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A STUDY ON INTERVAL-VALUED FUZZY GRAPHS

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Abstract

Fuzzy graphs which have revolutionized the analysis of problematic data to arrive at a better decision making power are of different kinds. Among them, the simplest and generalized form is the interval-valued fuzzy graph. The main purpose of this paper is to introduce the notion of an interval-valued p-morphism on interval-valued fuzzy graphs. The action of interval-valued p-morphism on interval-valued strong regular graphs are studied and proved elegant theorems on weak and co weak isomorphism. Also μ-complement of highly irregular product interval-valued fuzzy graphs, α-cut and strength cut graph of an interval-valued fuzzy graphs are discussed.

Keywords: *Interval-valued p-morphism, μ-complement of a product interval-valued fuzzy graph, α-cut and strength cut graph of an interval-valued fuzzy graph.*

1. Introduction

The moment science is involved in finding solutions to theoretical problems mathematical dependency increases. It has been proved by researchers that frameworks of analysis developed using mathematical models, especially those based on fuzzy logic were capable of handling uncertain data sets. The new mathematical models have shown their superiority over the conventional fuzzy logic based sets. Issues of doubt, data inconsistency and wrong or mismatched data were effectively analyzed with the help of graph theory in the various domains like mechanical engineering, electrical engineering, with special focus on machine systems. The self-learning feature of these new mathematical models has given the possibility of scaling the size of the operations to suit the requirement industrial requirement. In short, mathematical models based on fuzzy graph theory have simplified the handling of problematic to arrive at rational conclusions.

The problem of Konigsberg bridge, in eighteenth century laid the foundation to graph theory, where Euler strongly suggested that there is a solution using graph theory. In 1965, Zadeh [18-20] developed a fuzzy set theory using the real numbers between 0 and 1. Akram and Dudek [1] extended the fuzzy set theory to interval-valued fuzzy sets. Further Rashamanlou et al. [5-8] studied degree of a vertex, regular, highly irregular, strong, complete, balanced, antipodal interval-valued fuzzy graphs. Talebi and Rashamanlou [17] investigated homomorphism, isomorphism, weak, co weak isomorphism, self complement of interval-valued fuzzy graphs. Samanta and pal [9-16] studied strong edge, weak edge of an interval-valued fuzzy graph, degree of a vertex in bipolar fuzzy graph, regular, irregular bipolar fuzzy graphs, bipolar fuzzy hyper graphs, fuzzy k-competition, p-competition of fuzzy graphs, m-step fuzzy competition graphs, fuzzy planar graphs, tolerance, threshold fuzzy graphs. Pramanik et al. [4] introduced interval valued fuzzy planar graphs. In 2016 Narayanan and Maheswari [3] studied strongly edge irregular interval-valued fuzzy graphs. Borzooei et al. [2] studied regularity of vague graphs.

2. Preliminaries

Some definitions and conventions used in this paper are discussed in this section. Literature review is available in [1, 4, 5, 17].

Definition 2.1 A graph $G=(V,E)$ is an ordered pair consisting of a non-empty vertex set V, an edge set E and a connection that associates with every edge between two vertices (not as a matter of course particular) called its end points.

ISSN 1512-1232

Definition 2.2 Let $G=(V,E)$ be a graph. Then $S=(N,L)$ is said to be a subgraph of G if $N \subseteq V$ and $L \subseteq E$.

Definition 2.3 A fuzzy set A on a universal set X is characterized by function $m: X \to [0,1]$, which is called the membership function. A fuzzy set is denoted by $A=(X,m)$.

Definition 2.4 A fuzzy graph $\delta=(V,\sigma,\mu)$ is a non-empty set V together with a pair of functions $\sigma: V \to [0,1]$ and $\mu: V \times V \to [0,1]$ such that for all $m,n \in V$, $\mu(mn) \leq$ min $\{\sigma(m),\sigma(n)\}$, where $\sigma(m)$ and $\mu(mn)$ represent the membership values of the vertex m and of the edge mn in δ respectively. The underlying crisp graph of the fuzzy graph $\delta=(V,\sigma,\mu)$ is denoted as $\delta^*=(V,\sigma^*,\mu^*)$, where $\sigma^*=\{x \in V / \sigma(x)>0\}$ and $\mu^*=\{xy \in V \times V / \mu(xy)>0\}$. Thus for underlying fuzzy graph $\sigma^*=V$.

Definition 2.5 A fuzzy graph $\delta=(V,\sigma,\mu)$ is complete if $\mu(xy)=\min\{\sigma(x),\sigma(y)\}$ for all $x,y \in V$, where xy denotes the edge between the vertices x and y. The fuzzy graph $\delta_a=(V,\sigma_a,\mu_a)$ is called a fuzzy subgraph of $\delta=(V,\sigma,\mu)$ if $\sigma_a(m) \leq \sigma(m)$ for all $m \in V$ and $\mu_a(mn) \leq \mu(mn)$ for all edges $mn \in E$.

Definition 2.6 The interval-valued fuzzy set W in V is defined by $W=\left\{\left(x,\left[\left[\mu_W^-(x),\mu_W^+(x)\right]/x\in V\right]\right)\right\}$. where $\mu_W^-(x)$, $\mu_W^+(x)$ are fuzzy subsets of V such that $\mu_W^-(x) \leq \mu_W^+(x)$ for all $x \in V$. If $G^*=(V,E)$ is a crisp graph, then by an interval -valued fuzzy relation F on a set E we mean an interval-valued fuzzy set such that $\mu_F^-(xy) \leq \min\left\{\mu_W^-(x),\mu_W^-(y)\right\}$, $\mu_F^+(xy) \leq \min\left\{\mu_W^+(x),\mu_W^+(y)\right\}$, for all $x, y \in V$.

Definition 2.7 The interval-valued fuzzy graph is a pair $G=(W,F)$ of a graph $G^*=(V,E)$, where $W=\left[\mu_W^-,\mu_W^+\right]$ is an interval-valued fuzzy set on V and $F=\left[\mu_F^-,\mu_F^+\right]$ is an interval-valued fuzzy relation on E such that $\mu_F^-(xy) \leq \min\left\{\mu_W^-(x),\mu_W^-(y)\right\}$, $\mu_F^+(xy) \leq \min\left\{\mu_W^+(x),\mu_W^+(y)\right\}$ for all $xy \in E$.

