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MATHEMATICAL AND COMPUTER MODELING OF THREE-PARTY ELECTIONS

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Abstract

In this paper, the development of our previously proposed two-party electoral models, is proposed the nonlinear mathematical model with variable coefficients in the case of three-party elections, that describes the dynamics of the quantitative change of the votes of the pro-government and two opposition parties from election to election. The model considers four objects: state and administrative structures, acting by means of administrative resources for opposition-minded voters with the aim to win their support for the pro-government party; voters who support the first opposition party; voters who support the second opposition party; voters who support the pro-government party.

The model takes into account the change in the total number of voters in the period from election to election, i.e. the so-called demographic factor during the elections is taken into account. We have considered two cases: when the elections are held without falsification and when there are cases of falsification by the Election Commission in favor of the pro-government party. The model considered the cases with variable coefficients. In particular, we assume that in the period between elections coefficients of "attracting" voters are exponentially increasing function of time.

In the particular case we obtain exact analytical solutions. The conditions have been identified under which the opposition can win the forthcoming elections, and in some cases, the pro-government party can stay in power.

In general Cauchy problem was solved numerically using the MATLAB software package.

We get different variations of the outcome of the election based on voter turnout, the possible falsification of elections and demographic factors.

The proposed mathematical and computer model has both theoretical and practical importance. Political opponents (the power and opposition parties) can use our results: to choose a strategy, to calculate its abilities (selecting parameters) in order to achieve the set goal.

Keywords*: nonlinear model of elections, pro-government party, the oppositions parties , falsification, voter turnout, demographic factor.*

Introduction

During the last decade mathematical and computer modeling has been widely recognized in such disciplines as sociology, political science, and others [1, 2]. There is an interest in creation of a mathematical model, which would give the opportunity to determine the dynamics of changes in the number of voters of political subjects during the election period. Elections can be divided into two parts: the two-party and multi-party elections.

In [3-5] quantities of information streams by means of new mathematical models of information warfare are studied. By information warfare the authors mean an antagonism by means

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of mass media (an electronic and printing press, the Internet) between the two states or the two associations of states, or the economic structures (consortiums) conducting purposeful misinformation, propagation against each other. It was shown that in case of high aggression of the contradictory countries, not preventive image the operating peacekeeping organizations won't be able to extinguish the expanding information warfare.

In works [6,7] linear and nonlinear mathematical models of information warfare, and also optimizing problems are considered.

In [8] the new nonlinear mathematical and computer model of information warfare with participation of interstate authoritative institutes is offered. The model is described by Cauchy's problem for nonlinear non-homogeneous system of the differential equations. Confronting sides in extend of provocative statements, the third side (the peacekeeping international organizations) extends of soothing statements, interstate authoritative institutes the peacekeeping statements call the sides for the termination of information warfare. In that specific case, modes of information warfare "aggressor- victim”, for the third peacekeeping side are received exact analytical solutions, and functions defining number of the provocative statements distributed by the antagonistic sides satisfy to Cauchy's problems for Riccati certain equations which are solved by a numerical method. For the general model computer modeling is carried out and shown that irrespective of high aggression of confronting sides, interstate authoritative institutes will be able to extinguish information warfare and when for this purpose efforts of only the international organizations insufficiently.

In [9] consider the nonlinear mathematical model of bilateral assimilation without demographic factor. It was shown that the most part of the population talking in the third language is assimilated by that widespread language which speaks bigger number of people (linear assimilation). Also it was shown that in case of zero demographic factor of all three subjects, the population with less widespread language completely assimilates the states with two various widespread languages, and the result of assimilation (redistribution of the assimilated population) is connected with initial quantities, technological and economic capabilities of the assimilating states.

