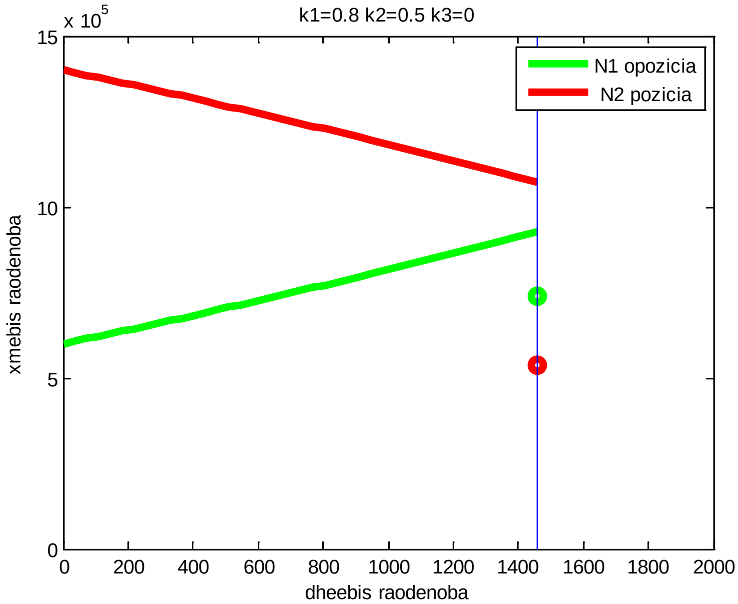

Fig.1 $f(t, N_1(t)) = b = const > 0$

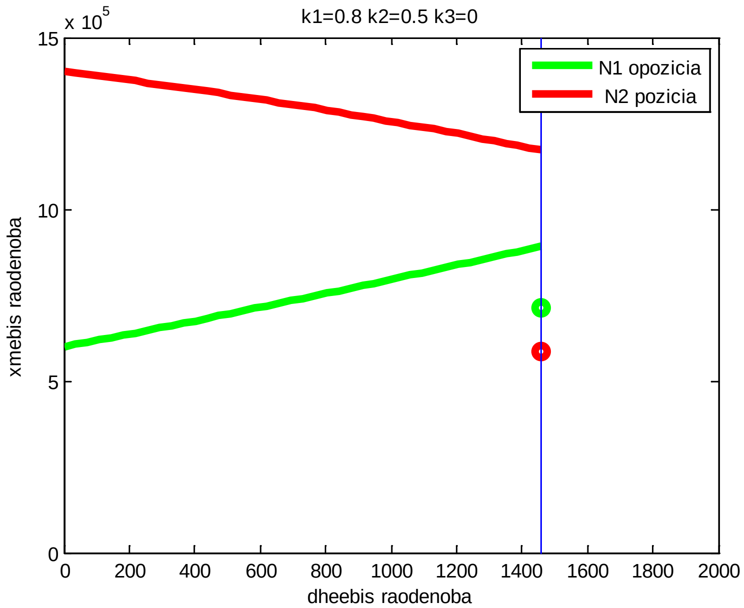

Fig.2 $f(t, N_1(t)) = b_1 N_1(t), \quad b_1 > 0$

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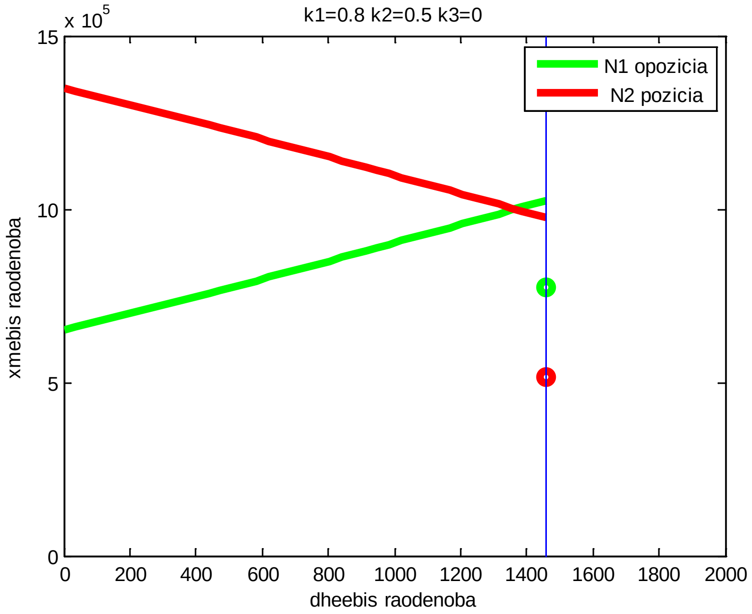

Fig.3 $f(t, N_1(t)) = F_0 e^{\delta \frac{t}{T}}$

2) $k_1 > k_2$, i.e. the relative importance of voters from all number of voters supporting the opposition party more than the relative importance of voters of the voters supporting the pro-government party. $N_1(T) = N_2(T)$) - the number of voters from both parties are equally at the time of elections. In this case, $k_1 * N_1(T) > k_2 * N_2(T)$, i.e. opposition party wins elections.