The underlying crisp graph of an interval-valued graph $G=(W,F)$ is the graph $G^*=(V,E)$ where $V=\left\{\left(w/\left[\left[\mu_W^-(w),\mu_W^+(w)\right]>[0,0]\right]\right)\right\}$ is called vertex set and $E=\left\{\left(wf/\left[(\mu_F^-(wf),\mu_F^+(wf))>[0,0]\right]\right)\right\}$ is called an edge set.

Definition 2.8 Let $G=(W,F)$ be an interval-valued fuzzy graph of $G^*=(V,E)$. Then $G=(W,F)$ is said to be strong if $\mu_F^-(xy)=\min\left\{\mu_W^-(x),\mu_W^-(y)\right\}$, $\mu_F^+(xy)=\min\left\{\mu_W^+(x),\mu_W^+(y)\right\}$, for all $xy \in E$.

Definition 2.9 Let $G=(W,F)$ be an interval-valued fuzzy graph of $G^*=(V,E)$. Then $G=(W,F)$ is said to be complete if $\mu_F^-(xy)=\min\left\{\mu_W^-(x),\mu_W^-(y)\right\}$, $\mu_F^+(xy)=\min\left\{\mu_W^+(x),\mu_W^+(y)\right\}$, for all $x,y \in V$.

Definition 2.10 The complement of an interval-valued fuzzy graph $G=(W,F)$ of a graph $G^*=(V,E)$ is an interval-valued fuzzy graph $\overline{G}=\left(\overline{W},\overline{F}\right)$, where $\overline{W}=W=\left[\mu_W^-(x),\mu_W^+(x)\right]$ and $\overline{F}=\left[\overline{\mu_F^-}(x),\overline{\mu_F^+}(x)\right]$, here

ISSN 1512-1232

$\overline{\mu_F^-}(xy) = \min\{\mu_W^-(x), \mu_W^-(y)\} - \mu_F^-(xy)$, $\overline{\mu_F^+}(xy) = \min\{\mu_W^+(x), \mu_W^+(y)\} - \mu_F^+(xy)$ for all $x, y \in V$.

Definition 2.11 Let $G_1 = (W_1, F_1)$ and $G_2 = (W_2, F_2)$ be two interval-valued fuzzy graphs on $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ respectively, such that

A homomorphism p from G_1 to G_2 is a mapping $p: V_1 \to V_2$ which satisfies the following conditions:

$\mu_{W_1}^-(u) \le \mu_{W_2}^-(p(u)), \mu_{W_1}^+(u) \le \mu_{W_2}^+(p(u)),\ \forall u \in V_1,$

$\mu_{F_1}^-(uv) \le \mu_{F_2}^-(p(u)p(v)), \mu_{F_1}^+(uv) \le \mu_{F_2}^+(p(u)p(v)),\ \forall uv \in E_1.$

An isomorphism p from G_1 to G_2 is a bijective mapping $p: V_1 \to V_2$ which satisfies the following conditions:

$\mu_{W_1}^-(u) = \mu_{W_2}^-(p(u)), \mu_{W_1}^+(u) = \mu_{W_2}^+(p(u)),\ \forall u \in V_1,$

$\mu_{F_1}^-(uv) = \mu_{F_2}^-(p(u)p(v)), \mu_{F_1}^+(uv) = \mu_{F_2}^+(p(u)p(v)),\ \forall uv \in E_1.$

A weak isomorphism p from G_1 to G_2 is a bijective mapping $p: V_1 \to V_2$ which satisfies the following conditions:

p is homomorphism, $\mu_{W_1}^-(u) = \mu_{W_2}^-(p(u)), \mu_{W_1}^+(u) = \mu_{W_2}^+(p(u)),\ \forall u \in V_1.$

A co weak isomorphism p from G_1 to G_2 is a bijective mapping $p: V_1 \to V_2$ which satisfies the following conditions:

p is homomorphism, $\mu_{F_1}^-(uv) = \mu_{F_2}^-(p(u)p(v)), \mu_{F_1}^+(uv) = \mu_{F_2}^+(p(u)p(v)),\ \forall uv \in E_1.$

Definition 2.12 Let $G = (W, F)$ be an interval-valued fuzzy graph, where $W = [\mu_W^-, \mu_W^+]$ and $F = [\mu_F^-, \mu_F^+]$. The degree of a vertex is defined as $d_G = (d_G^-(w), d_G^+(w))$ where, $d_G^-(w) = \sum \mu_F^-(wx),\ wx \in E,\ w \ne x$ is the negative degree of a vertex w and $d_G^+(w) = \sum \mu_F^+(wx),\ wx \in E,\ w \ne x$ is the positive degree of a vertex w.

Definition 2.13 The order of G is defined as $O(G) = \left(\sum_{w \in V} \mu_W^-(w), \sum_{w \in V} \mu_W^+(w) \right)$.

Definition 2.14 The size of G is defined as $S(G) = (S_G^-, S_G^+) = \left(\sum_{\substack{x \ne y \\ x,y \in V}} \mu_F^-(xy), \sum_{\substack{x \ne y \\ x,y \in V}} \mu_F^+(xy) \right)$.

3. Regularity on isomorphic interval-valued fuzzy graph

In this section, we introduced an interval-valued p-morphism on interval-valued fuzzy graphs and regular interval-valued fuzzy graph, we proved some important theorems on weak and co weak isomorphism.

Definition 3.1 Let $G_1 = (W_1, F_1)$ and $G_2 = (W_2, F_2)$ be two interval-valued fuzzy graphs on $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ respectively. A bijective function $P: V_1 \to V_2$ is called interval-valued morphism or interval-valued p-morphism if there exists two numbers $l_1 > 0$ and $l_2 > 0$, such that $\mu_{W_2}^-(p(u)) = l_1 \mu_{W_1}^-(u), \mu_{W_2}^+(p(u)) = l_1 \mu_{W_1}^+(u),\ \forall u \in V_1,$

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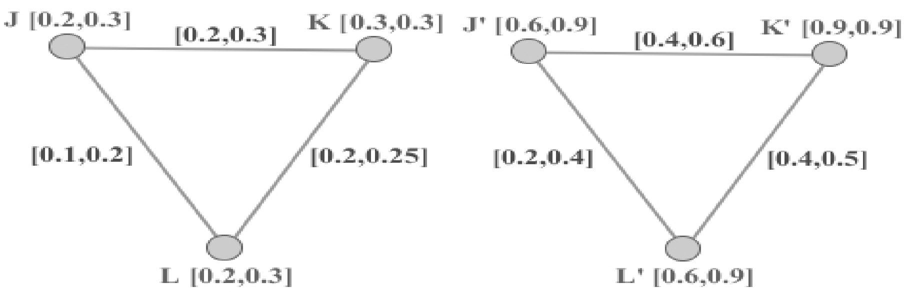
$\mu_{F_2}^-\left(p(u)p(v)\right)=l_2\mu_{F_1}^-(uv),\mu_{F_2}^+\left(p(u)p(v)\right)=l_2\mu_{F_1}^+(uv),\ \forall uv\in E_1$. In such a case, p will be called a (l_1,l_2) interval-valued p-morphism from G_1 to G_2. If $l_1=l_2=l$, we call p, an interval-valued p-morphism.