In [10] mathematical modeling of nonlinear process of assimilation taking into account demographic factor is offered. In considered model taking into account demographic factor natural decrease in the population of the assimilating states and a natural increase of the population which has undergone bilateral assimilation is supposed. At some ratios between coefficients of natural change of the population of the assimilating states, and also assimilation coefficients, for nonlinear system of three differential equations are received the two first integral. Cases of two powerful states assimilating the population of small state formation (autonomy), with different number of the population, both with identical and with various economic and technological capabilities are considered. It is shown that in the first case the problem is actually reduced to nonlinear system of two differential equations describing the classical model "predator - the victim", thus, naturally a role of the victim plays the population which has undergone assimilation, and a predator role the population of one of the assimilating states. The population of the second assimilating state in the first case changes in proportion (the coefficient of proportionality is equal to the relation of the population of assimilators in an initial time point) to the population of the first assimilating side. In the second case the problem is actually reduced to nonlinear system of two differential equations describing type model "a predator – the victim", with the closed integrated curves on the phase plane. In both cases there is no full assimilation of the population to less widespread language. Intervals of change of number of the population of all three objects of model are found. The considered mathematical models which in some approach can model real situations, with the real assimilating countries and the state formations (an autonomy or formation with the unrecognized status), undergone to bilateral assimilation, show that for them the only possibility to avoid from assimilation is the natural demographic increase in population and hope for natural decrease in the population of the assimilating states.

In [11] mathematical modeling of nonlinear process of the assimilation taking into

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account positive demographic factor which underwent bilateral assimilation of the side and zero demographic factor of the assimilating sides is considered. In model three objects are considered: the population and government institutions with widespread first language, influencing by means of state and administrative resources on the population of the third state formation for the purpose of their assimilation; the population and government institutions with widespread second language, influencing by means of state and administrative resources on the population of the third state formation for the purpose of their assimilation; population of the third state formation which is exposed to bilateral assimilation from two powerful states or the coalitions.

For nonlinear system of three differential equations of the first order are received the two first integral. Special cases of two powerful states assimilating the population of small state formation (autonomy), with different initial number of the population, both with identical and with various economic and technological capabilities are considered. It is shown that in all cases there is a full assimilation of the population to less widespread language. Thus, proportions in which assimilate the powerful states the population of small state formation are found.

In works [12 - 14] the mathematical model of political rivalry devoted to the description of fight occurring in imperious elite competing (but not necessarily antagonistic) political forces, for example, power branches is considered. It is supposed that each of the sides has ideas of "number" of the power which this side would like to have itself, and about "number" of the power which she would like to have for the partner.

These papers [15 - 19] present the nonlinear mathematical model of the public or the administrative management (or the macro and micro model). The cases of both constant and variable pressure forces on freethinking people were analyzed. Exact analytical decisions which determine dynamics of a spirit both free-thinking people, and operated (conformists) of people by time are received. During this analyses various governance systems were considered: a liberal, democratic, semi dictatorial and dictatorial.

These works [20 - 26] considered a two or three-party (one pro-government and two opposition parties) nonlinear mathematical model of elections when coefficients are constant. The assumption was made that the number of voters remain the same between two consecutive elections (zero demographic factor of voters). The exact analytical solutions were received. The conditions under which opposition party can win the upcoming elections were established.

These works [27 - 30] considered a two-party (pro-government and opposition parties) nonlinear mathematical model of elections with variable coefficients.

In [28] the nonlinear mathematical model describing dynamics of voters of pro-government and oppositional parties is offered. The case when coefficients of attraction of votes of pro-government and oppositional parties are exponential increasing functions from elections to elections is considered. Cauchy's task for nonlinear system of the differential equations with variable coefficients of model is solved by means of the program Matlab environment. Cases as maximum and certain voter turnout on elections, and also the set falsification of voices of opposition party are considered. The following qualitatively various results are received:

- despite superiority of coefficient of attraction of votes of opposition party over pro-governmental, due to administrative impact on voters of opposition party from government institutions, the pro-government party will win the next elections;
- despite superiority of the voters supporting opposition party by the election day due to the best mobilization on elections of the voters, the pro-government party will win the next elections;
- despite superiority of the voters supporting opposition party by the election day at an identical voter turnout on elections, due to a certain falsification of elections, the pro-

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government party will win the next elections;
- the opposition party, despite the best appearance on elections of voters of pro-government party and a certain falsification of elections, nevertheless will win the next elections.

In this publication the nonlinear mathematical model with variable coefficients in case of three-party elections which describes dynamics of quantitative change of votes of pro-government and two oppositional parties is presented. The model takes into account the change in the total number of voters in the period from election to election, i.e. the so-called demographic factor during the elections is taken into account. In model four objects are considered:

1. The state and administrative structures that utilize state resources in order to have an influence on the pro-oppositions voters with the aim to gain their support for the pro-government party.
2. Voters who support first opposition party.
3. Voters who support second opposition party.
4. Voters who support the pro-government party.