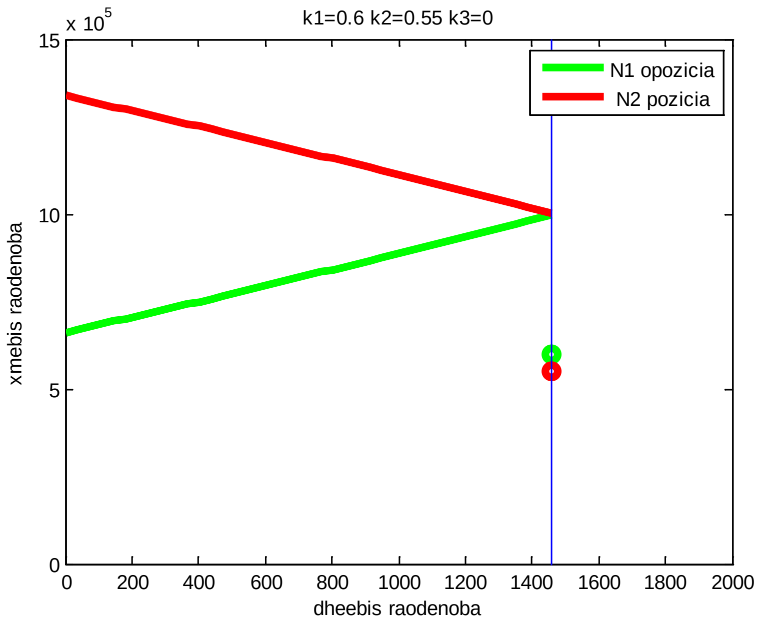

Fig.4 $f(t, N_1(t)) = b = const > 0$

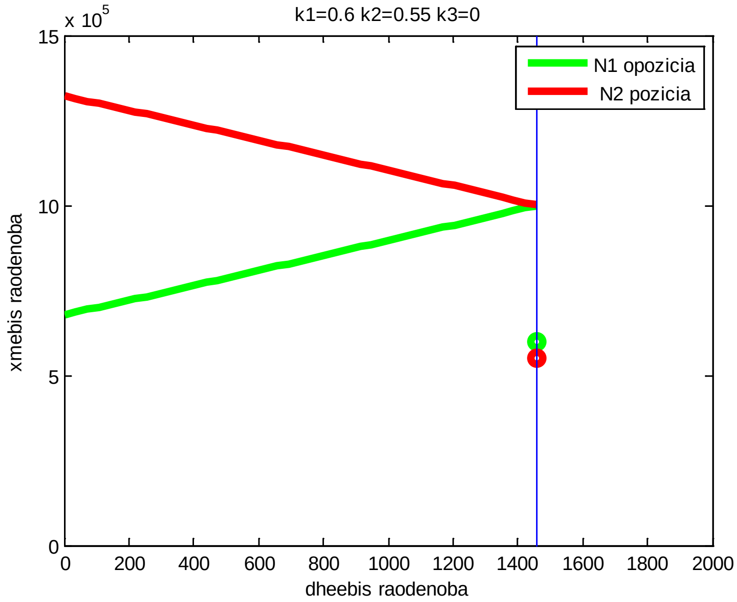

Fig.5 $f(t, N_1(t)) = b_1 N_1(t), \quad b_1 > 0$

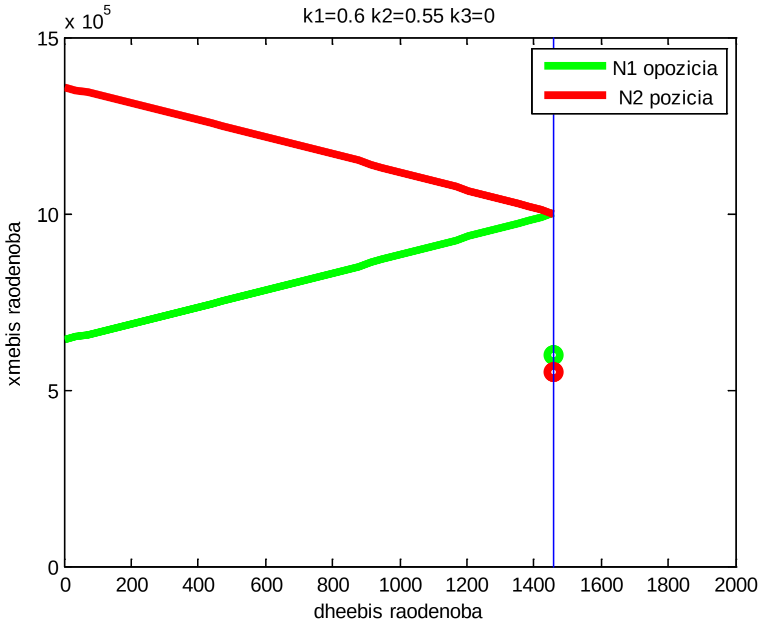

Fig.6 $f(t, N_1(t)) = F_0 e^{\delta \frac{t}{T}}$

3) $k_2 > k_1$, i.e. the relative importance of voters from all number of voters supporting pro-government parties more than the relative importance of voters of the voters supporting the opposition party. $N_1(T) > N_2(T)$- the number of voters in support of an opposition party at the time of the election is more than the number of voters in support the ruling party. By solving the inequality we find $k_2^* = \min k_2$, such that $k_2^* * N_2(T) > k_1 * N_1(T)$, i.e. define a minimum number of voters from the pro-government party must vote to ensure the victory of his party in the elections.