Example 3.1 Consider an interval-valued fuzzy graph $G_1=(W_1,F_1)$ shown in Figure 1(a). Then

$$W_1=\left\{\frac{[0.2,0.3]}{J},\frac{[0.3,0.3]}{K},\frac{[0.2,0.3]}{L}\right\}\text{and } F_1=\left\{\frac{[0.2,0.3]}{JK},\frac{[0.2,0.25]}{KL},\frac{[0.1,0.2]}{LJ}\right\}.$$

Again consider another interval-valued fuzzy graph $G_2=(W_2,F_2)$ shown in Figure 1(b). Then

$$W_2=\left\{\frac{[0.6,0.9]}{J'},\frac{[0.9,0.9]}{K'},\frac{[0.6,0.9]}{L'}\right\}\text{and } F_2=\left\{\frac{[0.4,0.6]}{J'K'},\frac{[0.4,0.5]}{K'L'},\frac{[0.2,0.4]}{L'J'}\right\}.$$



(a) G_1 **(b)** G_2

Figure 1: p-morphism between interval-valued fuzzy graphs G_1 and G_2

Here, there is an interval-valued p-morphism such that $p(J)=J',p(K)=K',p(L)=L',l_1=3,\ l_2=2$.

Definition 3.2 Let $G=(W,F)$ be a connected interval-valued fuzzy graph .Then G is said to be a highly irregular interval-valued fuzzy graph if every vertex of G is adjacent to vertices with distinct degrees.

Example 3.2 Consider an interval-valued fuzzy graph $G=(W,F)$ shown in Figure 2.

$$W=\left\{\frac{[0.3,0.4]}{J},\frac{[0.2,0.5]}{K},\frac{[0.5,0.5]}{L},\frac{[0.4,0.7]}{M}\right\}\text{and}$$

$$F=\left\{\frac{[0.1,0.3]}{JK},\frac{[0.1,0.2]}{KL},\frac{[0.3,0.4]}{LM},\frac{[0.2,0.3]}{MJ}\right\}.$$

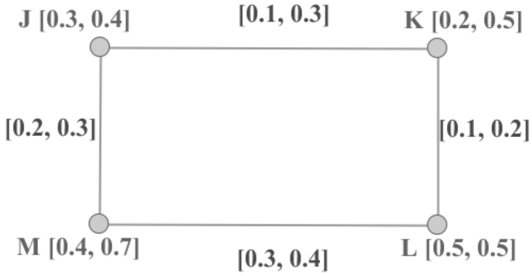


Figure 2:A highly irregular interval-valued fuzzy graph

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Then $d_G(J)=(0.3,0.6), d_G(K)=(0.2,0.5), d_G(L)=(0.4,0.6), d_G(M)=(0.5,0.7)$.

We see that every vertex of G is adjacent to vertices with distinct degrees. Hence $G=(W,F)$ is highly irregular interval-valued fuzzy graph.

Theorem 3.1 The relation p-morphism is an equivalence relation in the collection of interval-valued fuzzy graphs.

Proof Consider the collection of all interval-valued fuzzy graphs. Define the relation $G_1 \approx G_2$ if there exists a (l_1, l_2) p-morphism from G_1 to G_2 where both $l_1 \neq 0$ and $l_2 \neq 0$. Consider the identity morphism G_1 to G_1. It is a $(1, 1)$ morphism from G_1 to G_1 and hence $\approx$ is reflexive.

Let $G_1 \approx G_2$. Then there exists a $(l_1, l_2)-$morphism from G_1 to G_2 for some $l_1 \neq 0$ and $l_2 \neq 0$. Therefore $\mu^-_{w_2}(p(u)) = l_1\mu^-_{w_1}(u)$, $\mu^+_{w_2}(p(u)) = l_1\mu^+_{w_1}(u)$, $\forall u \in V_1$ and $\mu^-_{F_2}(p(u)p(v)) = l_2\mu^-_{F_1}(uv)$, $\mu^+_{F_2}(p(u)p(v)) = l_2\mu^+_{F_1}(uv)$, $\forall uv \in E_1$.

Consider $p^{-1}: G_2 \to G_1$. Let $m, n \in V_2$. Since p^{-1} is bijective, we have

$m = p(u)$, $n = p(v)$, for some $u, v \in V_2$.

Now $\mu^-_{W_1}(p^{-1}(m)) = \mu^-_{W_1}(p^{-1}(p(u))) = \mu^-_{W_1}(u) = \frac{1}{l_1}\mu^-_{W_2}(p(u)) = \frac{1}{l_1}\mu^-_{W_2}(m)$,

$$\mu^+_{W_1}(p^{-1}(m)) = \mu^+_{W_1}(p^{-1}(p(u))) = \mu^+_{W_1}(u) = \frac{1}{l_1}\mu^+_{W_2}(p(u)) = \frac{1}{l_1}\mu^+_{W_2}(m).$$

$$\mu^-_{F_1}(p^{-1}(m)p^{-1}(n)) = \mu^-_{F_1}(p^{-1}(p(u))p^{-1}(p(v))) = \mu^-_{F_1}(uv) = \frac{1}{l_2}\mu^-_{F_2}(p(u)p(v)) = \frac{1}{l_2}\mu^-_{F_2}(mn),$$

$$\mu^+_{F_1}(p^{-1}(m)p^{-1}(n)) = \mu^+_{F_1}(p^{-1}(p(u))p^{-1}(p(v))) = \mu^+_{F_1}(uv) = \frac{1}{l_2}\mu^+_{F_2}(p(u)p(v)) = \frac{1}{l_2}\mu^+_{F_2}(mn).$$

Thus, there exists $\left(\frac{1}{l_1}, \frac{1}{l_2}\right)-$morphism from G_2 to G_1. Therefore, $G_2 \approx G_1$ and hence $\approx$ is symmetric.

