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Mathematical Modeling of Nonlinear Processes Bilateral Assimilation

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Abstract

In work mathematical modeling of nonlinear process of the assimilation taking into account positive demographic factor which underwent bilateral assimilation of the side and zero demographic factor of the assimilating sides is considered. In model three objects are considered: the population and government institutions with widespread first language, influencing by means of state and administrative resources on the population of the third state formation for the purpose of their assimilation; the population and government institutions with widespread second language, influencing by means of state and administrative resources on the population of the third state formation for the purpose of their assimilation; population of the third state formation which is exposed to bilateral assimilation from two powerful states or the coalitions.

For nonlinear system of three differential equations of the first order are received the two first integral. Special cases of two powerful states assimilating the population of small state formation (autonomy), with different initial number of the population, both with identical and with various economic and technological capabilities are considered. It is shown that in all cases there is a full assimilation of the population to less widespread language. Thus, proportions in which assimilate the powerful states the population of small state formation are found.

Key words: *bilateral assimilation, nonlinear system, first integral.*

Introduction

Mathematical modeling and computing experiment in the last decades gained all-round recognition in science as the new methodology which is roughly developing and widely introduced not only in natural science and technological spheres, but also in economy, sociology, political science and other public disciplines [1 - 4].

In [5 - 7] the mathematical model of political rivalry devoted to the description of fight occurring in imperious elite competing (but not necessarily antagonistic) political forces, for example, power branches is considered. It is supposed that each of the sides has ideas of "number" of the power which this side would like to have itself, and about "number" of the power which she would like to have for the partner.

Works [8 - 12] are devoted to creation of mathematical model of such social process what administrative (state) management is. The last can be carried out as at macro-level (for example, the state) and at micro-level (for example, an educational or research institution, industrial or financial facility, etc.).

In work [13] computer research of a trajectory of development of three ethnos living in one territory is conducted. Thus assimilation is supposed as a result of mixed marriages.

In work [14] we consider the nonlinear mathematical model of bilateral assimilation without demographic factor. It was shown that the most part of the population talking in the third language is assimilated by that widespread language which speaks bigger number of people (linear assimilation). Also it was shown that in case of zero demographic factor of all three subjects, the population with less widespread language completely assimilates the states with two various widespread languages, and the result of assimilation (redistribution of the assimilated population) is connected with initial quantities, technological and economic capabilities of the assimilating states.

In works[15,16] mathematical modeling of nonlinear process of assimilation taking into account demographic factor is offered. In considered model taking into account demographic factor natural decrease in the population of the assimilating states and a natural increase of the population which has undergone bilateral assimilation is supposed. At some ratios between coefficients of natural change of the population of the assimilating states, and also assimilation coefficients, for nonlinear system of three differential equations are received the two first integral. Cases of two powerful states assimilating the population of small state formation (autonomy), with different number of the population, both with identical and with various economic and technological capabilities are considered. It is shown that in the first case the problem is actually reduced to nonlinear system of two differential equations describing the classical model "predator - the victim", thus, naturally a role of the victim plays the population which has undergone assimilation, and a predator role the population of one of the assimilating states. The population of the second assimilating state in the first case changes in proportion (the coefficient of proportionality is equal to the relation of the population of assimilators in an initial time point) to the population of the first assimilator. In the second case the problem is actually reduced to nonlinear system of two differential equations describing type model "a predator – the victim", with the closed integrated curves on the phase plane. In both cases there is no full assimilation of the population to less widespread language. Intervals of change of number of the population of all three objects of model are found. The considered mathematical models which in some approach can model real situations, with the real assimilating countries and the state formations (an autonomy or formation with the unrecognized status), undergone to bilateral assimilation, show that for them the only possibility to avoid from assimilation is the natural demographic increase in population and hope for natural decrease in the population of the assimilating states.

In work [17] mathematical modeling of nonlinear process of the bilateral assimilation with zero demographic factor of the assimilating sides is considered.

1 . System of the equations and initial conditions

For the description of dynamics of the population of three associations (the state, to the coalition of the states with the same state language), speaking different languages on prevalence and opportunities of the respective states, we offer the following nonlinear mathematical model:

$$\begin{cases} \dfrac{du(t)}{dt} = \alpha_1 u(t) + \beta_1 u(t) w(t) \\ \\ \dfrac{dv(t)}{dt} = \alpha_2 v(t) + \beta_2 v(t) w(t) \\ \\ \dfrac{dw(t)}{dt} = \alpha_3 w(t) - \beta_3 u(t) w(t) - \beta_4 v(t) w(t) \end{cases} \tag{1.1}$$

$$v(0) = v_0, u(0) = u_0, w(0) = w_0, \tag{1.2}$$

$u(t)$ – number of people at present time t, talking in the first widespread language;
$v(t)$ – number of people at present time t, talking in the second widespread language;

$w(t)$ – number of people at present time t, talking in the language which is exposed from two widespread languages of assimilation;
$\beta_1 > 0, \beta_2 > 0, \beta_3 > 0, \beta_4 > 0$ – respectively coefficients of distribution (assimilation) of the first and second languages (assimilating impact on the people talking on third, not widespread language);
$\alpha_1, \alpha_2, \alpha_3$ - respectively coefficients of natural change (increase, reduction or constancy) populations of the people talking in the first, second and third languages.