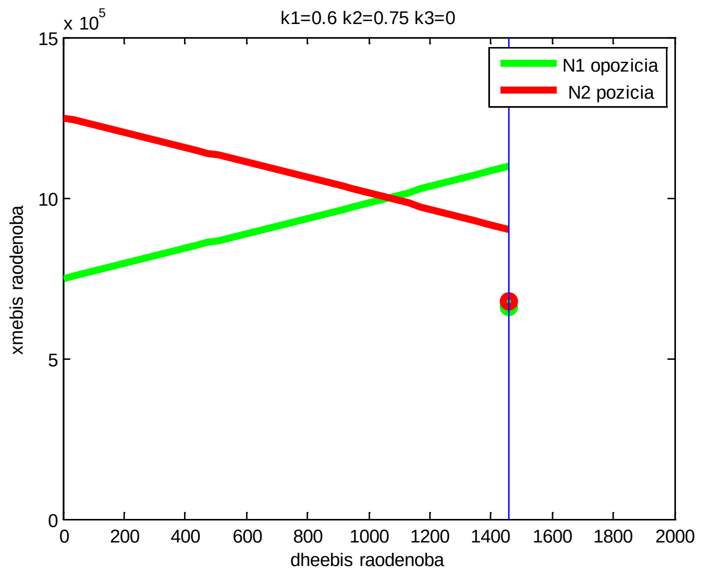

Fig.7 $f\big(t, N_1(t)\big) = b = const > 0$

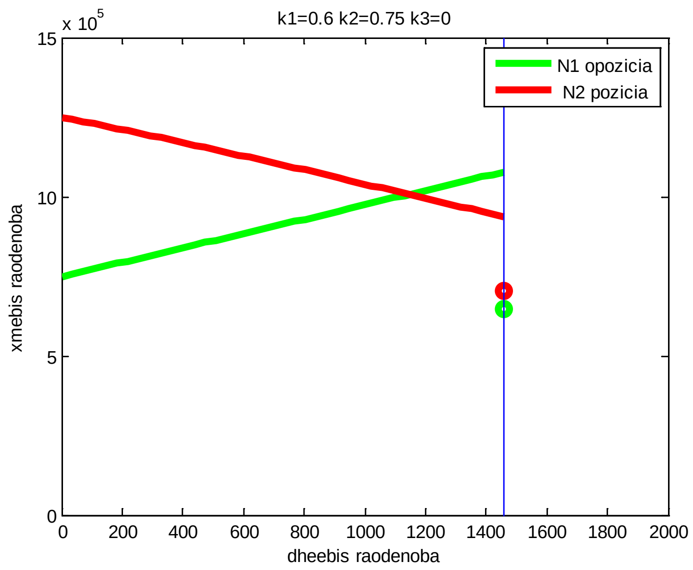

Fig.8 $f(t, N_1(t)) = b_1 N_1(t), \quad b_1 > 0$

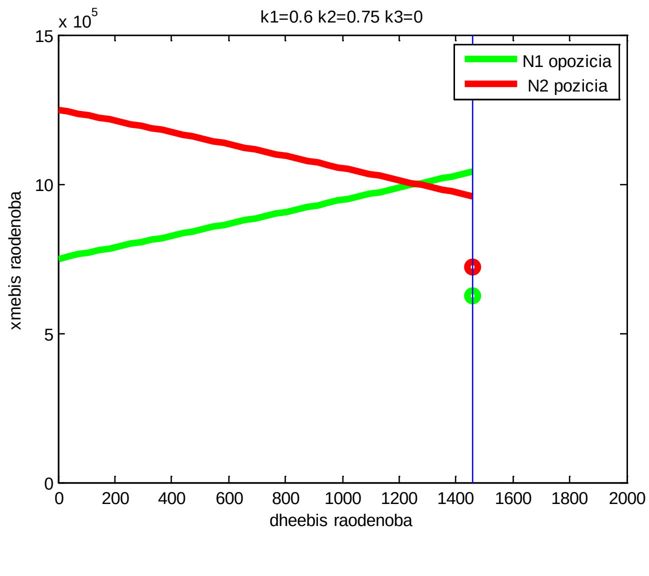

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Fig.9 $f(t, N_1(t)) = F_0 e^{\delta \frac{t}{T}}$

Now consider the case when, during the election held falsification (***k_3>0***):

1)***k_1> k_2***,, i.e. relative importance of voters from the total number of voters in support of the opposition party more than the relative importance of voters from the total number of the voters supporting the pro-government party. ***$N_2(T)$>$N_1(T)$*** --- the number of voters supporting pro-government party at the time of the election more than the number of voters supporting the opposition party. However, due to the activity of opposition voters ***(k_1> k_2)*** have: ***$k_1 * N_1(T) > k_2 * N_2(T)$,*** which means that the advantage of the opposition party in the elections. Solving simple inequality, we find ***k_3^*= min k_3***, such that ***$k_1 * (1-k_3^*) * N_1(T) < k_2 * N_2(T)$***. I.e. we find a minimum percentage of falsification is sufficient (if known ***k_1*** and ***k_2***) for ruling party again won the elections.

The yellow dot on the graph (and all subsequent) shows the number of ballots of opposition supporters after the application of ballot fraud.