Let $G_1 \approx G_2$ and $G_2 \approx G_3$. Then, there exists a $(l_1, l_2)-$morphism from G_1 to G_2 say p for some $l_1 \neq 0$, $l_2 \neq 0$ and there exists $(l_3, l_4)-$ morphism from G_2 to G_3 say q for some $l_3 \neq 0$ and $l_4 \neq 0$. So $\mu^-_{w_3}(q(x)) = l_3\mu^-_{w_2}(x)$, $\mu^+_{w_3}(q(x)) = l_3\mu^+_{w_2}(x)$, $\forall x \in V_2$,

and $\mu^-_{F_3}(q(x)q(y)) = l_4\mu^-_{F_2}(xy)$, $\mu^+_{F_3}(q(x)q(y)) = l_4\mu^+_{F_2}(xy)$, $\forall xy \in E_2$.

Let $r: q \circ p: G_1 \to G_3$ be a mapping.

Now, $\mu^-_{w_3}(r(u)) = \mu^-_{w_3}((q \circ p)(u)) = \mu^-_{w_3}(q(p(u))) = l_3\mu^-_{w_2}(p(u)) = l_3l_1\mu^-_{w_1}(u)$,

$\mu^+_{w_3}(r(u)) = \mu^+_{w_3}((q \circ p)(u)) = \mu^+_{w_3}(q(p(u))) = l_3\mu^+_{w_2}(p(u)) = l_3l_1\mu^+_{w_1}(u)$.

$\mu^-_{F_3}(r(u)r(v)) = \mu^-_{F_3}((q \circ p)(u)(q \circ p)(v)) = \mu^-_{F_3}(q(p(u))q(p(v)))$

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$$= l_4 \mu_{F_2}^{-}\left(p(u)p(v)\right) = l_4 l_2 \mu_{F_1}^{-}(uv),$$

$$\mu_{F_3}^{+}\left(r(u)r(v)\right) = \mu_{F_3}^{+}\left((q\circ p)(u)(q\circ p)(v)\right) = \mu_{F_3}^{+}\left(q\left(p\left(u\right)\right)q\left(p\left(v\right)\right)\right)$$

$$= l_4 \mu_{F_2}^{+}\left(p(u)p(v)\right) = l_4 l_2 \mu_{F_1}^{+}(uv).$$

Thus, there exists $(l_3 l_1, l_4 l_2)-$morphism r from G_1 to G_3. Therefore, $G_1 \approx G_3$ and hence, $\approx$ is transitive.

So the relation p -morphism is an equivalence relation in the collection of all interval-valued fuzzy graphs.

Theorem 3.2 Let G_1 and G_2 be two interval-valued fuzzy graphs such that G_1 is (l_1, l_2) interval-valued morphism to G_2 for some $l_1 \neq 0$ and $l_2 \neq 0$ .The image of strong edge in G_1 is also strong edge in G_2 if and only if $l_1 = l_2$.

Proof Let xy be strong edge in G_1 such that $p(x)p(y)$ is also strong edge in G_2. Now as $G_1 \approx G_2$, we have

$$\begin{aligned} l_2 \mu_{F_1}^{-}(xy) = \mu_{F_2}^{-}\left(p(x)p(y)\right) &= \mu_{W_2}^{-}\left(p(x) \wedge p(y)\right) \\ &= l_1 \mu_{W_1}^{-}(x) \wedge l_1 \mu_{W_1}^{-}(y) \\ &= l_1\left(\mu_{W_1}^{-}(x) \wedge \mu_{W_1}^{-}(y)\right) = l_1 \mu_{F_1}^{-}(xy) \end{aligned}$$

Hence, $l_2 \mu_{F_1}^{-}(xy) = l_1 \mu_{F_1}^{-}(xy), \quad \forall xy \in E_1$.

$$\begin{aligned} l_2 \mu_{F_1}^{+}(xy) = \mu_{F_2}^{+}\left(p(x)p(y)\right) &= \mu_{W_2}^{+}\left(p(x) \wedge p(y)\right) \\ &= l_1 \mu_{W_1}^{+}(x) \wedge l_1 \mu_{W_1}^{+}(y) \\ &= l_1\left(\mu_{W_1}^{+}(x) \wedge \mu_{W_1}^{+}(y)\right) = l_1 \mu_{F_1}^{+}(xy) \end{aligned}$$

Hence, $l_2 \mu_{F_1}^{+}(xy) = l_1 \mu_{F_1}^{+}(xy), \quad \forall xy \in E_1$.

The equation holds if and only if $l_1 = l_2$.

Theorem 3.3 If the interval-valued fuzzy graph G_1 is co weak isomorphic to G_2 and if G_1 is regular then G_2 is also regular.

Proof As an interval-valued fuzzy graph G_1 is co weak isomorphic to G_2. Then there exists a co-weak isomorphism $p: G_1 \to G_2$ which is bijective that satisfies $\mu_{W_1}^{-}(u) \le \mu_{W_2}^{-}\left(p(u)\right), \mu_{W_1}^{+}(u) \le \mu_{W_2}^{+}\left(p(u)\right), \ \forall u \in V_1,$
$\mu_{F_1}^{-}(uv) = \mu_{F_2}^{-}\left(p(u)p(v)\right), \mu_{F_1}^{+}(uv) = \mu_{F_2}^{+}\left(p(u)p(v)\right), \ \forall uv \in E_1.$ As G_1 is regular, for $u \in V_1$, $\sum_{u \neq v, v \in V_1} \mu_{F_1}^{-}(uv) =$ constant and $\sum_{u \neq v, v \in V_1} \mu_{F_1}^{+}(uv) =$ constant.

ISSN 1512-1232

Now $\sum_{p(u)\neq p(v)} \mu_{F_2}^{-}\left(p(u)p(v)\right) = \sum_{u\neq v, v\in V_1} \mu_{F_1}^{-}(uv) =$ constant and

$\sum_{p(u)\neq p(v)} \mu_{F_2}^{+}\left(p(u)p(v)\right) = \sum_{u\neq v, v\in V_1} \mu_{F_1}^{+}(uv) =$ constant. Therefore G_2 is regular.