In model there are various indicators of a voter turnout in the election day, and also falsification chances in advantage the pro-government party.

1. A system of equations and initial conditions.

To describe the dynamics of choosing between three election subjects (pro-go-vernment and two opposition parties), we propose the following nonlinear mathematical model:

$$\begin{cases} \dfrac{dN_1(t)}{dt} = (\alpha_1(t)-\alpha_2(t))N_1(t)N_2(t) + (\alpha_1(t)-\alpha_3(t))N_1(t)N_3(t) - \beta_1(t)N_1(t) + \gamma_1(t)N_1(t) \\ \dfrac{dN_2(t)}{dt} = (\alpha_2(t)-\alpha_1(t))N_1(t)N_2(t) + (\alpha_2(t)-\alpha_3(t))N_2(t)N_3(t) - \beta_2(t)N_2(t) + \gamma_2(t)N_2(t) \\ \dfrac{dN_3(t)}{dt} = (\alpha_3(t)-\alpha_1(t))N_1(t)N_3(t) + (\alpha_3(t)-\alpha_2(t))N_2(t)N_3(t) + \beta_1(t)N_1(t) + \beta_2(t)N_2(t) + \gamma_3(t)N_3(t) \end{cases} \qquad (1.1)$$

The system of the equations (1.1) is considered on an interval $t \in (0,T]$, corresponding initial conditions (Cauchy's condition) at the moment of $t = 0$

$$N_{10} = N_1(0), N_{20} = N_2(0), N_{30} = N_3(0). \qquad (1.2)$$

The solution of a task of Cauchy (1.1), (1.2) we look for on a segment $t \in [0,T]$ in a class of continuously differentiable functions

$$N_1(t), N_2(t), N_3[t] \in C^1[0,T] \quad . \qquad (1.3)$$

In system of the equations (1.1): $N_1(t), N_2(t), N_3(t) -$ the number of voters in support of the first, second opposition and pro-government parties, respectively, at time $t, t \in [0,T]$; $t = 0$ — moment the last elections, in consequence of which party won the elections and became the pro-government party ($N_{10} + N_{20} < N_{30}$); $t = T$ — time of the next elections (usually T = 4 years or 1460 days); $\alpha_1(t), \alpha_2(t), \alpha_3(t)$ - coefficients corresponding to the activity to attract the votes of the first, second opposition and the pro-government parties, respectively, at time *t*, depending on the program of action, financial and information capabilities of these parties; $\beta_1(t), \beta_2(t)$ — continuous positive functions which characterize the scale of use of the administrative resources directed to oppositional voters for the purpose of their attraction on the party of pro-government party byvarious, perhaps

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non-democratic methods; $\gamma_1(t), \gamma_2(t), \gamma_3(t)$ -- coefficients of the accounting of so-called selective demographic change of the parties.

2. Three-party nonlinear mathematical model with constant coefficients. Exact analytical solutions.

We will consider a special case, without so-called demographic factor, or such case when from elections to elections the number of voters is invariable, i.e. during this period the number of the died voters is equal to the number of voters for the first time acquired the suffrage. In that case we have model with zero demographic factor of voters, i.e.in system (1.1) it is necessary to put

$$\gamma_i(t) \equiv 0, \; i \in \overline{1,3}, \; t \in [0,T] \quad . \tag{2.1}$$

We will consider a case with constant coefficients

$$\alpha_i(t) = \alpha_i = const > 0, i = \overline{1,3},$$
$$\beta_j(t) = \beta_j = const > 0, \; j = 1,2,$$

then the system (1.1) and initial conditions (1.2) will take a form

$$\begin{cases} \dfrac{dN_1(t)}{dt} = (\alpha_1 - \alpha_2)N_1(t)N_2(t) + (\alpha_1 - \alpha_3)N_1(t)N_3(t) - \beta_1 N_1(t) \\ \dfrac{dN_2(t)}{dt} = (\alpha_2 - \alpha_1)N_1(t)N_2(t) + (\alpha_2 - \alpha_3)N_2(t)N_3(t) - \beta_2 N_2(t) \\ \dfrac{dN_3(t)}{dt} = (\alpha_3 - \alpha_1)N_1(t)N_3(t) + (\alpha_3 - \alpha_2)N_2(t)N_3(t) + \beta_1 N_1(t) + \beta_2 N_2(t) \end{cases} \tag{2.2}$$

$N_{10} = N_1(0), N_{20} = N_2(0), N_{30} = N_3(0)$, $N_{10} + N_{20} < N_{30}$, $N_1(t), N_2(t), N_3(t) \in C^1[0,T]$.