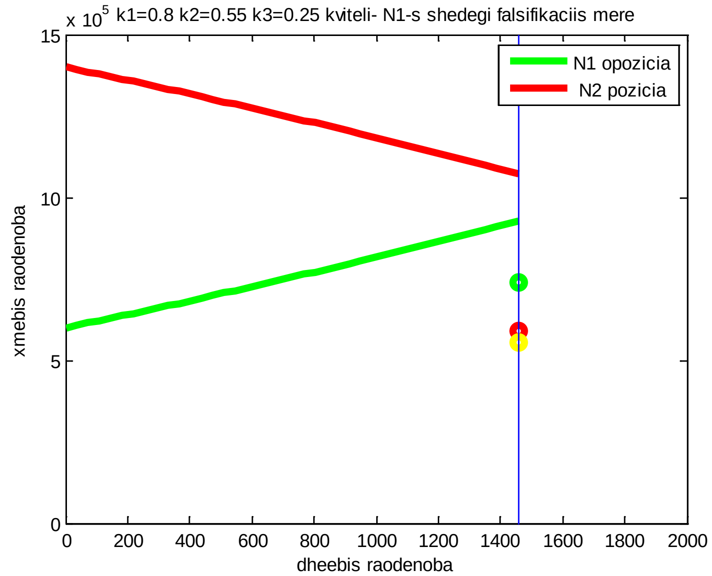

Fig.10 $f(t, N_1(t)) = b = const > 0$

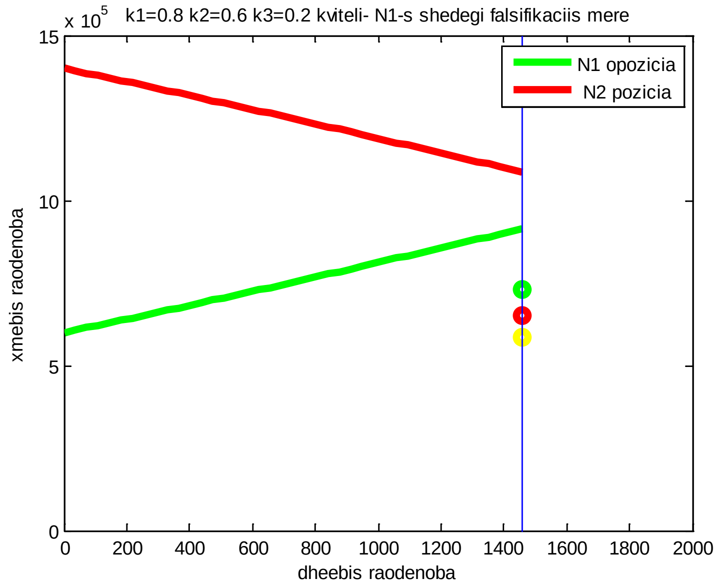

Fig.11 $f(t, N_1(t)) = b_1 N_1(t), \quad b_1 > 0$

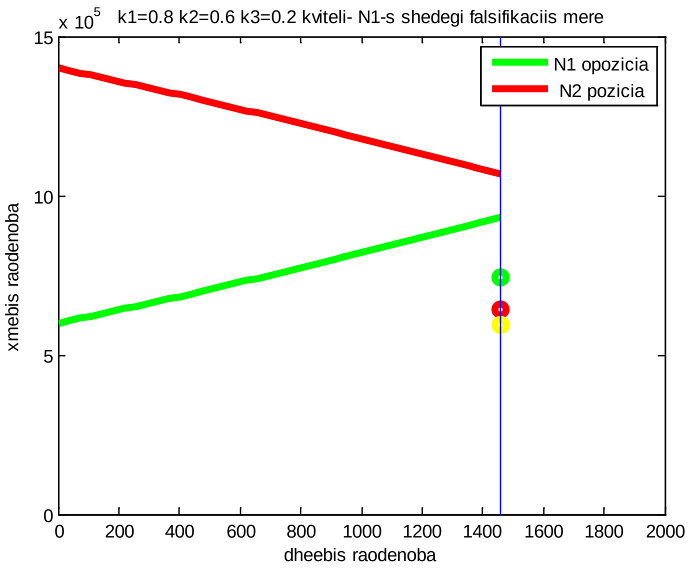

Fig.12 $f(t, N_1(t)) = F_0 e^{\delta \frac{t}{T}}$

2) $\boldsymbol{k_1 > k_2}$, i.e. the relative importance of voters from the total number of voters in support of the opposition party more than the relative importance of voters from the total number of the voters supporting the pro-government party. $\boldsymbol{N_2(T)>N_1(T)}$ --- the number of voters in support of the ruling party at the time of the election more than the number of voters supporting the opposition party. Thanks to the activity of voters support the opposition party $\boldsymbol{(k_1 > k_2)}$ have: $\boldsymbol{k_1 * N_1(T) > k_2 * N_2(T)}$, which means that the advantage of the opposition party in the elections. In the case of a predetermined coefficient $\boldsymbol{k_3}$, find $\boldsymbol{k_1^* = \min k_1}$, such that $\boldsymbol{k_1^* * (1-k_3) * N_1(T) > k_2 * N_2(T)}$. I.e. at a pre-known maximum scale falsification is possible to determine the smallest number of voters an opposition party for the election victory of his party.

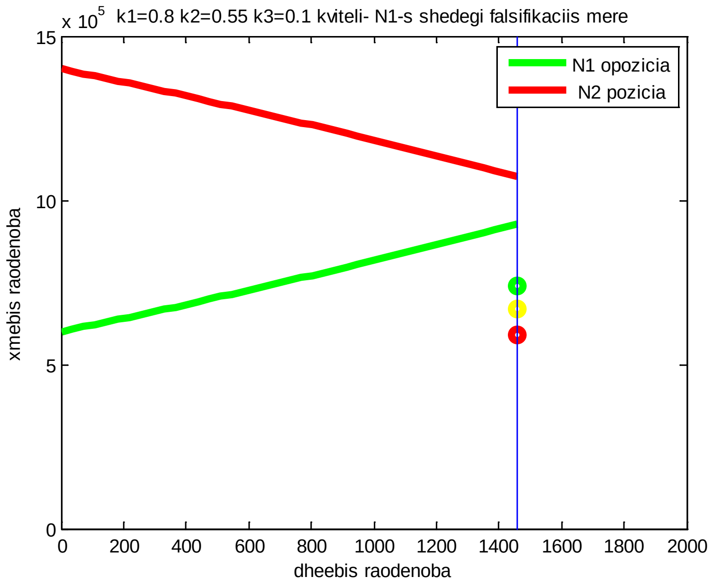

Fig.13 $f(t, N_1(t)) = b = const > 0$

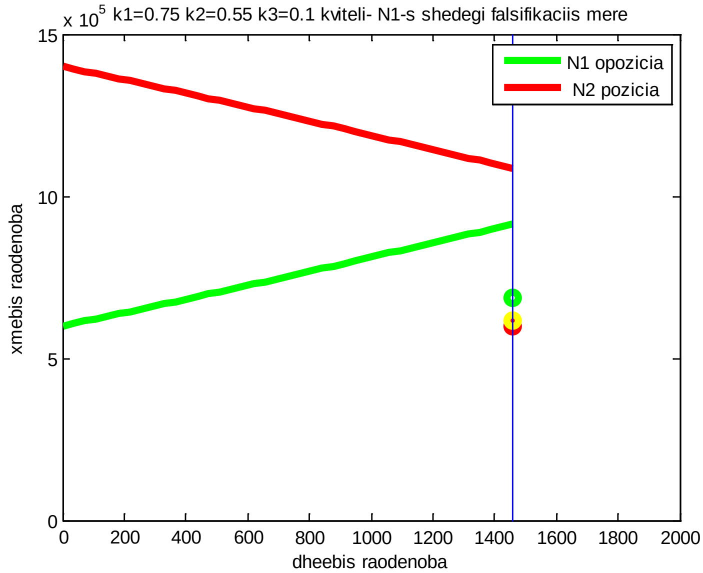

Fig.14 $f(t, N_1(t)) = b_1 N_1(t), \quad b_1 > 0$

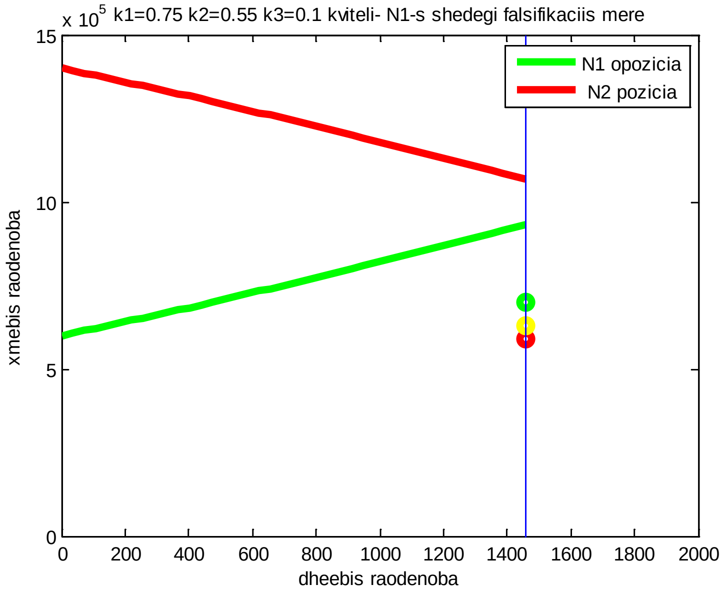

Fig.15 $f(t, N_1(t)) = F_0 e^{\delta \frac{t}{T}}$

3) $\boldsymbol{k_1 > k_2}$, i.e. the relative importance of voters from the total number of voters in support of the opposition party more than the relative importance of voters from the total number of the voters support the ruling party. $\boldsymbol{N_1(T)=N_2(T)}$- - the number of voters from both parties are equally at the time of elections. Thus have $\boldsymbol{k_1 * N_1(T) > k_2 * N_2(T)}$, which means that the advantage of the opposition party in the elections. It means an advantage in elections is an opposition party. By solving the inequality, we find $\boldsymbol{k_3^* = \min k_3}$, such, that $\boldsymbol{k_1 * (1-k_3^*) * N_1(T) < k_2 * N_2(T)}$. I.e. find minimum percentage of falsification is sufficient (if known $\boldsymbol{k_1}$и$\boldsymbol{k_2}$) to the pro-government party won the elections.