Theorem 3.4 Let G_1 and G_2 be two interval-valued fuzzy graphs. If G_1 is weak isomorphic to G_2 and if G_1 is strong then G_2 is also strong.

Proof As G_1 is an interval-valued fuzzy graph is weak isomorphic with G_2 .Then there exists a weak isomorphism $p: G_1 \to G_2$ which is bijective that satisfies
$\mu_{w_1}^{-}(u) = \mu_{w_2}^{-}\left(p(u)\right), \mu_{w_1}^{+}(u) = \mu_{w_2}^{+}\left(p(u)\right),\ \forall u \in V_1$ and
$\mu_{F_1}^{-}(uv) \leq \mu_{F_2}^{-}\left(p(u)p(v)\right), \mu_{F_1}^{+}(uv) \leq \mu_{F_2}^{+}\left(p(u)p(v)\right),\ \forall uv \in E_1$. As G_1 is strong, we have $\mu_{F_1}^{-}(uv) = \min\left(\mu_{W_1}^{-}(u), \mu_{W_1}^{-}(v)\right)$ and $\mu_{F_1}^{+}(uv) = \min\left(\mu_{W_1}^{+}(u), \mu_{W_1}^{+}(v)\right)$. Now, we get

$\mu_{F_2}^{-}\left(p(u)p(v)\right) \geq \mu_{F_1}^{-}(uv) = \min\left(\mu_{W_1}^{-}(u), \mu_{W_1}^{-}(v)\right) = \min\left(\mu_{W_2}^{-}(p(u)), \mu_{W_2}^{-}(p(v))\right).$

By the definition, $\mu_{F_2}^{-}\left(p(u)p(v)\right) \leq \min\left(\mu_{W_2}^{-}(p(u)), \mu_{W_2}^{-}(p(v))\right)$.

Therefore, $\mu_{F_2}^{-}\left(p(u)p(v)\right) = \min\left(\mu_{W_2}^{-}(p(u)), \mu_{W_2}^{-}(p(v))\right).$

Similarly, $\mu_{F_2}^{+}\left(p(u)p(v)\right) \geq \mu_{F_1}^{+}(uv) = \min\left(\mu_{W_1}^{+}(u), \mu_{W_1}^{+}(v)\right) = \min\left(\mu_{W_2}^{+}(p(u)), \mu_{W_2}^{+}(p(v))\right).$

By the definition, $\mu_{F_2}^{+}\left(p(u)p(v)\right) \leq \min\left(\mu_{W_2}^{+}(p(u)), \mu_{W_2}^{+}(p(v))\right).$

Therefore $\mu_{F_2}^{+}\left(p(u)p(v)\right) = \min\left(\mu_{W_2}^{+}(p(u)), \mu_{W_2}^{+}(p(v))\right).$ So G_2 is strong.

Theorem 3.5 If the interval-valued fuzzy graph G_1 is co weak isomorphic with a strong regular interval-valued fuzzy graph G_2, then G_1 is strong regular interval-valued fuzzy graph.

Proof As an interval-valued fuzzy graph G_1 is co weak isomorphic to G_2 , there exists a co-weak isomorphism $p: G_1 \to G_2$ which is bijective that satisfies
$\mu_{w_1}^{-}(u) \leq \mu_{w_2}^{-}\left(p(u)\right), \mu_{w_1}^{+}(u) \leq \mu_{w_2}^{+}\left(p(u)\right),\ \forall u \in V_1$ and
$\mu_{F_1}^{-}(uv) = \mu_{F_2}^{-}\left(p(u)p(v)\right), \mu_{F_1}^{+}(uv) = \mu_{F_2}^{+}\left(p(u)p(v)\right),\ \forall uv \in E_1$.
Now, we get

$\mu_{F_1}^{-}(uv) = \mu_{F_2}^{-}\left(p(u)p(v)\right) = \min\left(\mu_{w_2}^{-}\left(p(u)\right), \mu_{w_2}^{-}\left(p(v)\right)\right) \geq \min\left(\mu_{w_1}^{-}(u), \mu_{w_1}^{-}(v)\right)$

$\mu_{F_1}^{+}(uv) = \mu_{F_2}^{+}\left(p(u)p(v)\right) = \min\left(\mu_{w_2}^{+}(p(u)), \mu_{w_2}^{+}(p(v))\right) \geq \min\left(\mu_{w_1}^{+}(u), \mu_{w_1}^{+}(v)\right).$

But by the definition, $\mu_{F_1}^{-}(uv) \leq \min\left(\mu_{w_1}^{-}(u), \mu_{w_1}^{-}(v)\right),$

$\mu_{F_1}^{+}(uv) \leq \min\left(\mu_{w_1}^{+}(u), \mu_{w_1}^{+}(v)\right).$

ISSN 1512-1232

Hence $\mu_{F_1}^-(uv)=\min(\mu_{w_1}^-(u),\mu_{w_1}^-(v))$ and $\mu_{F_1}^+(uv)=\min(\mu_{w_1}^+(u),\mu_{w_1}^+(v))$. Therefore G_1 is strong. Also for $u\in V_1$, $\sum_{u\neq v, v\in V_1}\mu_{F_1}^-(uv)=\sum\mu_{F_2}^-(p(u)p(v))=$ constant as G_2 is regular and $\sum_{u\neq v}\mu_{F_1}^+(uv)=\sum\mu_{F_2}^+(p(u)p(v))=$ constant as G_2 is regular. Therefore G_1 is regular.

Theorem 3.6 Let G_1 and G_2 be two isomorphic interval-valued fuzzy graphs then G_1 is strong regular if and only if G_2 is strong regular.