If in system we put (2.2) all equations, it is very simple to receive the first first integral

$$N_1(t) + N_2(t) + N_3(t) = N_{10} + N_{20} + N_{30} = a \tag{2.3}$$

From (2.2), (2.3) we will receive

$$\frac{dN_1(t)}{dN_2(t)} = \frac{N_1(t)[(\alpha_3 - \alpha_1)N_1(t) + (\alpha_3 - \alpha_2)N_2(t) + (\alpha_1 - \alpha_3)a - \beta_1]}{N_2(t)[(\alpha_3 - \alpha_1)N_1(t) + (\alpha_3 - \alpha_2)N_2(t) + (\alpha_2 - \alpha_3)a - \beta_2]} \; . \tag{2.4}$$

It is easy to receive the second first integral if we make an assumption

$$(\alpha_1 - \alpha_2)a = \beta_1 - \beta_2, \tag{2.5}$$

then

$$a(\alpha_1 - \alpha_3) - \beta_1 = a(\alpha_2 - \alpha_3) - \beta_2 \, . \tag{2.6}$$

If equality is executed

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$$(\alpha_3-\alpha_1)N_{10}+(\alpha_3-\alpha_2)N_{20}+(\alpha_1-\alpha_3)a-\beta_1=0,$$

then we will receive the trivial decision, constancy of the voters supporting parties and on the following elections, in this case, there will be just the same result, as on the previous elections.

$$N_1(t)=N_{10},\ N_2(t)=N_{20},\ N_3(t)=N_{30},\ t\in[0,T]$$

if

$$(\alpha_3-\alpha_1)N_1(t)+(\alpha_3-\alpha_2)N_2(t)+(\alpha_1-\alpha_3)a-\beta_1\neq 0,$$

then from (2.4) and (2.6) we will receive

$$\frac{dN_1(t)}{dN_2(t)}=\frac{N_1(t)}{N_2(t)},$$

thus

$$N_1(t)=\frac{1}{p}N_2(t),\ p=\frac{N_{20}}{N_{10}}, \tag{2.7}$$

taking into account (2.2) and (2.7) we will receive

$$\frac{dN_1(t)}{dt}=[p(\alpha_1-\alpha_2)-(\alpha_1-\alpha_3)(p+1)]N_1^{\,2}(t)+[(\alpha_1-\alpha_3)a-\beta_1]N_1(t).$$

We will finally receive the exact solution of a task of Cauchy (2.2) in case of performance (2.5)

$$\begin{cases} N_1(t)=\dfrac{\dfrac{(\alpha_1-\alpha_3)a-\beta_1}{\alpha_1-\alpha_3+p(\alpha_2-\alpha_3)}N_{10}e^{[(\alpha_1-\alpha_3)a-\beta_1]t}}{\dfrac{(\alpha_1-\alpha_3)a-\beta_1}{\alpha_1-\alpha_3+p(\alpha_2-\alpha_3)}+N_{10}(e^{[(\alpha_1-\alpha_3)a-\beta_1]t}-1)} \\ N_2(t)=pN_1(t) \\ N_3(t)=a-(p+1)N_1(t) \end{cases} \tag{2.8}$$

The nontrivial model (previously not predicted result) will be in a case

$$\dot{N}_1(t)>0, \dot{N}_2(t)>0, \dot{N}_3(t)<0\ . \tag{2.9}$$

We will enter designation

$$\delta=(\alpha_1-\alpha_3)a-\beta_1,, \tag{2.10}$$

$$q=\frac{(\alpha_1-\alpha_3)a-\beta_1}{\alpha_1-\alpha_3+p(\alpha_2-\alpha_3)} \tag{2.11}$$

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Then from (2.8) we will receive

$$\dot{N}_1(t) = \frac{qN_{10}[\delta e^{\delta t}(q - N_{10} + N_{10}e^{\delta t}) - \delta N_{10}e^{2\delta t}]}{(q - N_{10} + N_{10}e^{\delta t})^2} = \frac{q\delta(q - N_{10})N_{10}e^{\delta t}}{(q - N_{10} + N_{10}e^{\delta t})^2} . \qquad (2.12)$$