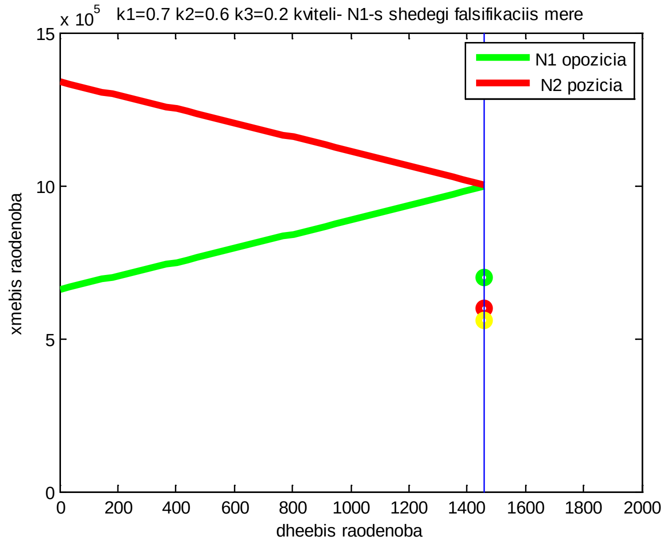

Fig.16 $f(t, N_1(t)) = b = const > 0$

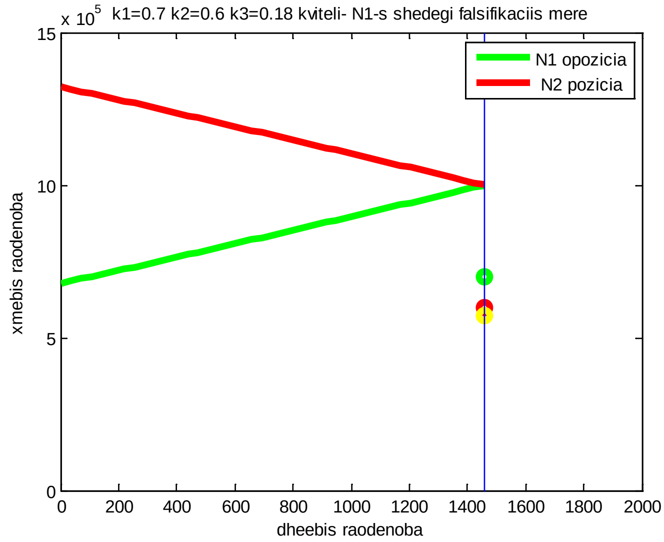

Fig.17 $f(t, N_1(t)) = b_1 N_1(t), \quad b_1 > 0$

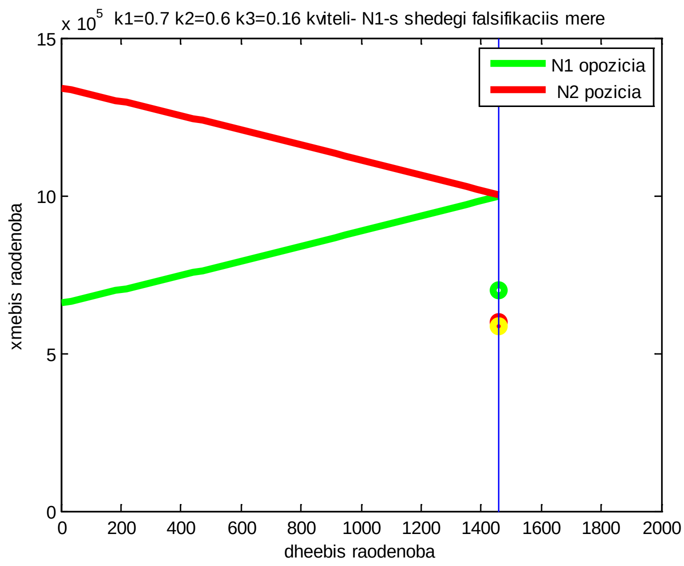

Fig.18 $f(t,N_1(t)) = F_0 e^{\delta \frac{t}{T}}$

4) $\boldsymbol{k_1 > k_2}$, i.e. the relative importance of voters from the total number of voters in support of the opposition party more than the relative importance of voters from the total number of the voters support the ruling party. $\boldsymbol{N_1(T)=N_2(T)}$ - the number of voters from both parties are equally at the time of elections. Thus have $\boldsymbol{k_1 * N_1(T) > k_2 * N_2(T)}$, which means that the advantage of the opposition party in the elections. But given the possible falsification $\boldsymbol{(k_3>0)}$, in the case of a predetermined coefficient $\boldsymbol{k_3}$ find $\boldsymbol{k_1^* = min\, k_1}$, that $\boldsymbol{k_1^* * (1-k_3) * N_1(T) > k_2 * N_2(T)}$. I.e. at a pre-known maximum scale falsification is possible to determine the smallest number of voters joining an opposition party for the election victory of his party.