In this work we assume that the first two assimilating sides have zero demographic factor, and the third side has positive demographic factor

$$\alpha_1 = 0, \qquad \alpha_2 = 0, \qquad \alpha_3 > 0 \ . \tag{1.3}$$

In case of (1.3) system (1.1) will assume an air

$$\begin{cases} \dfrac{du(t)}{dt} = \beta_1 u(t) w(t) \\[2ex] \dfrac{dv(t)}{dt} = \beta_2 v(t) w(t) \\[2ex] \dfrac{dw(t)}{dt} = \alpha_3 w(t) - \beta_3 u(t) w(t) - \beta_4 v(t) w(t) \end{cases} \tag{1.4}$$

Having divided the first equation of system (1.4) into the second equation, taking into account initial conditions (1.2), it is easy to receive the first integral of system of the differential equations

$$u(t) = p v^{\gamma_1}(t) \tag{1.5}$$

$$p = \frac{u_0}{v_0^{\gamma_1}} \ . \qquad \gamma_1 = \frac{\beta_1}{\beta_2} \ .$$

Now we will receive the second first integral of a system (1.4).

From (1.5) it is easy to receive

$$v(t) = q u^{1/\gamma_1}(t) , \tag{1.6}$$

$$q = p^{-1/\gamma_1} \ .$$

Having divided the first equation of system (1.5) into the third equation, taking into account (1.6) we will receive

$$\frac{du}{dw} = \frac{\beta_1 u w}{\alpha_3 w - \beta_3 u w - \beta_4 v w} = \frac{\beta_1 u}{\alpha_3 - \beta_3 u - \beta_4 v}$$

$$\frac{du}{dw} = \frac{\beta_1 u}{\alpha_3 - \beta_3 u - \beta_4 q u^{1/\gamma_1}} \ . \tag{1.7}$$

Having divided variables in (1.7) and integrating taking into account initial conditions (1.2), we will receive

$$\int_{u_0}^{u} \frac{\left(\alpha_3 - \beta_3 u - \beta_4 q u^{1/\gamma_1}\right) du}{u} = \beta_1 \int_{w_0}^{w} dw \tag{1.8}$$

Simplifying (1.8) we will receive some dependence of functions $w(t)$, $u(t)$

$$\alpha_3 \ln\frac{u}{u_0} - \beta_3(u-u_0) - \beta_4 \cdot q\ \gamma_1 \left(u^{1/\gamma_1} - u_0^{1/\gamma_1}\right) = \beta_1(w - w_0) \tag{1.9}$$

Having expressed function $w(t)$ through function $u(t)$ we will receive

$$w = \frac{\alpha_3}{\beta_1}\ln\frac{u}{u_0} - \frac{\beta_3}{\beta_1}(u-u_0) - \frac{\beta_4}{\beta_1} \cdot q\ \gamma_1 \left(u^{1/\gamma_1} - u_0^{1/\gamma_1}\right) + w_0 \tag{1.10}$$

We will enter designations:

$$k = \frac{\alpha_3}{\beta_1}, \qquad m = \frac{\beta_3}{\beta_1}, \qquad l = \frac{\beta_4}{\beta_1} \cdot q\ \gamma_1 \tag{1.11}$$

Taking into account (1.11), (1.10) will assume an air

$$w = k\ln\frac{u}{u_0} - m(u-u_0) - l\left(u^{1/\gamma_1} - u_0^{1/\gamma_1}\right) + w_0 \tag{1.12}$$