It agrees (2.12) obviously that

$$sign\dot{N}_1(t) = sign(q\delta(q - N_{10})) \ , \qquad (2.13)$$

taking into account (2.10) and (2.11) we will receive

$$q\delta = \frac{[(\alpha_1 - \alpha_3)a - \beta_1]^2}{\alpha_1 - \alpha_3 + (\alpha_2 - \alpha_3)p} = \frac{\delta^2}{\alpha_1 - \alpha_3 + (\alpha_2 - \alpha_3)p} \ . \qquad (2.14)$$

We will consider several various cases

1. $\alpha_1 \geq \alpha_2 > \alpha_3 > 0$

i.e. we have a case of two strong opposition parties and taking into account (2.14) we will receive

$$q\delta > 0,$$

then from (2.13), we will receive

$$sign\dot{N}_1(t) = sign(q - N_{10}),$$

thus, for performance of a condition (2.9), it is necessary

$$q > N_{10},$$

or

$$\frac{(\alpha_1 - \alpha_3)a - \beta_1}{\alpha_1 - \alpha_3 + p(\alpha_2 - \alpha_3)} > N_{10},$$

$$(\alpha_1 - \alpha_3)a - \beta_1 > (\alpha_1 - \alpha_3)N_{10} + N_{20}(\alpha_2 - \alpha_3),$$

taking into account (2.3) and (2.7), we will receive

$$\beta_1 < (\alpha_1 - \alpha_2)N_{20} + (\alpha_1 - \alpha_3)N_{30},$$

at the same time according to an assumption (2.5)

$$\beta_1 > (\alpha_1 - \alpha_2)a \ ,$$

thus we will receive nontrivial model when performing the following double inequality

$$(\alpha_1 - \alpha_2)a < \beta_1 < (\alpha_1 - \alpha_2)N_{20} + (\alpha_1 - \alpha_3)N_{30} . \qquad (2.16)$$

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2. $\alpha_1 > \alpha_3, \alpha_2 = \alpha_3$

In this case from (2.14) we will receive

$$q\delta = \frac{\delta^2}{\alpha_1 - \alpha_3} > 0,$$

and again

$$sign\dot{N}_1(t) = sign(q - N_{10}),$$

that the condition (2.9) was satisfied, it is necessary

$$q > N_{10},$$

and from (2.11) we will receive

$$q = \frac{(\alpha_1 - \alpha_3)a - \beta_1}{\alpha_1 - \alpha_3}.$$

We will consider when it is carried out

$$q = \frac{(\alpha_1 - \alpha_3)a - \beta_1}{\alpha_1 - \alpha_3} > N_{10},$$

$$(\alpha_1 - \alpha_3)a - \beta_1 > (\alpha_1 - \alpha_3)N_{10},$$

and from (2.3) we will receive

$$(\alpha_1 - \alpha_3)(N_{20} + N_{30}) - \beta_1 > 0. \quad (2.17)$$

If $\alpha_2 = \alpha_3$ then from (2.5) we will receive

$$\beta_1 = (\alpha_1 - \alpha_3)a + \beta_2, \quad (2.18)$$

taking into account (2.3), (2.17), (2.18) we will receive

$$(\alpha_1 - \alpha_3)(N_{20} + N_{30}) - [(\alpha_1 - \alpha_3)a + \beta_2] > 0,$$

or

$$-\beta_2 - (\alpha_1 - \alpha_3)N_{10} > 0.$$

have come to an obvious contradiction that means that in this case we won't receive nontrivial model.

3. $\alpha_1 > \alpha_3 > \alpha_2$

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$$q\delta = \frac{\delta^2}{\alpha_1 - \alpha_3 + (\alpha_2 - \alpha_3)p}$$

the condition

$$q\delta > 0,$$

is satisfied if we assume that

$$\alpha_1 - \alpha_3 > (\alpha_3 - \alpha_2)p\,,$$

then again we will receive

$$sign\dot{N}_1(t) = sign(q - N_{10}),$$

receiving nontrivial model requires performance

$$q > N_{10}.$$

Thus performance of conditions is necessary at the same time

1) $\dfrac{(\alpha_1 - \alpha_3)a - \beta_1}{\alpha_1 - \alpha_3 + p(\alpha_2 - \alpha_3)} > N_{10}$

2) $(\alpha_1 - \alpha_2)a = \beta_1 - \beta_2$

3) $\alpha_1 - \alpha_3 > (\alpha_2 - \alpha_3)p$.