Proof As an interval-valued fuzzy graph G_1 is isomorphic with an interval-valued fuzzy graph G_2, there exists an isomorphism $p:G_1\to G_2$ which is bijective and satisfies $\mu_{w_1}^-(u)=\mu_{w_2}^-(p(u)),\mu_{w_1}^+(u)=\mu_{w_2}^+(p(u)),\ \forall u\in V_1$,
$\mu_{F_1}^-(uv)=\mu_{F_2}^-(p(u)p(v)),\mu_{F_1}^+(uv)=\mu_{F_2}^+(p(u)p(v)),\ \forall uv\in E_1$. Now, G_1 is strong if and only if $\mu_{F_1}^-(uv)=\min(\mu_{w_1}^-(u),\mu_{w_1}^-(v))$ and $\mu_{F_1}^+(uv)=\min(\mu_{w_1}^+(u),\mu_{w_1}^+(v))$ if and only if $\mu_{F_2}^-(p(u)p(v))=\min(\mu_{w_2}^-(p(u)),\mu_{w_2}^-(p(v)))$, $\mu_{F_2}^+(p(u)p(v))=\min(\mu_{w_2}^+(p(u)),\mu_{w_2}^+(p(v)))$ if and only if G_2 is strong. G_1 is regular if and only if for $u\in V_1$, $\sum_{u\neq v,v\in V_1}\mu_{F_1}^-(uv)=$ constant and $\sum_{u\neq v,v\in V_1}\mu_{F_1}^+(uv)=$ constant, if and only if $\sum_{p(u)\neq p(v)}\mu_{F_2}^-(p(u)p(v))=$ constant and $\sum_{p(u)\neq p(v),p(v)\in V_2}\mu_{F_2}^+(p(u)p(v))=$constant, for all $p(u)\in V_2$ if and only if G_2 is regular.

Theorem 3.7 An interval-valued fuzzy graph G is strong regular if and only if its complement $\overline{G}$ is strong regular interval -valued fuzzy graph also.

Proof. The complement of an interval-valued fuzzy graph is defined as $\mu_w^-=\overline{\mu_w^-},\mu_w^+=\overline{\mu_w^+}$, $\overline{\mu_F^-}(xy)=\min\{\mu_W^-(x),\mu_W^-(y)\}-\mu_F^-(xy)$, $\overline{\mu_F^+}(xy)=\min\{\mu_W^+(x),\mu_W^+(y)\}-\mu_F^+(xy)$. As G is strong regular if and only if

$\overline{\mu_F^-}(xy)=\min\{\mu_W^-(x),\mu_W^-(y)\}-\mu_F^-(xy)=\mu_F^-(xy)-\mu_F^-(xy)=0,$

$\overline{\mu_F^+}(xy)=\min\{\mu_W^+(x),\mu_W^+(y)\}-\mu_F^+(xy)=\mu_F^+(xy)-\mu_F^+(xy)=0$ if and only if $\overline{\mu_F^-}(xy)=0$ and $\overline{\mu_F^+}(xy)=0$ if and only if $\overline{G}$ is strong regular interval-valued fuzzy graph.

Theorem 3.8 For any two isomorphic highly irregular interval-valued fuzzy graphs, their order and size are same.

Proof If $p:G_1\to G_2$ is an isomorphism between the two highly irregular interval-valued fuzzy graphs G_1 and G_2 with the underlying sets V_1 and V_2 respectively. Then

$\mu_{w_1}^-(u)=\mu_{w_2}^-(p(u)),\mu_{w_1}^+(u)=\mu_{w_2}^+(p(u)),\ \forall u\in V_1,$

$\mu_{F_1}^-(uv)=\mu_{F_2}^-(p(u)p(v)),\mu_{F_1}^+(uv)=\mu_{F_2}^+(p(u)p(v)),\ \forall uv\in E_1.$

So, we get

ISSN 1512-1232

$$O(G_1)=\left(\sum_{x_1\in V_1}\mu_{w_1}^{-}(x_1),\sum_{x_1\in V_1}\mu_{w_1}^{+}(x_1)\right)=\left(\sum_{x_1\in V_1}\mu_{w_2}^{-}(p(x_1)),\sum_{x_1\in V_1}\mu_{w_2}^{+}(p(x_1))\right)$$

$$=\left(\sum_{x_2\in V_2}\mu_{w_2}^{-}(x_2),\sum_{x_2\in V_2}\mu_{w_2}^{+}(x_2)\right)=O(G_2).$$

$$S(G_1)=\left(\sum_{x_1y_1\in E_1}\mu_{F_1}^{-}(x_1y_1),\sum_{x_1y_1\in E_1}\mu_{F_1}^{+}(x_1y_1)\right)=\left(\sum_{x_1\ y_1\in V_1}\mu_{F_2}^{-}(p(x_1)p(y_1)),\sum_{u_1\ y_1\in V_1}\mu_{F_2}^{+}(p(x_1)p(y_1))\right)$$

$$=\left(\sum_{x_2y_2\in E_2}\mu_{F_2}^{-}(x_2y_2),\sum_{x_2y_2\in E_2}\mu_{F_2}^{+}(x_2y_2)\right)=S(G_2).$$

Theorem 3.9 If G_1 and G_2 are isomorphic highly irregular interval-valued fuzzy graphs. Then, the degrees of the corresponding vertices u and $p(u)$ are preserved.

Proof If $p:G_1\to G_2$ is an isomorphism between the highly irregular interval-valued fuzzy graphs G_1 and G_2 with the underlying sets V_1 and V_2 respectively.

Then $\mu_{F_1}^{-}(x_1y_1)=\mu_{F_2}^{-}(p(x_1)p(y_1))$ and $\mu_{F_1}^{+}(x_1y_1)=\mu_{F_2}^{+}(p(x_1)p(y_1))\ \ \forall x_1,y_1\in V_1$.

Therefore $d_{G_1}^{-}(x_1)=\sum_{x_1,y_1\in V_1}\mu_{F_1}^{-}(x_1y_1)=\sum_{p(x_1)p(y_1)\in V_2}\mu_{F_2}^{-}(p(x_1)p(y_1))=d_{G_2}^{-}(p(x_1))$,

$$d_{G_1}^{+}(x_1)=\sum_{x_1,y_1\in V_1}\mu_{F_1}^{+}(x_1y_1)=\sum_{p(x_1)p(y_1)\in V_2}\mu_{F_2}^{+}(p(x_1)p(y_1))=d_{G_2}^{+}(p(x_1))\forall x_1\in V_1 .$$

i.e. the degrees of the corresponding vertices of G_1 and G_2 are same.

4. μ-complement of a product interval-Valued fuzzy graph

Definition 4.1 The product interval-valued fuzzy graph is a pair $G=(W,F)$ of a graph $G^*=(V,E)$ where $W=[\mu_W^-,\mu_W^+]$ is an interval-valued fuzzy set on V and $F=[\mu_F^-,\mu_F^+]$ is an interval-valued fuzzy relation on E such that $\mu_F^-(xy)\le\mu_W^-(x)\times\mu_W^-(y),\ \mu_F^+(xy)\le\mu_W^+(x)\times\mu_W^+(y)$, for all $xy\in E$.