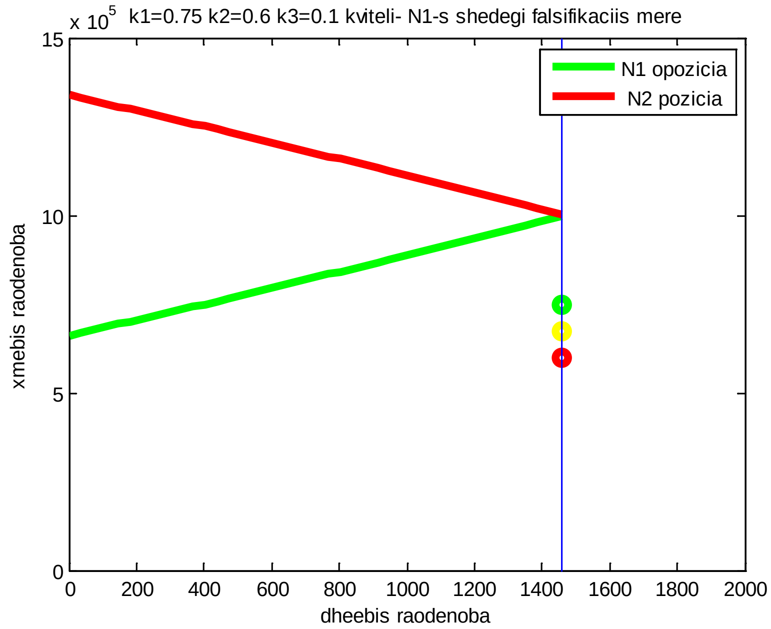

Fig.19 $f(t,N_1(t)) = b = const > 0$

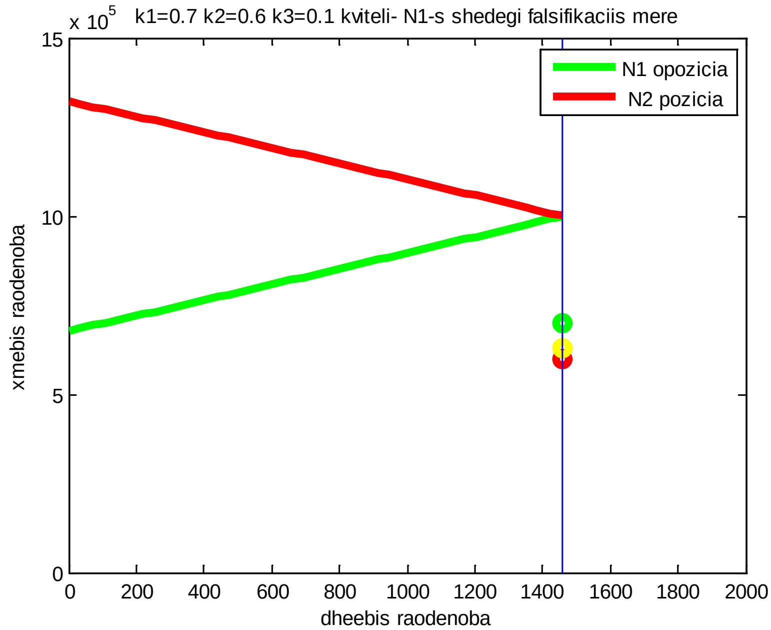

Fig.20 $f(t,N_1(t)) = b_1 N_1(t), \quad b_1 > 0$

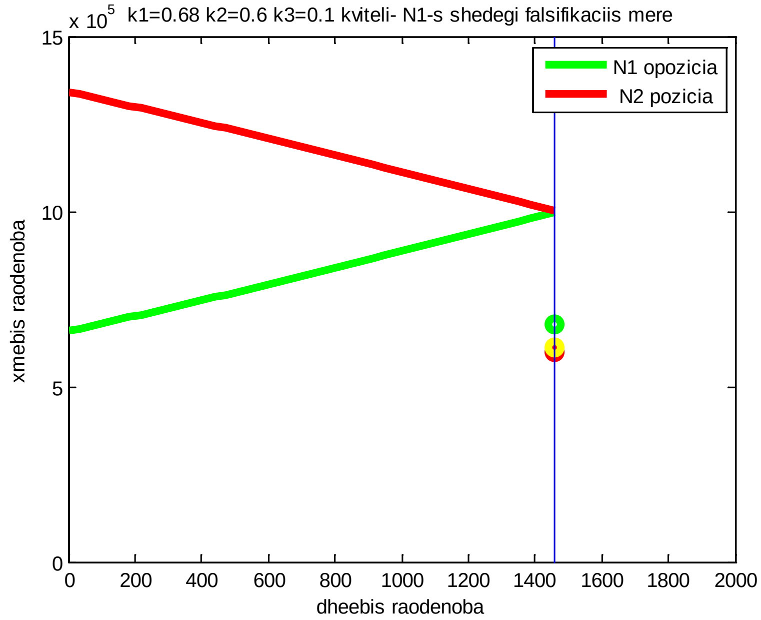

Fig.21 $f(t,N_1(t)) = F_0 e^{\delta \frac{t}{T}}$

5) $\boldsymbol{k_1 > k_2}$, i.e. the relative importance of voters from the total number of voters in support of the opposition party more than the relative importance of voters from the total number of the voters support the ruling party. $\boldsymbol{N_1(T) > N_2(T)}$ - the number of voters in support of an opposition party at the time of the election more than the number of voters support the ruling party. Thus have $\boldsymbol{k_1 * N_1(T) > k_2 * N_2(T)}$, which means a possible election victory of the opposition party. By solving the inequality we find $\boldsymbol{k_3^* = \min k_3}$, satisfying the condition: $\boldsymbol{k_1 * (1-k_3^*) * N_1(T) < k_2 * N_2(T)}$. I.e. find, a minimum percentage of falsification is sufficient (if known $\boldsymbol{k_1}$ and $\boldsymbol{k_2}$) to the pro-government party won the elections.