We will consider special cases

1. $\beta_1 = \beta_2, \qquad \gamma_1 = 1$

Then equality (1.12) will assume an air

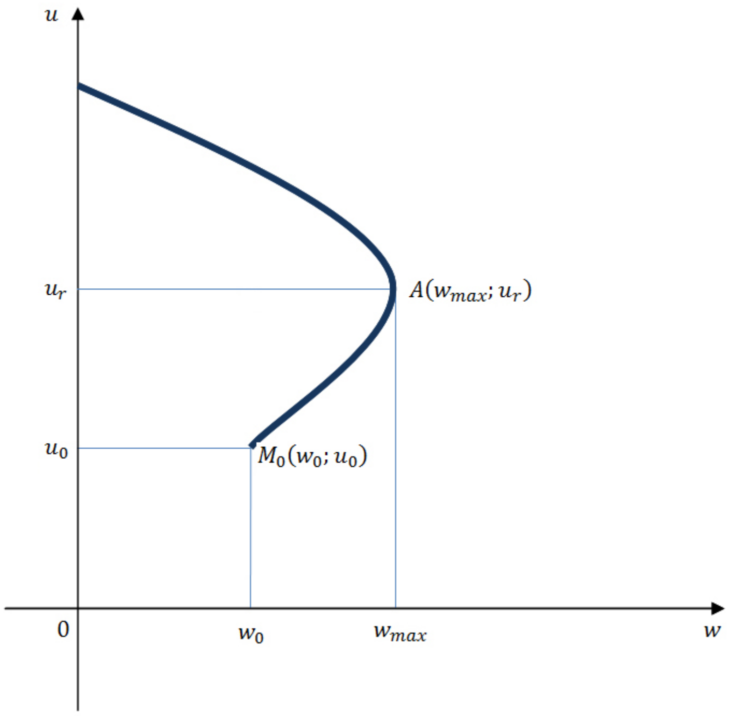
$$w = k\ln\frac{u}{u_0} - (m+l)(u-u_0) + w_0 \tag{1.13}$$

Investigating function $w(t)$ (1.13) on the phase plane Ou, w , it is easy to receive that it reaches the maximum in a point

$$u_{cr} = \frac{\alpha_3}{\beta_3 + \beta_4 \frac{v_0}{u_0}} \tag{1.14}$$

$$w_{max} = \frac{\alpha_3}{\beta_1}\ln\frac{u_{cr}}{u_0} - \frac{1}{\beta_1}\left(\beta_3 + \frac{\beta_4 v_0}{u_0}\right)(u_{cr} - u_0) + w_0 \tag{1.15}$$

Further at increase u function w decreases and at some value addresses in zero (full assimilation of the third side).



For finding of the maximum value of function u we have the following transcendental equation

$$\frac{\alpha_3}{\beta_1}\ln\frac{u_{max}}{u_0}-\frac{1}{\beta_1}\left(\beta_3+\frac{\beta_4 v_0}{u_0}\right)(u_{max}-u_0)+w_0=0 \qquad (1.16)$$

Thus, having found maximum value of function u from the transcendental equation (1.16), taking into account (1.5) it is easy to receive the maximum value of function v

$$v_{\max}=\frac{v_0}{u_0}u_{\max}\ . \qquad (1.17)$$

Thus, eventually, we will receive full of assimilation of the third side

$$\begin{cases} u=u_{\max} \\ v=v_{\max} \\ w=0 \end{cases} \qquad (1.18)$$

2. $2\beta_1=\beta_2,\qquad \gamma_1=\frac{1}{2}$

Then equality (1.12) will assume an air

$$w=k\ln\frac{u}{u_0}-m(u-u_0)-l(u^2-{u_0}^2)+w_0 \qquad (1.19)$$

Investigating function $w(t)$ (1.19) on the phase plane Ou,w , it is easy to receive that it reaches the maximum in a point

$$u_{cr}=\frac{(\sqrt{\beta_3^2u_0^2+4\beta_4\alpha_3v_0}-\beta_3u_0)u_0}{2\beta_4v_0} \qquad (1.20)$$

$$w_{\max 1}=k\ln\frac{u_{cr}}{u_0}-m(u_{cr}-u_0)-l(u_{cr}^2-u_0^2)+w_0$$

Further at increase u function w decreases and at some value addresses in zero (full assimilation of the third side). For finding of the maximum value of function u we have the following transcendental equation

$$k\ln\frac{u_{\max 1}}{u_0}-m(u_{\max 1}-u_0)-l(u_{\max 1}^2-u_0^2)+w_0=0\ . \qquad (1.21)$$

Thus, having found maximum value of function u from the transcendental equation (1.21), taking into account (1.5) it is easy to receive the maximum value of function v

$$v_{\max 1}=\frac{v_0}{u_0^2}u_{\max 1}^2\ . \qquad (1.22)$$

Thus, eventually, we will receive full of assimilation of the third side

$$\begin{cases} u=u_{\max 1} \\ v=v_{\max 1} \\ w=0 \end{cases} \qquad (1.23)$$

It is shown that in all cases (1.18), (1.23) there is a full assimilation of the population to less widespread language. Thus, proportions in which assimilate the powerful states the population of small state formation are found (1.16), (1.17), (1.21), (1.22).

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