We will consider as far as it is possible

$$(\alpha_1 - \alpha_3)a - \beta_1 > (\alpha_1 - \alpha_3)N_{10} + (\alpha_2 - \alpha_3)pN_{10}$$

$$(\alpha_1 - \alpha_3)a - (\alpha_1 - \alpha_2)a - \beta_2 > (\alpha_1 - \alpha_3)N_{10} + (\alpha_2 - \alpha_3)pN_{10}$$

$$(\alpha_2 - \alpha_3)a - \beta_2 > N_{10}[(\alpha_1 - \alpha_3) + p(\alpha_2 - \alpha_3)],$$

but $\alpha_2 > \alpha_3$
and

$$(\alpha_2 - \alpha_3)a - \beta_2 < 0,$$

when takes place

$$N_{10}[(\alpha_1 - \alpha_3) + p(\alpha_2 - \alpha_3)] > 0.$$

Have again come to a contradiction, thus and in this case we won't receive nontrivial model.

Finally, have come to a conclusion that the model (2.2) at an assumption (2.5) will be nontrivial if conditions are satisfied :

$$\begin{cases} \alpha_1 \geq \alpha_2 > \alpha_3 > 0 \\ (\alpha_1 - \alpha_2)a < \beta_1 < (\alpha_1 - \alpha_2)N_{20} + (\alpha_1 - \alpha_3)N_{30} \end{cases} . \qquad (2.19)$$

3. Computer modeling of three-party elections

We will consider Cauchy's task (1.1), (1.2) generally when change model parameters, and also change of so-called selective demographic factor is considered.

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As it has been already noted, the decision of nonlinear system of the differential equations (1.1), (1.2) gives the chance to show dynamics of possible, estimated votes from elections to elections and to define estimated voices of three selective subjects in case of 100% of a voter turnout $(N_1(T), N_2(T), N_3(T))$.

It is clear that such voter turnout on elections isn't real. Therefore it is necessary to take into account an assessment of the indicator of appearances at elections and in case of the non-democratic countries some falsification of voices of opposition parties.

Computer modeling of the offered mathematical model allows to consider these processes and to make the corresponding amendments of the received results.

In model the case of variable coefficients is considered, in particular, we assume that from choices to choices coefficients of attraction of votes are exponential increasing function of time.

$$\alpha_i(t) = \alpha_{i0} e^{\delta_i \frac{t}{T}}, i = \overline{1,3}, \tag{3.1}$$

where

$$\alpha_{i0} > 0, \delta_i > 0, i = \overline{1,3} \ .$$

For $\beta_1(t), \beta_2(t)$ functions of use of administrative resources and so-called demographic coefficients $\gamma_1(t), \gamma_2(t), \gamma_3(t)$ cases are considered:

$$\beta_j(t) = \beta_{j0} e^{\delta_j \frac{t}{T}}, \beta_j > 0, \delta_j > 0, j = \overline{1,2}. \tag{3.2}$$

$$\gamma_i(t) = \gamma_{i0} e^{\delta_i \frac{t}{T}}, \delta_i > 0, i = \overline{1,3}.$$

Then the system (1.1) will take a form

$$\begin{cases} \dfrac{dN_1(t)}{dt} = N_1(t)((\alpha_{10} e^{\delta_1 \frac{t}{T}} - \alpha_{20} e^{\delta_2 \frac{t}{T}}) N_2(t) + (\alpha_{10} e^{\delta_1 \frac{t}{T}} - \alpha_{30} e^{\delta_3 \frac{t}{T}}) N_3(t) - \beta_{10} e^{\delta_1 \frac{t}{T}} + \gamma_{10} e^{\delta_1 \frac{t}{T}}) \\ \dfrac{dN_2(t)}{dt} = N_2(t)((\alpha_{20} e^{\delta_2 \frac{t}{T}} - \alpha_{10} e^{\delta_1 \frac{t}{T}}) N_1(t) + (\alpha_{20} e^{\delta_2 \frac{t}{T}} - \alpha_{30} e^{\delta_3 \frac{t}{T}}) N_3(t) - \beta_{20} e^{\delta_2 \frac{t}{T}} + \gamma_{20} e^{\delta_2 \frac{t}{T}}) \\ \dfrac{dN_3(t)}{dt} = N_3(t)((\alpha_{30} e^{\delta_3 \frac{t}{T}} - \alpha_{10} e^{\delta_1 \frac{t}{T}}) N_1(t) + (\alpha_{30} e^{\delta_3 \frac{t}{T}} - \alpha_{20} e^{\delta_2 \frac{t}{T}}) N_2(t) + \gamma_{30} e^{\delta_3 \frac{t}{T}}) + \beta_{10} e^{\delta_1 \frac{t}{T}} N_1(t) + \beta_{20} e^{\delta_2 \frac{t}{T}} N_2(t) \end{cases} \tag{3.3}$$