Every product interval-valued fuzzy graph is also an interval-valued fuzzy graph.

Definition 4.2 Let $G=(W,F)$ be a product interval-valued fuzzy graph. Then μ-complement of G is defined as $G^\mu=(V,W,F^\mu)$, where $F^\mu=\left(\mu_F^{-^\mu},\mu_F^{+^\mu}\right)$ and

$$\mu_F^{-^\mu}(xy)=\begin{cases}\mu_W^-(x)\times\mu_W^-(y)-\mu_F^-(xy) & \text{if } \mu_F^-(xy)>0\\ 0 & \text{if } \mu_F^-(xy)=0\end{cases}$$

$$\mu_F^{+^\mu}(xy)=\begin{cases}\mu_W^+(x)\times\mu_W^+(y)-\mu_F^+(xy) & \text{if } \mu_F^+(xy)>0\\ 0 & \text{if } \mu_F^+(xy)=0.\end{cases}$$

ISSN 1512-1232

Example 4.1 Consider a product interval-valued fuzzy graph $G=(W,F)$ as shown in Figure 3(a)., where

$$W=\left\{\frac{[0.2,0.4]}{J},\frac{[0.2,0.38]}{K},\frac{[0.3,\,0.4]}{L}\right\} \text{ and}$$

$$F=\left\{\frac{[0.1,\,0.15]}{JK},\frac{[0.1,\,0.2]}{KL},\frac{[0.2,\,0.4]}{LJ}\right\}.$$

(a) Product Interval-valued fuzzy graph G **(b) μ - Complement of G.**

Figure 3: μ -complement of a product interval-valued fuzzy graph G

Then μ -Complement of a product interval-valued fuzzy graph G is shown in Figure 3 (b).

Theorem 4.1 Let G be a highly irregular product interval-valued fuzzy graph then its μ - complement need not be highly irregular.

Proof Let G be a highly irregular product interval-valued fuzzy graph. In G, for every vertex, the adjacent vertices with distinct degrees or the non-adjacent vertices with distinct degrees may happen to be adjacent vertices with same degrees in its μ -complement. This contradicts the definition of highly irregular product interval-valued fuzzy graph.

Theorem 4.2 Let G_1 and G_2 be two highly irregular product interval-valued fuzzy graphs. If G_1 and G_2 are isomorphic, then μ -complements of G_1 and G_2 are isomorphic and vice versa.

Proof Suppose that G_1 and G_2 are isomorphic, there exists a bijective map $p:V_1\to V_2$

which satisfies $\mu_{W_1}^{-}(u)=\mu_{W_2}^{-}(p(u)),\ \mu_{W_1}^{+}(u)=\mu_{W_2}^{+}(p(u)),\ \forall u\in V_1$
and $\mu_{F_1}^{-}(uv)=\mu_{F_2}^{-}(p(u)p(v)),\ \mu_{F_1}^{+}(uv)=\mu_{F_2}^{+}(p(u)p(v)),\ \forall uv\in E_1$.

By the definition of μ -complement, we have

$$\mu_{F_1}^{-^{\mu}}(xy)=\mu_{W_1}^{-}(x)\times\mu_{W_1}^{-}(y)-\mu_{F_1}^{-}(xy)=\mu_{W_2}^{-}(p(x))\times\mu_{W_2}^{-}(p(y))-\mu_{F_2}^{-}((p(x))(p(y))),$$

$$\mu_{F_1}^{+^{\mu}}(xy)=\mu_{W_1}^{+}(x)\times\mu_{W_1}^{+}(y)-\mu_{F_1}^{+}(xy)=\mu_{W_2}^{+}(p(x))\times\mu_{W_1}^{+}(p(y))-\mu_{F_2}^{+}((p(x))(p(y)))$$

for all $xy\in E_1$. Hence, $G_1^{\mu}\cong G_2^{\mu}$.

Similarly, we can prove the converse.

5. Strength cut graph of an interval-valued fuzzy graph.

In this part α -strength cut graph of an interval-valued fuzzy graph G is defined with an example.

For an interval-valued fuzzy graph $G=(W,F)$, an edge $mn\in E$ is said to be effective if $(0.5)\min\{\mu_W^{+}(m),\mu_W^{+}(n)\}\le\mu_F^{-}(mn)$ and $(0.5)\min\{\mu_W^{+}(m),\mu_W^{+}(n)\}\le\mu_F^{+}(mn)$ and it is non effective otherwise. The strength of an edge pq in an interval-valued fuzzy graph $G=(W,F)$ is denoted by

τ_{pq} and is defined as $\tau_{pq} = \left[\tau_{pq}^-, \tau_{pq}^+\right]$ where $\tau_{(p,q)}^- = \dfrac{\mu_F^-(pq)}{\min\left\{\mu_W^+(p), \mu_W^+(q)\right\}}$ and $\tau_{pq}^+ = \dfrac{\mu_F^+(pq)}{\min\left\{\mu_W^+(p), \mu_W^+(q)\right\}}$. Again strength of a vertex w is denoted by τ_w and is defined as $\tau_w = \left[\tau_w^-, \tau_w^+\right]$ where τ_w^- is the greatest value along its membership value $\mu_W^-(w)$ and the strengths τ_{wx}^- of edges wx incident to w and τ_w^+ is the greatest value along its membership value $\mu_W^+(w)$ and the strengths τ_{wx}^+ of edges wx incident to w.

Definition 5.1 Suppose $G = (W, F)$ is an interval-valued fuzzy graph. For any $0 \le \alpha \le 1$, α -strength cut graph of G is defined to be the crisp graph $G^\alpha = \left(V^\alpha, E^\alpha\right)$ such that $V^\alpha = \left\{p \in V / \tau_p \ge [\alpha, \alpha]\right\}$ and $E^\alpha = \left\{pq \in E, p, q \in V / \tau_{pq} \ge [\alpha, \alpha]\right\}$.