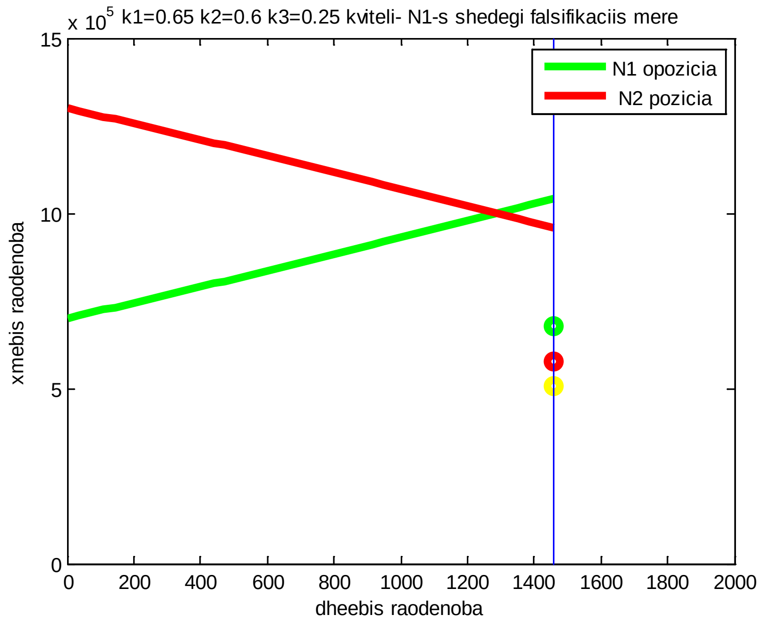

Fig.22 $f(t,N_1(t)) = b = const > 0$

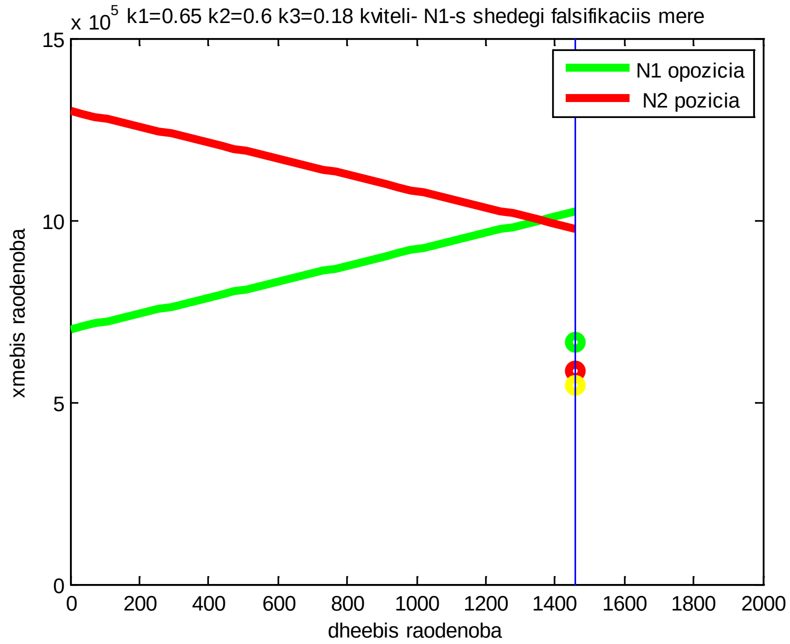

Fig.23 $f(t, N_1(t)) = b_1 N_1(t), \quad b_1 > 0$

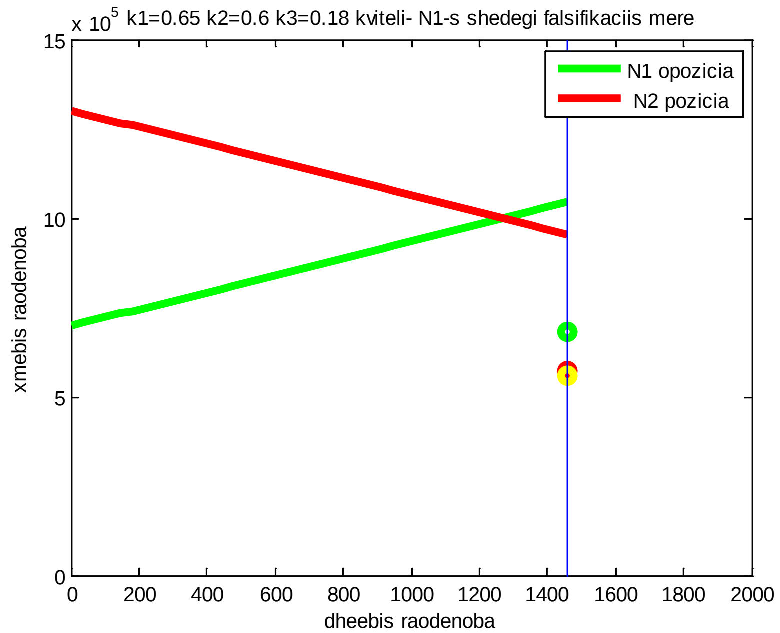

Fig.24 $f(t, N_1(t)) = F_0 e^{\delta \frac{t}{T}}$

6) $\boldsymbol{k_1 = k_2}$, i.e. the relative number of voters from both parties equally; $\boldsymbol{N_1(T) > N_2(T)}$ -- the number of voters in support of an opposition party at the time of the election more than the number of voters support the ruling party. Thus have $\boldsymbol{k_1 * N_1(T) > k_2 * N_2(T)}$, which means that the advantage of the opposition party in the elections. The opposition party has to walk through increased its activity (increase coefficient $\alpha_1(t)$), that despite the possible falsification ($\boldsymbol{k_3 > 0}$) to achieve the final victory in the elections: $\boldsymbol{k_1^{*}(1-k_3) * N_1(T) > k_2 * N_2(T)}$.