We will enter designations:

k_1 — relative value of the supporters of first opposition who voted in the election day from all number of the voters supporting first opposition party;

k_2 — relative value of the supporters of second opposition who voted in the election day from all number of the voters supporting second opposition party;

k_3 — relative value of the supporters of ruling party who voted in the election day from all number of the voters supporting pro-government party;

f_1 — relative value of the forged voices of the first opposition party;

f_2 — relative value of the forged voices of the second opposition party.

We consider two cases: when the elections are held, without falsification, and when in the falsification takes place during the elections (scenario of model, picture 1.).

Numerous numerical experiment (1.2), (3.1)-(3.3) is made and the corresponding graphics (visualization) are received. For example, in fig. 1 and fig. 2 cases of various relative turnout of voters of three parties are given, falsification and so-called demographic factor.

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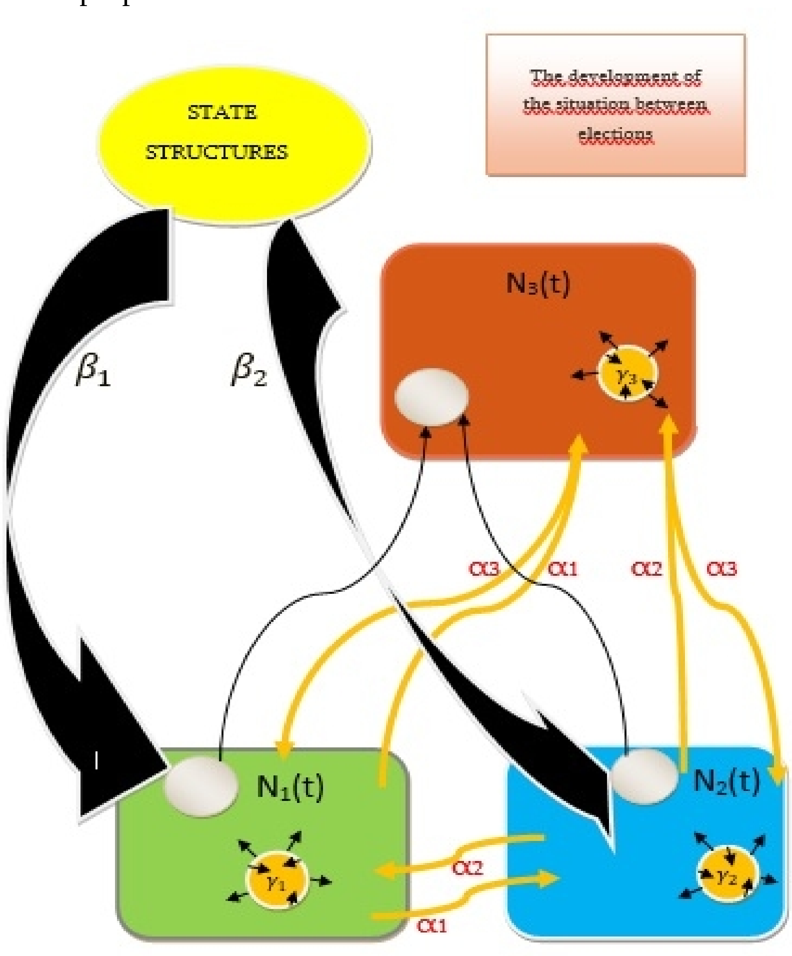
We proposed a mathematical model has both theoretical and practical importance.Political opponents (the power and opposition parties) by means of intellectual (programs of action), financial (sponsors), information (the print and electronic media which is under their department) means can widely use the results received by us and calculate parameters and choose strategy for achievement of the desired purposes.