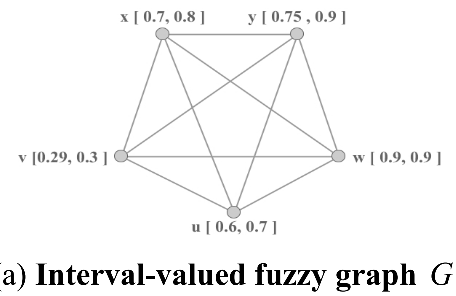
Definition 5.2 For $0 \le \alpha \le 1$, α – cut graph of an interval-valued fuzzy graph $G = (W, F)$, where $W = \left[\mu_W^-, \mu_W^+\right]$, $F = \left[\mu_F^-, \mu_F^+\right]$ is a crisp graph $G_\alpha = (V_\alpha, E_\alpha)$ such that $V_\alpha = \left\{x \in V / \left[\mu_W^-(x), \mu_W^+(x)\right] \ge [\alpha, \alpha]\right\}$ and $E_\alpha = \left\{xy \in E / \left[\mu_F^-(xy), \mu_F^+(xy)\right] \ge [\alpha, \alpha]\right\}$.

Example 5.1 Let $G = (W, F)$ be an interval-valued fuzzy graph of $G^* = (V, E)$, where

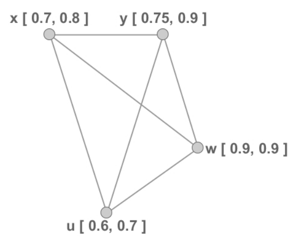
$$W = \left\{\frac{u}{[0.6, 0.7]}, \frac{v}{[0.29, 0.3]}, \frac{w}{[0.9, 0.9]}, \frac{x}{[0.7, 0.8]}, \frac{y}{[0.75, 0.9]}\right\} \text{ and}$$

$$F = \left\{\frac{uv}{[0.2, 0.25]}, \frac{uw}{[0.5, 0.6]}, \frac{ux}{[0.5, 0.6]}, \frac{uy}{[0.55, 0.69]}, \frac{vw}{[0.09, 0.09]}, \frac{vx}{[0.22, 0.26]}, \frac{vy}{[0.2, 0.2]}, \frac{wx}{[0.4, 0.6]}, \frac{wy}{[0.65, 0.89]}, \frac{xy}{[0.69, 0.79]}\right\}.$$

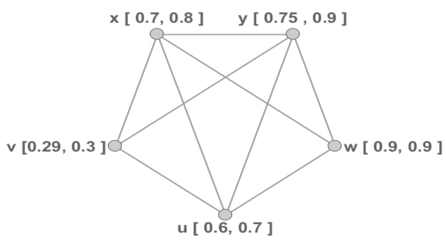
Then, the resultant 0.5 –cut graph and 0.5 –strength cut graph are shown in Figure 4(b) and Figure 4(c) respectively.



(a) **Interval-valued fuzzy graph** G



(b) **0.5–cut graph of** G



(c) **0.5–strength cut graph of** G

Figure 4: Example of a 0.5–strength cut graph

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Theorem 5.1 Let G be an interval-valued fuzzy graph. If $0 \le \alpha \le \beta \le 1$, then $G^{\beta} \subseteq G^{\alpha}$.

Proof Suppose $G = (W, F)$ is an interval-valued fuzzy graph and $0 \le \alpha \le \beta \le 1$.

Then $G^{\alpha} = \left(V^{\alpha}, E^{\alpha}\right)$ such that $V^{\alpha} = \left\{p \in V / \tau_p \ge [\alpha, \alpha]\right\}$ and $E^{\alpha} = \left\{pq \in E, p, q \in V / \tau_{pq} \ge [\alpha, \alpha]\right\}$.

Also, $G^{\beta} = \left(V^{\beta}, E^{\beta}\right)$ such that $V^{\beta} = \left\{p \in V / \tau_p \ge [\beta, \beta]\right\}$ and $E^{\beta} = \left\{pq \in E, p, q \in V / \tau_{pq} \ge [\beta, \beta]\right\}$.

Let $m \in V^{\beta}$. Then $\tau_m \ge \beta \ge \alpha$. Therefore, $m \in V^{\alpha}$. In the same way, for any element $mn \in E^{\beta}$ gives that $mn \in E^{\alpha}$. Therefore, $G^{\beta} \subseteq G^{\alpha}$.

Theorem 5.2 Let G be an interval-valued fuzzy graph. If $0 \le \alpha \le 1$, then $G_{\alpha} \subseteq G^{\alpha}$.

Proof Suppose $G = (W, F)$ is an interval-valued fuzzy graph. Then

$G_{\alpha} = (V_{\alpha}, E_{\alpha})$ such that $V_{\alpha} = \left\{p \in V / \left[\mu_W^-(p), \mu_W^+(p)\right] \ge [\alpha, \alpha]\right\}$

and $E_{\alpha} = \left\{pq \in E, p, q \in V / \left[\mu_F^-(pq), \mu_F^+(pq)\right] \ge [\alpha, \alpha]\right\}$.

Again, $G^{\alpha} = \left(V^{\alpha}, E^{\alpha}\right)$ such that $V^{\alpha} = \left\{p \in V / \tau_p \ge [\alpha, \alpha]\right\}$ and $E^{\alpha} = \left\{pq \in E, p, q \in V / \tau_{pq} \ge [\alpha, \alpha]\right\}$.

Let $p, q \in V_{\alpha}$ and $pq \in E_{\alpha}$.

Then $\left[\mu_W^-(p), \mu_W^+(p)\right] \ge [\alpha, \alpha]$ and $\left[\mu_F^-(pq), \mu_F^+(pq)\right] \ge [\alpha, \alpha]$.

These results along with $\alpha \le 1$, gives that

$$\left[\frac{\mu_F^-(pq)}{\min\left\{\mu_W^+(p), \mu_W^+(q)\right\}}, \frac{\mu_F^+(pq)}{\min\left\{\mu_W^+(p), \mu_W^+(q)\right\}}\right] \ge [\alpha, \alpha].$$

Hence $\tau_{pq} \ge [\alpha, \alpha]$ and $pq \in E^{\alpha}$. Thus for every edge of G_{α}, there exists an edge in G^{α}. By the strength of vertices definition, $V_{\alpha} \subseteq V^{\alpha}$. So the result $G_{\alpha} \subseteq G^{\alpha}$ is true.

Conclusions

Fuzzy graph theory concepts are used in computer science, decision making, mining data. An interval-valued fuzzy model is generalisation of the fuzzy model, we have discussed p-morphism, μ -complement, α -cut of an interval-valued fuzzy graph and μ -complement of a product interval valued fuzzy graph. In future we are extending our research work to interval-valued fuzzy planar graphs, interval–valued fuzzy hyper graphs.

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