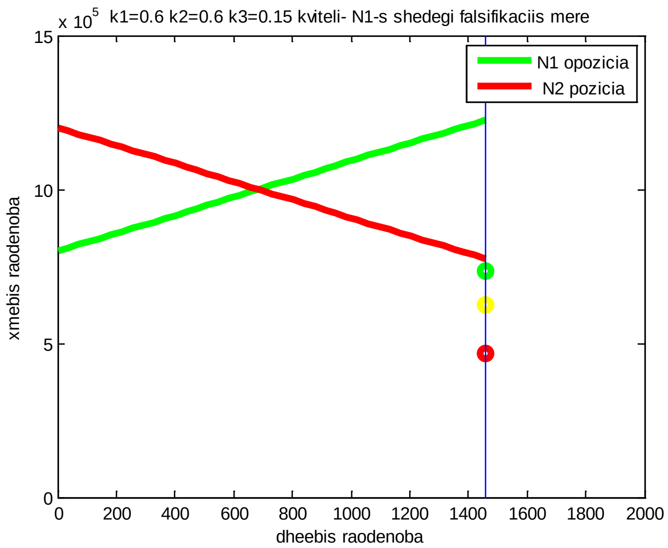

Fig.25 $f(t, N_1(t)) = b = const > 0$

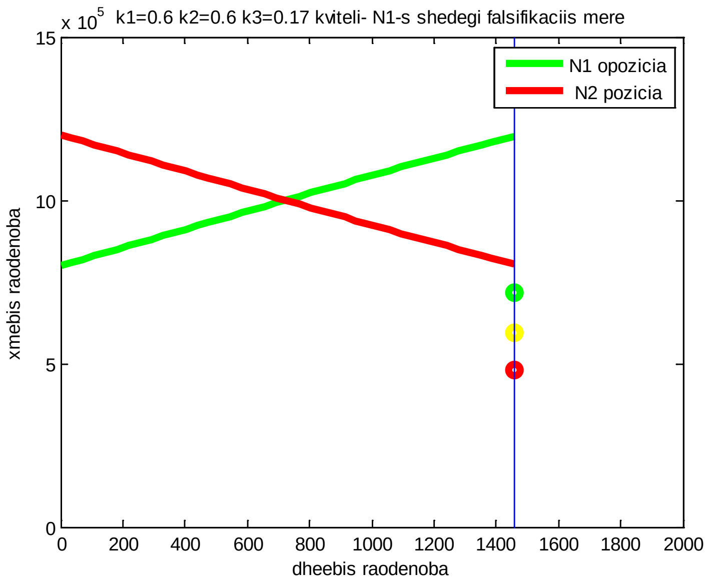

Fig.26 $f(t, N_1(t)) = b_1 N_1(t),\ b_1 > 0$

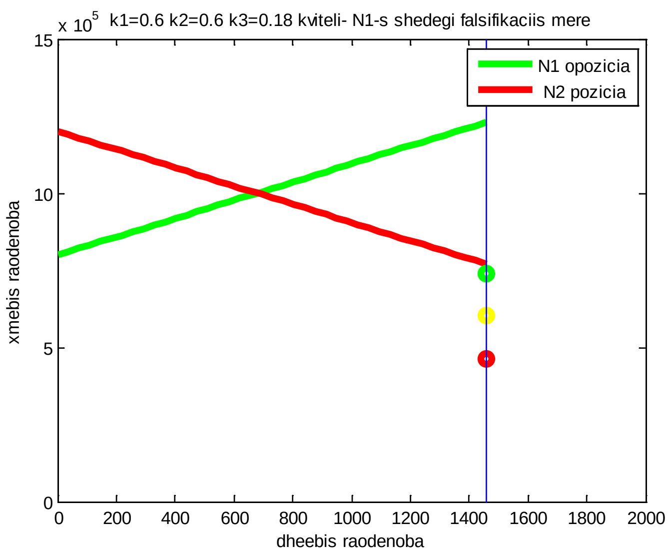

Fig.27 $f(t, N_1(t)) = F_0 e^{\delta \frac{t}{T}}$

7) $\boldsymbol{k_2 > k_1}$, i.e. the relative importance of voters from all number of voters supporting pro-government parties over the relative importance of voters from all number of the voters supporting the opposition party. $\boldsymbol{N_1(T)}$>N2(T), i.e. the number of voters in support of an opposition party at the time of the election more than the number of voters supporting pro-government parties. In the case of a predetermined coefficient $\boldsymbol{k_3}$, find $\boldsymbol{k_1^* = \min k_1}$, such that $\boldsymbol{k_1^* * (1-k_3) * N_1(T) > k_2 * N_2(T)}$. I.e. at a pre-known maximum scale falsification is possible to determine the smallest number of voters joining an opposition party for the election victory of his party.

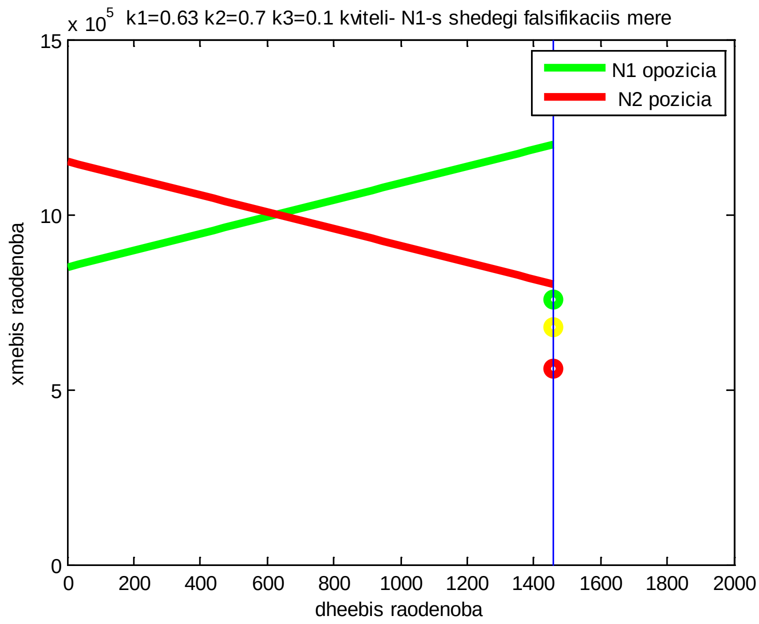

Fig.28 $f\left(t, N_1(t)\right) = b = const > 0$

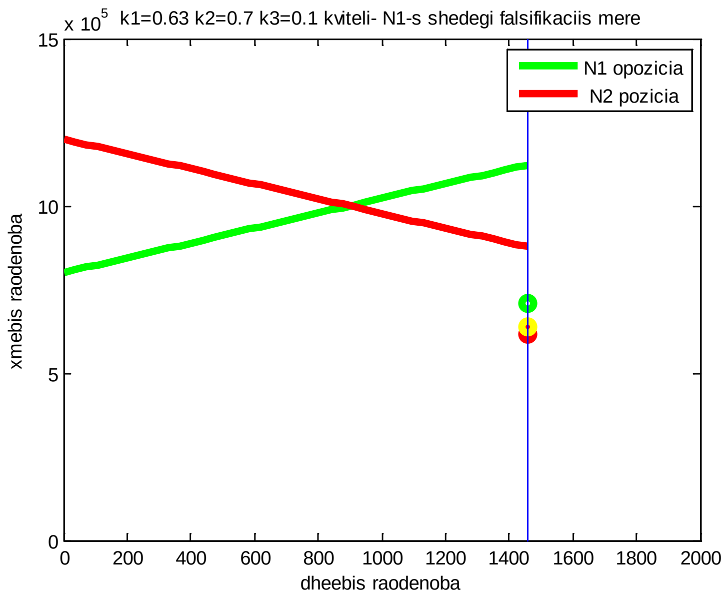

Fig.29 $f(t, N_1(t)) = b_1 N_1(t), \quad b_1 > 0$

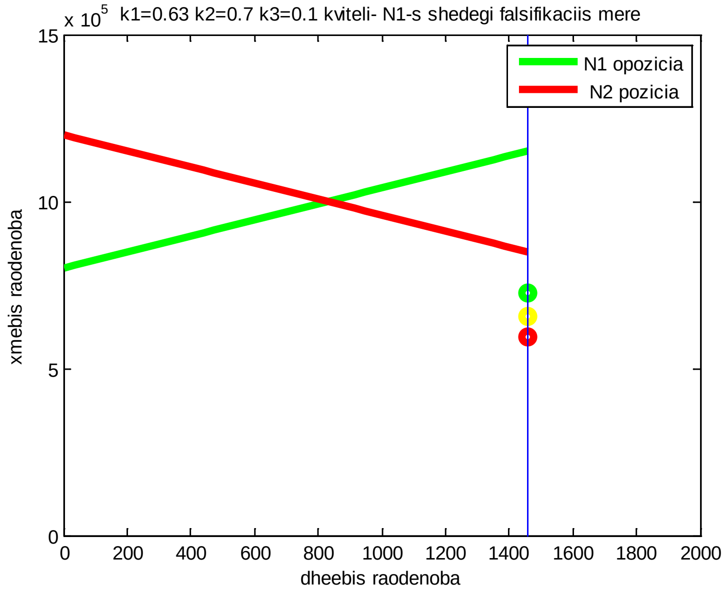

Fig.30 $f(t,N_1(t)) = F_0 e^{\delta\frac{t}{T}}$

We proposed a mathematical model has both theoretical and practical importance. Political opponents (government and opposition) can widely use the results: to choose a strategy, select options and to pursue the goal.

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