Picture 1. Scenario of model

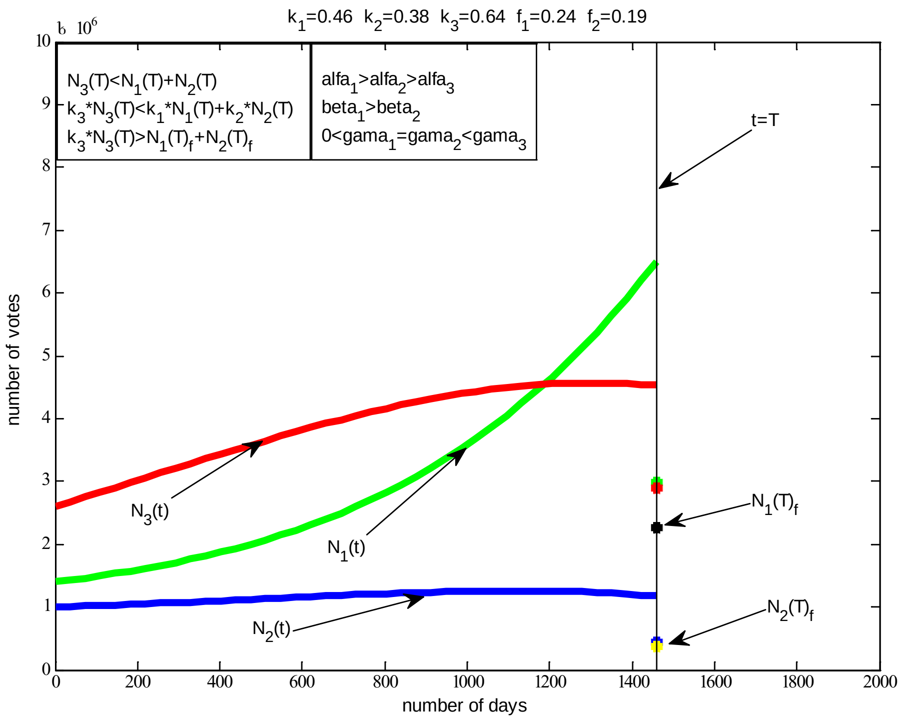


Fig.1

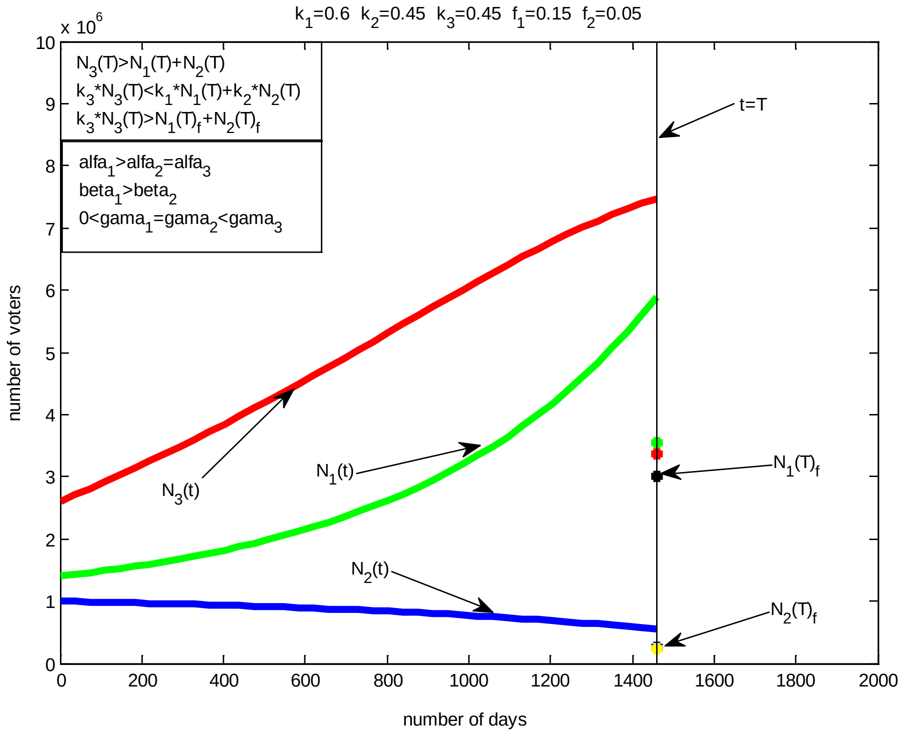


Fig.2

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